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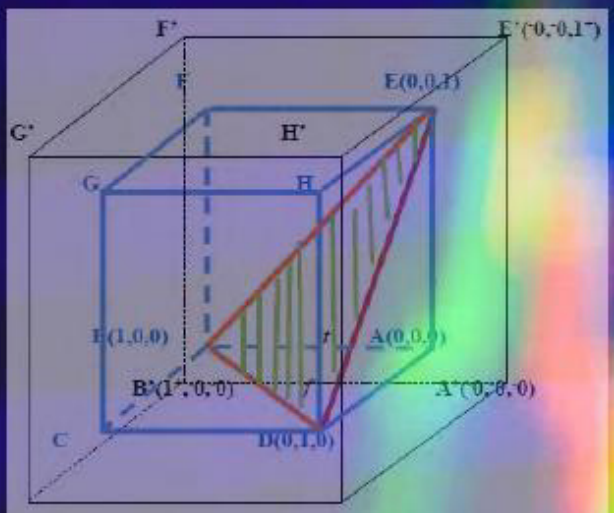

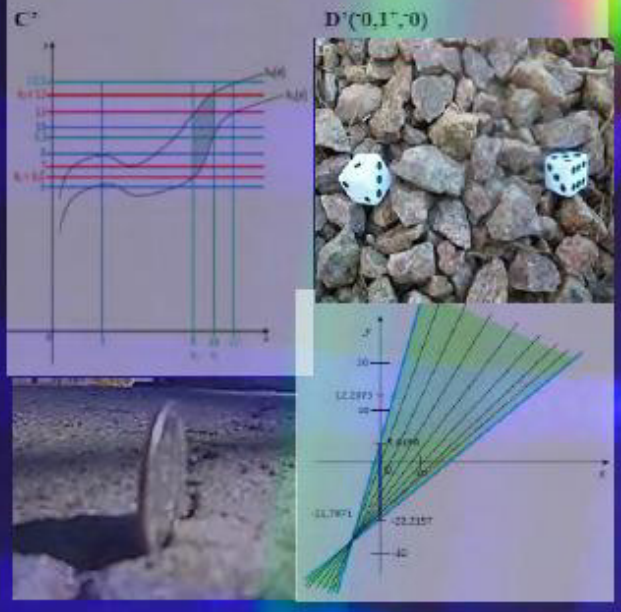

‹A› ‹neutA› ‹antiA›

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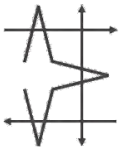

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"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their inter actions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of]-0, 1+[.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.
Neutrosophic Statistics is a generalization of the classical statistics.
What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, igno- rance, imprecision, etc.

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Neutrosophic Sets and Systems {Special Issue: Proceedings of the International Conference Advances in SuperHyperStructures and Applied Neutrosophic Theories, Universidad de Guayaquil, Ecuador, 28-29 November 2024} Vol. 74.

This volume contains the proceedings of the conference held at the University of Guayaquil on November 28 and 29, 2024, featuring contributions from researchers representing Colombia, Cuba, Ecuador, Spain, the United States, Greece, Japan, Mexico, and Peru.

The conference focused on SuperHyperStructures and Applied Neutrosophic Theories, commemorating the 30th anniversary of neutrosophic theories and their extensive applications. The topic of SuperHyperStructures and Neutrosophic SuperHyperStructures explores advanced mathematical frameworks built on powersets of a set H, extending to higher orders $P_n(H)$. SuperHyperStructures are constructed using all non-empty subsets of H, while Neutrosophic SuperHyperStructures incorporate the empty set ϕ, representing indeterminacy.

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These structures model real-world systems where elements are organized hierarchically, from sets to sub-sets and beyond, enabling the analysis of complex and indeterminate relationships.

Held under the motto "Uniendo Saberes para la Innovación Multidisciplinaria" ("Uniting Knowledge for Multidisciplinary Innovation"), the event was graced by the presence of key authorities from the University of Guayaquil. Established on December 1, 1867, this esteemed institution is the largest university in Ecuador. Over its distinguished history, it has produced 11 Presidents, eight Vice Presidents, and numerous senators and representatives, who have been either students or faculty members, solidifying its reputation as a cradle of leadership.

The conference began with a keynote lecture by Prof. Florentin Smarandache, titled "Neutrosophic and Plithogenic Theories: Origin and Applications". This captivating presentation provided a comprehensive overview of the origins, evolution, and diverse applications of neutrosophic and plithogenic theories.

Special recognition was extended to Dr. Florentin Smarandache for his groundbreaking contributions to the advancement of neutrosophic sciences worldwide and his remarkable legacy in academia and research.

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On behalf of the International Association of Neutrosophic Science and the Latin American Association of Neutrosophic Science, Dr. Smarandache presented special awards to Dr. Francisco Morán Peña, Rector of the University of Guayaquil, and Dr. Sofía Lovato Torres for their significant efforts in promoting neutrosophic sciences at the university and across Ecuador. Additionally, Dr. Lorenzo Cevallos Torres was honored for his pioneering contributions to neutrosophic research at both institutional and national levels.

Presentations during the conference spanned a wide array of disciplines, including statistics, business administration, social sciences, and industrial engineering. Moreover, less frequently

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explored areas, such as chemistry, were addressed, showcasing the versatility and expanding scope of neutrosophic applications.

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The contributions outlined in this volume highlight the innovative use of neutrosophic ideas across various domains, such as healthcare, environmental management, education, and business. Notable studies include the application of neutrosophic frameworks to improve medical outcomes for diabetic patients, the analysis of social competence and gender disparities in entrepreneurship, and the optimization of agricultural practices through sustainable methodologies. Research also delved into indigenous rights and environmental conservation using neutrosophic multi-criteria decision-making and explored novel applications in chemistry and bioinsecticides.

Several studies examined the connection between neutrosophic theories and the philosophical principles of Incan and indigenous South American cultures, emphasizing their shared focus on harmony, balance, and holistic problem-solving. These works underscore the adaptability of neutrosophic science in addressing complex challenges across diverse fields.

We hope that the research presented in this volume will further advance the understanding and application of neutrosophic theories, serving as a vital resource for scholars. This special issue celebrates both the 30th anniversary of neutrosophic theory and the 70th birthday of its founder, Dr. Florentin Smarandache.

November 2024

Florentin Smarandache
Maikel Y. Leyva Vázquez
Mohamed Abdel-Basset

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INDEX

Maikel Leyva Vázquez. Preface

1. Lisbeth J. Reales-Chacón, María E. Lucena de Ustáriz, Francisco J. Ustáriz-Fajardo, Andrea C. Peña-fiel Luna, Gabriela J. Bonilla-Ayala, Pablo Djabayan-Djibeyan, José L. Erazo-Parra, Mónica A.Valdiviezo-Maygua. **Study of the Efficacy of Neural Mobilizations to Improve Sensory and Functional Responses of Lower Extremities in Older Adults with Diabetic Peripheral Neuropathy Using Plithogenic n-SuperHyperGraphs** 1

2. María Eugenia Lucena de Ustáriz, Francisco Javier Ustáriz-Fajardo, Adriana Monserrath Monge Moreno, Adriana Isabel Rodríguez Basantes, Lisbeth Josefina Reales Chacón, Rosa Elisa Cruz Tenempaguay, Verónica Paulina Cáceres Manzano, Nathaly Kassandra Moscoso Moreno. **Antibacterial and Antifungal Properties of Hedyosmum cuatrecazanum Occhioni Essential Oil: A Promising Natural Alternative Studied Using Neutrosophic SuperHyperSoft Sets** 14

3. Huberto Tito Hinostroza Robles, María Teresa Cabanillas López, Fidel Denis Huasco Espinoza. **Analysis of the Evolution of Social Competence in Students Through Research Methods Based on Neutrosophic Sets** 24

4. Lorena Gisella Reyes Tomala, Lilia Esther Valencia Cruzaty, Arturo Gustavo Benavides Rodríguez, Jacqueline Del Rocío Bacilio Bejeguen. **Gender Gap in SME Ownership: An Analysis of Female Entrepreneurship in Santa Elena Canton, Ecuador, Using TreeSoft Sets** 37

5. Yenson Vinicio Mogro Cepeda, Marco Antonio Riofrío Guevara, Emerson Javier Jácome Mogro, Rachele Piovanelli Tizano. **Impact of Irrigation Water Technification on Seven Directories of the San Juan-Patoa River Using Plithogenic n-SuperHyperGraphs Based on Environmental Indicators in the Canton of Pujilí, 2021** 47

6. Emerson Jácome Mogro, Jaime Rojas Molina, Gustavo José Sandoval Cañas, Pablo Herrera Soria. **Impact of Irrigation Water Technification on Seven Directories of the San Juan-Patoa River Using Plithogenic n-SuperHyperGraphs Based on Environmental Indicators in the Canton of Pujilí, 2021** 57

7. Evelyn Alexandra Tovar Molina, Angelita Elizabeth Romero Poveda, Bryan Marcelo Barragán Pazmiño, Kelly Nicole Punina Salazar. **Analysis of the Impact of Soft Skills on Organizational Competitiveness in Cotopaxi through Modeling of the Plithogenic IADOV** 66

8. Dennys Raúl Dupotey Hernández, Darvin Manuel Ramírez Guerra. **Neutrosophic statistical analysis of cardiovascular disease prevention from a physical culture in urban communities** 75

9. Oswaldo Edison García Brito, Andrea Sofía Ribadeneira Vacacela, Carmen Hortensia Sánchez Burneo, Mónica Cecilia Jimbo Galarza. **English for specific purposes in the medical sciences to strengthen the professional profile of the higher education medicine student: a knowledge representation using SuperHyperSoft Sets** 84

10. Wilson Eduardo Faz Cevallos, Andrea Nicole Faz Tapia, Valeria Jessnia Faz Tapia, Clara de las Mercedes Razo Ascazubi. **Plithogenic Statistical Analysis of Labor Demand and its Impact on Stability in the Floricultural Sector** 92

11. Myrian del Rocío Hidalgo Achig, Mayra Alexandra Chicaiza Herrera, Bryan Marcelo Barragán Pazmiño. **A Neutrosophic Approach to Analyzing Determining Factors in RIMPE Tax Collection in Ecuador** 100

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12.	Teófilo Meza Taipe, Jacobo Romero Quispe, Eladio Guzmán Villa, Patricio Aparicio Santiago Saturnino, Edwin Collazos Paucar, Adrián Alejandro Flores Konja. **Integration of Fuzzy Set Theory and Neutrosophic Sets for Qualitative and Quantitative Analysis in Inclusive Service to Native Communities of Ucayali**	109
13.	José Luis Agreda Oña, Andrés Sebastián Moreno Ávila, Matius Rodolfo Mendoza Poma. **Study of Sound Pressure Levels through the Creation of Noise Maps in the Urban Area of Latacunga City using Plithogenic n-Superhypergraphs**	119
14.	Takaaki Fujita, Florentin Smarandache. **Uncertain Automata and Uncertain Graph Grammar**	128
15.	Karina Paola Marín Quevedo, Cristian Santiago Jiménez Jacome, Ángel Rubén Murillo Ilbay, Isabel Chávez. **Evaluation of Agronomic Performance of two Varieties of Faba Bean (Vicia faba L.), INIAP 440 - Quitumbe and INIAP 442 - Sultana, using Neutrosophic Superhypersoft sets**	192
16.	Antonios Paraskevas, Florentin Smarandache. **A Neutrosophic Framework for Artificial Immune Systems**	202
17.	Julio Barzola-Monteses, Rosangela Caicedo-Quiroz, Franklin Parrales-Bravo, Cristhian Medina-Suarez, Wendy Yanez-Pazmino, David Zabala-Blanco, Maikel Y. Leyva-Vazquez. **Detection of Cardiovascular Diseases Using Predictive Models Based on Deep Learning Techniques: A Hybrid Neutrosophic AHP-TOPSIS Approach for Model Selection**	210
18.	Giovana Paulina Parra Gallardo, Alicia Maribel Gualan Gualan, María Monserrath Morales Padilla. **Pre- and Post-harvest Application of Ethylene in Bulb Onion (Allium Cepa l.) Hybrid 'Burguesa' Using Plithogenic n-SuperHyperGraphs**	227
19.	Alexander Santos Silva Bernardo, Jeri Gloria Ramón Ruffner, David Sánchez Cruz, Patricia Padilla-Vento. **Evaluation of the Impact of Government Audit on Quality Management, Based on Neutroalgebra Generated by the PROSPECTOR Function**	236
20.	Marcela Ximena Parra Pérez, Arleth M. Márquez Bermeo. **Creative Exploration of Yagé in Intrapersonal Discovery through the Plithogenic IADOV Method**	250
21.	Efrén Gonzalo Montenegro Cueva, Milena Alejandra Napa Alcívar. **Neutrosophic Analysis of Strategies to Improve Productivity in an Restaurant Chain**	259
22.	Lorenzo Cevallos-Torres, Roger Martínez, Rosangela Caicedo-Quiroz, Rosa Hernández-Magallanes, Douglas Iturburu-Salvador, Franklin Parrales-Bravo, Maikel Leyva-Vázquez. **Assessment of Academic Integrity in University Students Using a Hybrid Fuzzy-Neutrosophic Model under Uncertainty**	267
23.	Jorge Fabián Troya Sarzosa, Julio Cesar Alegre Orihuela. **Evaluation of Sustainable Agricultural Practices Using Organic Fertilizers for Soil Recovery in Salache: A Neutrosophic Superhypersoft Set Approach**	275
24.	Erick Félix Quesquén Alarcón, Hermenegildo Chaccara Huachaca, Luz Marina Sito Justiniano, Yessika Madelaine Abarca Arias , Mary Liz Mendoza Hidalgo, Dante Manuel Macazana Fernández. **Sentiment Analysis and NeutroAlgebra to Evaluate Organizational Strategies and Performance Levels of Basic Education Teachers**	285
25.	Diego Fernando Coka Flores, Ignacio Fernando Barcos Arias, María Elena Infante Miranda, Omar Mar Cornelio. **Applying Neutrosophic Natural Language Processing to Analyze Complex Phenomena in Interdisciplinary Contexts**	297
26.	Giovanna Fernanda Vinueza Arroyo, Tanya Tupamara Camargo Martinez, Manuel Augusto Suarez Albiñol, Joffre R. Paladines Rodriguez. **Application of the Plithogenic**	306

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	IADOV Approach to Evaluate Satisfaction with the Implementation of Indigenous Rights	306
27.	Juan Alberto Rojas Cárdenas, Nelson Francisco Freire Sánchez, Alipio Absalón Cadena Pozo, Karina Pérez Teruel. **Applying Neutrosophic Natural Language Processing to Analyze Complex Phenomena in Interdisciplinary Contexts**	321
28.	Oswaldo Lıber Andrade Salazar, Lenı Cecılıa Campaña Muñoz , Second Huacón Farm, Angel Braulio Martinez Vasquez. **Using the neutrosophic N-AHP method to prioritize variables and expert analysis**	332
29.	Pablo Andrés Jaramillo Arias, Carlos Javier Torres Miño, Jorge Israel Rivadeneira Cepeda, Hugo Xavier Cuesta Subia. **Analysis of Genetic Components of Late Blight Resistance (Phytophthora infestans) in Potatoes (Solanum phureja) from Cutuglahua, Pichincha, Using Neutrosophic SuperHyperSoft Sets**	344
30.	Magaly Lucia Benalcázar Luna, Ana Lucía Naranjo, Luis Fernando Muisin Salazar. **Analyzing Internal and External Factors for Ecological Preservation in San Juan de Pastocalle: A Combined SWOT and Neutrosophic AHP Approach**	353
31.	Alex Fabian Solano Moreno, Carlos Javier Lizcano Charpeta, Beatrice Del Carmen Viteri Naranjo, Jean Pierre Ramos-Carpio. **Use of Delphi-AHP neutrosophic methods for the analysis of school indicators**	361
32.	Bryan Joel Morales Sánchez, Cristian Fernando Benavides Salazar, Julio Cesar Benavides Salazar , Erika Estefanía Barreiro Cedeño, Yusleidy Marlie Gordo Gómez. **Neutrosophic cognitive map for the analysis of substances subject to control**	373
33.	Roberto Carlos Jiménez Martinez, María José Calderón Velásquez, Manuel Augusto Suarez Albiñol, Yusleidy Marlie Gordo Gómez. **Insertion and use of multicriteria methods for neutrosophic judicial evaluation**	385
34.	Freddy Anaximandro Álvarez Lema, Manuel Antonio Abarca Zaquinaula, Milton Alberto Sampedro Arrieta, Rodolfo Matius Mendoza Poma. **Optimizing Tourism Offers with COPRAS-SVNS for Stress Reduction and Sustainability Promotion**	393
35.	Rosa María Criollo Delgado, Julio Cesar Vidal Rischmoller, Eluard Alexander Mendoza Zenozain. **Plithogenic Hypothesis on the Influence of Electronic Commerce on Retail Sales Dynamics in Shopping Centers**	402
36.	Vanessa Lisseth Montenegro Altamirano, Jaime Rodrigo Cadena Morillo, Carmen Marina Méndez Cabrita, Ricardo Sánchez Casanova. **Application of Neutrosophic Multi-Criteria Decision-Making Techniques for Assessing Violations of Child Support Rights**	412
37.	Sofía G. Lovato-Torres, María T. Ortiz-Luzuriaga, Gissela M. Saltos-Santana, Verónica Coronel-Pérez. **The Tourism Infrastructure of Rural Areas and Its Relationship with Visitor Satisfaction at the Tourist Destination Incorporating Neutrosophic Analysis of Uncertainty**	420
38.	Dante Manuel Macazana Fernandez , Wilder Fabio Ramos Palacios, Daniel Amílcar Pinto Pagaza, Mary Liz Mendoza Hidalgo, Tula Margarita Espinoza Moreno, Mayra Susan Albán Taípe. **Enhancing the Teaching-Learning Process through Neutrosophic Statistical Analysis of Professional Competencies and Metacognitive Strategies**	430
39.	Lorenzo Cevallos-Torres, Jefferson Núñez-Gaibor, Maikel Leyva-Vasquez, Víctor Gómez-Rodríguez, Franklin Parrales-Bravo, Jesús Hechavarría-Hernández. **NCC: Neutrosophic Control Charts, a didactic way to detect Cardiac Arrhythmias from reading Electrocardiograms**	441
40.	Takaaki Fujita, Florentin Smarandache. **Mixed Graph in Fuzzy, Neutrosophic, and Plithogenic Graphs**	457

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41. Nelly Valeria Vino Ocho, Teresa De Jesús Molino Gutiérrez, Luis Fernando Piñas Piñas, Oscar José Alejo Machado. **Neutrosophic evaluation through the PESTEL method** 480

42. Luis Fernando Blanco Ayala, Ricardo Edilberto Palacios Pérez, Jean Pierre Wong Silva, Bertha Silva Narvaste, Josué Joel Ríos Herrera, Viviana Pastora Panchi Mayo. **Evaluation of the Impact of the Use of Digital Skills on the Substantial Increase in Learning through the Neutrosophic PEST-SWOT approach** 489

43. Rosa María Criollo Delgado, Julio Cesar Vidal Rischmoller, Sergio Marcelo Palomino Criollo. **Applying Neutroalgebra and Neutrosophic Sentiment Analysis to Assess Last Mile Logistics Impact on Customer Experience** 505

44. Benjhall Jordan Camargo Vilcapoma, Valia Ximena Huaman Coras, Rosa María Criollo Delgado. **Neutrosophic Set Theory in E-Commerce Evaluation: A Novel Approach to Sales Strategy Optimization** 517

45. Christian Esteban Gómez Carrión, Ruth Asela Saravia Alviar, Jessica Haydée Saravia Alviar, Brenda Roxana Vergara Pinto, Elca Rocío del Águila Gastelu, Secundino Marrero Ramírez. **Evaluation of anxiety-producing factors in the context of dental care using Delphi-AHP neutrosophic methods** 528

46. Alan Christian López Castillo, Edward Usaqui Barbaran, Miguel Oscar López Orellana, Aníbal Campos Barreto, Llesica Soria Ramirez, Wilmer Ortega Chávez, Víctor Tedy López Panaifo, Alex Davis Astohuaman Huaranga. **Media Pressure and Pretrial Detention Requirements in Ucayali, Peru: A Study Based on Neutrosophic Measures** 538

47. Danny Joseph Anthony Flores Villarruel, Rosa María Criollo Delgado. **Effect of the 7P's of the Marketing Mix on the Sales Strategies of a Private University: a comprehensive analysis using the neutrosophic PEST-SWOT approach** 549

48. Júlio Cesar Vidal Rischmoller, Rosa María Criollo Delgado, Eluar Alexander Mendoza Zenozain. **Evaluation of Public Policy Management in Strengthening Citizen Security through the Plithogenic Hypothesis Approach** 560

49. Marco Antonio Tarifeño Ramírez, Ada Lucía Gallegos Ruiz Conejo, Héctor Félix Cerna Maguiña, Nancy Elizabeth Alberca Pintado, Carlos Edwin Rojas Saldívar, Secundino Marrero Ramírez. **2-Tuple Linguistic Neutrosophic Numbers and Variance Analysis to Examine Citizenship Formation in the General Studies Programs at the Universities of Lima** 570

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Study of the Efficacy of Neural Mobilizations to Improve Sensory and Functional Responses of Lower Extremities in Older Adults with Diabetic Peripheral Neuropathy Using Plithogenic n-SuperHyperGraphs

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Abstract. Diabetic Peripheral Neuropathy (DPN) is an aggravation of diabetes mellitus, which consists of ulcers and loss of sensitivity in the lower limbs. In its most severe stage, the person's limb may be amputated, which causes disability. This is a disease with multiple causes and whose treatment requires several solutions, including drug treatment, physiotherapy, and re-education of the person to change to a healthy lifestyle, if necessary, where the need for adequate nutritional habits appears. It is due to such complexity that we selected the Plithogenic n- SuperHyperGraph as a tool to represent the collected data, which were processed with the help of the multi-way log-linear model for contingency tables. For the first time, these three theories are combined to solve a real-life problem.

Keywords: Diabetes Mellitus, Diabetic Peripheral Neuropathy, Plithogeny, SuperHyperStructures, Plithogenic n- SuperHyperGraph, Multi-way Contingency Tables, Log-Linear Model.

1 Introduction

Diabetes mellitus (DM) is an alteration of blood glucose levels. It has two main subtypes: type 1, considered an autoimmune disease of family inheritance, which mainly affects children and adolescents. Type 2 is the most common because it affects middle-aged and older adults due to their lifestyle and poor diet. Diabetes is considered the most common chronic disease worldwide, affecting 1 in 11 adults between 20 and 79 years of age. 90% of patients suffer from DM 2 [1].

Diabetic Peripheral Neuropathy (DPN) is the most common vascular complication of diabetes mellitus, increasing morbidity and disability due to ulceration and amputation. Peripheral neuropathy is considered a sensory-motor polyneuropathy caused by microvascular and metabolic changes of hyperglycemia. It also produces early segmental demyelination, decreasing nerve conduction velocity.

The prevalence of DPN ranges between 21.3% and 34.5% in type 2 DM worldwide, of which 54% of patients may be asymptomatic. Because DPN affects the peripheral nervous system, distal symmetrical polyneuropathy is the most common, associated with numbness, tingling, pain, or weakness that begins in the feet and ascends proximally [2].

Neuropathy is the leading cause of disability worldwide, affecting the quality of life of diabetic patients because it causes chronic pain, high-risk falls, foot ulcers, and amputations, and is associated with sleep disorders, anxiety, and depression. DPN is considered to be poorly documented in Latin America and the Caribbean due to the lack of studies conducted in each country. However, the prevalence is 46.5% on the continent with an increasing trend over time [3].

For the treatment of DPN, neural mobilization through sliding neurodynamics is considered, i.e. exercises are performed in which the patient is asked to make small movements of a specific joint for 10 seconds, during which they will feel a slight tension in the nerve pathway. These exercises have neurophysiological benefits, manage to minimize the negative effects on peripheral nerve roots, and improve autonomic nervous function.

The present research on the application of neural mobilization exercises [4] in the elderly with diabetic peripheral neuropathy is carried out because there is no major scientific evidence in Ecuador about the physiotherapeutic intervention in DPN for which a strength evaluation is carried out, Michigan Neuropathy test Screening Instrument **(**MNSI) that evaluates whether or not the patient has neuropathy, superficial and deep sensitivity evaluation. These evaluations help determine an appropriate intervention for older adults through neurodynamic exercises that help us improve or reduce the signs and symptoms of DPN.

The problem we address in this article could be analyzed using classical statistics. However, one of our objectives is to study in the greatest possible depth the aspects that influence the symptoms that patients suffer from. Among the characteristics that are pointed out is the presence of several factors of a different nature. In addition to the medical problem that comes with suffering from a disease such as diabetes, this takes on other dimensions, such as the family and state economic expense that the presence of the disease means in each country. There is also a social impact due to the presence of people who suffer from a health problem and who are not only unable to work but may also need a caregiver. There is psychological damage to the person who may feel useless and to the caregiver who must dedicate his or her life to supporting this person.

The educational aspect is very important among the solutions to this problem. The patient must change his lifestyle and diet, and this requires relearning. He or she must eliminate toxic habits, such as cigarettes and alcohol, that can worsen his or her health situation.

That is why the mathematical tool that could be used must contain all these aspects. We have taken Plithogeny as a model [5, 6, 7] to study the feasibility of applying neural mobilization to improve the functional and sensory response ofolder adults with diabetic peripheral neuropathy.

Plithogeny is the theory introduced by Smarandache to model the dynamic interaction between concepts. “Plithogeny is the genesis or origin, creation, formation, development, and evolution of new entities from dynamics and organic mergers of multiple contradictory and/or neutral and/or non-contradictory old entities. Plithogeny advocates connections and unification of theories and ideas in any field. We can take "knowledge" as "entities" in various fields, such as the soft sciences, the hard sciences, and the theories of the arts and letters, among others." [8-12].

“Therefore, Plithogeny is the dynamics of many types of opposites, and/or their neutrals, and/or non-opposites and their organic fusion. Plithogeny is a generalization of dialectics (dynamics of a kind of opposites: <*A*> and <*antiA*>), the neutrosophy (dynamics of a kind of opposites and their neutral: <*A*> and <*antiA*>, and <*neutA*>), because plithogeny studies the dynamics of many types of opposites and their neutral and not opposites (<*A*> and <*antiA*>, and <*neutA*>, <*B*> and <*antiB*> and <*neutB*>, etc.), and many that are not opposites (<*C*>, <*D*>, etc.) altogether. A particular application and concept derived from Plithogeny is the plithogenic set, as it is an extension of the classical set, fuzzy set, intuitionist fuzzy set, and neutrosophic set, and has many scientific applications.” [13, 14, 15].

We use the n-Super HyperGraphs combined with Plithogeny theory to model this problem. HyperStructures were introduced by Marty [16], which are structures where a HyperOperation is

defined from an Operation on the power set of the non-empty set H without taking into account the empty subset, where the HyperAxioms are also added. The SuperHyperStructure was introduced by Smarandache, where the domain is replaced by the power set of H not only the image. Here SuperHyperAxioms is defined in it as well [17-23].

A SuperHyperGraph or n-SuperHyperGraph is the structure defined by Smarandache, where there is a set of vertices (finite or infinite) such that a subset is made up of the Single Vertices (the classic ones), another by the Indeterminate Vertices (unclear, vague, partially know) and a third set is made up of the Null Vertices (totally unknown, empty). In them, the edges are defined on the n-power set of vertices, which are defined recursively as the power of the (n-1)-power set and so on [24-26].

The data collected and represented in the form of plithogenic n-SuperHyperGraphs are statistically processed using the log-linear model for multi-way contingency tables [27, 28]. These data were obtained from a group of older adults in two locations; the facilities in the Ambato canton, in the province of Tungurahua, Ecuador. It is the first time, to the authors' knowledge, that the two theories of plithogenic n-SuperHyperGraphs are combined with classical statistics of contingency tables [29, 30].

The paper consists of a Materials and Methods section, where the main concepts of Plithogeny n-SuperHyperGraphs and log-linear models are explained for multi-way contingency tables. In the following section, we present the results obtained. The last section is dedicated to giving conclusions.

2 Materials and Methods

This section contains two sub-sections, the first one is dedicated to explaining the basic notions of Plithogenic n-SuperHyperGraphs defined in [15]. Then, subsection 2.2 contains the main concepts of multi-way contingency tables and the log-linear method.

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in [15].

First, an n-SuperHyperGraph is defined as follows [24-26]:

Given $V = \{V_1, V_2, \cdots, V_{\mathrm{m}}\}$, where $1 \leq \mathrm{m} \leq \infty$ is a set of vertices, containing *Single Vertices* which are classical, *Indeterminate Vertices* which are unclear, vague, partially known, and the *Null Vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq \mathrm{n} \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq \mathrm{n} \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices as in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1)- SuperVertex,* and the others can be *r- SuperVertex* for $r \leq \mathrm{n}$.

E_n contains the following types of edges :

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

Within the framework of the theory of Plithogenic n-SuperHyperGraphs, we have the following concepts [4]:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervene,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights.

2.2 Multi-Way Contingency Tables

A multi-way contingency table is a contingency table defined for two or more cross-ratio classification variables. Two-dimensional tables are generally called contingency tables, while the term multi- is applied when the number of variables is at least three [26, 28].

A *Generic Multi-Way Table* is defined using $I = I_1 \times I_2 \cdots \times I_q$ as the set of indices for each variable to be studied $X_1, X_2, \cdots, X_q$, such that I_j is the set of indices corresponding to the possible classifications of the jth variable. Thus, $n_{i_1 i_2 \cdots i_q}$ is the frequency of occurrence of the classifications $i_1, i_2, \cdots, i_q$ for each of the corresponding variables.

The *Partial/Conditional Tables* involve fixing the category of one of the variables. Fixed variables are denoted in parentheses. For example, the XZ and YZ-partial tables are denoted by $n_{i(j)k}$ and $n_{(i)jk}$, respectively. Also, the *Partial/Conditional Probabilities* are calculated by $\pi_{ij(k)} = \pi_{ij/k} = Prob(X = i, Y = j/Z = k)$. *Partial/Conditional Proportions* are defined by $p_{ij(k)} = p_{ij/k} = \frac{\pi_{ijk}}{\pi_{++k}}$ for $k = 1, 2, \ldots, K$. Where π_{++k} is the frequency for i and j setting k, for more information see [26, 28].

Below, we briefly explain what log-linear models consist of. To simplify the exposition we take the case of the three-way contingency table. If X, Y, and Z are the variables, then the following possible models are obtained [16, 17]:

- Model (X, Y, Z): All variables are considered independent, the model is as follows:

$$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z \quad (1)$$

- Model (X, YZ): Only the YZ association is considered, while X is independent of the other two variables.
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{jk}^{YZ} \quad (2)$$
- Model (XY, YZ): X and Z are independent for each value of Y:
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ} \quad (3)$$
- Model (XY, YZ, XZ): There is a pair-wise association between all variables, but there is no joint association between the three.
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} \quad (4)$$
- Model (XYZ): If the above model does not fit the data well, then the association between the three variables should be considered:
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ} \quad (5)$$

To contrast two different models, the statistic called *likelihood ratio is used*, which is calculated as:

$$G^2 = 2\sum f ln(f/F) \quad (6)$$

Where f is the observed frequency and F is the expected frequency according to the model. This statistic is distributed according to a chi-square under the hypothesis that the model is correct, with degrees of freedom depending on the parameters used to fit the model.

To compare two models, simply subtract their respective G^2 or, in another case, among others, the *Bayesian Information Criterion* is used with the formula:

$$BIC = G^2 - \mathrm{df}\log N \quad (7)$$

Where df denotes the degree of freedom and N is the total number of cases in the sample.

3 The Study

In the research, 3 instruments were used to collect data, which were designed and validated by experts in the subject, based on their reliability during their application were the following:

Michigan Neuropathy Screening Instrument (MNSI)

Michigan Neuropathy Screening Instrument, created by Feldman and his colleagues in 1994, was developed to adapt and simplify the evaluation criteria for diabetic peripheral neuropathy. This is considered a rapid and objective detection tool with a qualitative evaluation system, which consists of 2 parts: a questionnaire and a clinical examination. The questionnaire consists of 15 questions and each of them will have a score of one point if positive or zero if negative. It consists of 2 false questions that will not be graded, the maximum score being 13.

The second part of MNSI performs a physical examination of the foot. It carefully evaluates the deformities of the skin, whether it presents dry skin, infection, or cracks. The score is obtained by adding the data of both limbs; if the score is greater than 2 it is considered as DPN.

Manual Dynamometry Test

The manual dynamometer test is performed to assess the muscle strength of different areas of the body. A dynamometer is used and is held by the evaluator over the patient's structure to be assessed. The assessment is performed with reduced gravity. The patient will exert a sustained force on the dynamometer for a period of 5 seconds while the evaluator holds the dynamometer stable. This measurement will be performed three times on each left and right limb. The procedure has been described for different muscle groups, which is why compliance with these procedures is recommended.

Superficial and deep sensitivity test

Sensitivity assessment has been standardized thanks to Dr. Jean Ayres. These are responsible for

identifying sensory modulation. During the examination, the physiotherapist will explain the procedure to the patient. The patient will be comfortable and blindfolded to avoid bias during the assessment. In this way, the recognition of the stimulus without a visual field provides sensory integrity. The test will be performed on both limbs for comparison. The superficial sensitivity test explores each dermatome (touch, temperature) and is carried out on the dermatomes of the upper and lower limbs.

To evaluate the vibratory sensitivity, a tuning fork is used on a bony surface. The patient will indicate what he or she feels, where he or she feels it, and when the stimulus stops. His or her response will determine:

- Hypopalesthesia: decreased vibratory sensitivity,
- Hyperpalesthesia: increased vibratory sensitivity,
- Apalesthesia: absence of vibratory sensitivity.

The investigation was carried out in two locations: the facilities of the Atahualpa Parish Council in the Ambato canton, and also the facilities of the old train station in the Cevallos canton, in the province of Tungurahua.

Population and Sample

The research population consisted of 34 adults, of which 17 participants completed the 8 weeks of intervention, with a pre- and post-execution evaluation. The 17 participants were eliminated because they did not comply with the 8-week treatment or because they did not attend the final evaluation, for which reason a confidence level of 95% was obtained, with a margin of error of 5% in patients with Diabetic Peripheral Neuropathy.

Inclusion criteria

- Male or female patients aged 65 years or older.
- Patients diagnosed with type 2 diabetes mellitus.
- Patients who have altered sensitivity in the lower limb.
- Patients who have independent mobility.
- Patients who have signs or symptoms of diabetic peripheral neuropathy.

Exclusion criteria

- Patients with high-risk diseases such as heart disease.
- Patients with vascular problems such as varicose veins.
- Adults who do not have mobility in their lower limbs due to amputation or injury.
- People who have had recent surgeries

The input object (V) in this study is *Patients*, where it is understood that they are the patients selected to study the effectiveness of neural mobilizations in improving DPN. The Enveloping Vertex (Super Enveloping Vertex or Dominant Enveloping Vertex or Dominant Super Enveloping Vertex) in this problem is related to the following attributes and sub-attributes:

V_1= Socio-Demographic Data, V_2= Lower Extremity Sensitivity, V_3= Michigan Questionnaire, V_4= Michigan Physical Examination.

Attribute sets = {Socio-Demographic Data, Lower Extremity Sensitivity, Michigan Questionnaire, Michigan Physical Examination}.

Socio-Demographic Data = {Age (V_{11}), Weight (V_{12}), Gender (V_{13})}.

Lower Limb Sensitivity = {Tactile Sensitivity (V_{21}), Thermal Sensitivity (V_{22}), Vibration (V_{23})}.

Age = {Older Adults (V_{111}), Adults (V_{112})}.

Weight = {Normal (V_{121}), Overweight (V_{122}), Obese (V_{123})}.

Gender = {Male (V_{131}), Female (V_{132})}.

Touch Sensitivity = {Initial (V_{211}), Final (V_{212})}.

Thermal Sensitivity = {Initial (V_{221}), Final (V_{222})}.

Vibration = {Initial (V_{231}), Final (V_{232})}.

Initial = {Absent, Altered, Normal}.

Final = {Absent, Altered, Normal}.

Michigan Questionnaire = {Initial (V_{31}), Final (V_{32})}.

Michigan Physical Examination = {Initial (V_{41}), Final (V_{42})}.

In the case of Michigan tests, it is determined:

Initial = {Altered, Normal}.

Final = {Altered, Normal}.

Table 1 summarizes these variables:

Table 1: Vertex, Vertex Attributes, Vertex Sub Attributes, and Vertex Sub Sub Attributes in the study carried out. The variable that represents them appears in parentheses.

Vertex	**Vertex Attributes**	**Vertex Sub Attributes**	**Vertex Sub Sub Attributes**
Socio-demographic data (V_1)	Age (V_{11})	Older adults (V_{111})	
		Adults (V_{112})	
	Weight (V_{12})	Normal (V_{121})	
		Overweight (V_{122})	
		Obesity (V_{123})	
	Gender (V_{13})	Male (V_{131})	
		Female (V_{132})	
Lower limb sensitivity (V_2)	Tactile Sensitivity (V_{21})	Initial (V_{211})	Absent (V_{2111})
			Altered (V_{2112})
			Normal (V_{2113})
		Final (V_{212})	Absent (V_{2121})
			Altered (V_{2122})
			Normal (V_{2123})
	Thermal Sensitivity (V_{22})	Initial (V_{221})	Absent (V_{2211})

Vertex	Vertex Attributes	Vertex Sub Attributes	Vertex Sub Sub Attributes
			Altered (V_{2212})
			Normal (V_{2213})
		Final (V_{222})	Absent (V_{2221})
			Altered (V_{2222})
			Normal (V_{2223})
	Vibration (V_{31})	Initial (V_{231})	Absent (V_{2311})
			Altered (V_{2312})
			Normal (V_{2313})
		Final (V_{232})	Absent (V_{2321})
			Altered (V_{2322})
			Normal (V_{2323})

Note that the sub-attributes V_{211}, V_{221}, and V_{231} are input enveloping vertex, while V_{212}, V_{222}, *and* V_{232} are output enveloping vertex. The intervene enveloping vertex are the others.

Table 2 contains the absolute frequency of each of the variables:

Table 2: Absolute frequency obtained for each of the variables. They appear in parentheses.

Vertex	Vertex Attributes	Vertex Sub Attributes	Vertex Sub Sub Attributes
Socio-demographic data (17)	Age (17)	Older Adults (13)	
		Adults (4)	
	Weight (17)	Normal (2)	
		Overweight (8)	
		Obesity (7)	
	Gender (17)	Male (4)	
		Female (13)	
Lower Limb Sensitivity (17)	Tactile Sensitivity (17)	Initial (17)	Absent (0)
			Altered (2)

Vertex	Vertex Attributes	Vertex Sub Attributes	Vertex Sub Sub Attributes
			Normal (15)
		Final (17)	Absent (0)
			Altered (0)
			Normal (17)
	Thermal Sensitivity (17)	Initial (17)	Absent (0)
			Altered (4)
			Normal (13)
		Final (17)	Absent (0)
			Altered (0)
			Normal (17)
	Vibration (17)	Initial (17)	Absent (2)
			Altered (3)
			Normal (12)
		Final (17)	Absent (0)
			Altered (2)
			Normal (15)

Note that each of the absolute frequencies can be converted to relative frequencies by dividing by 17.

Table 3 contains the vertices corresponding to the Michigan examination, which is written separately for space reasons and to make it more understandable; however, it should be understood as a continuation of Table 1:

Table 3: Vertex, Vertex Attributes and Vertex Sub Attributes in the study carried out for the Michigan tests. The variable that represents them appears in parentheses.

Vertex	Vertex Attributes	Vertex Sub Attributes
Michigan Questionnaire (V_3)	Initial (V_{31})	Altered (V_{311})
		Normal (V_{312})

	Final (V_{32})	Altered (V_{321})
		Normal (V_{322})
Michigan Physical Examination (V_4)	Initial (V_{41})	Altered (V_{411})
		Normal (V_{412})
	Final (V_{42})	Altered (V_{421})
		Normal (V_{422})

Note that the sub-attributes V_{31}, V_{41} are input enveloping vertex, while V_{41}, V_{42} are output enveloping vertex.

Table 4 contains the absolute frequencies of the variables in Table 3.

Table 4: Absolute frequency obtained for each of the variables corresponding to the Michigan examination. The variable that represents them appears in parentheses.

Vertex	**Vertex Attributes**	**Vertex Sub Attributes**
Michigan Questionnaire (17)	Initial (17)	Altered (3)
		Normal (14)
	Final (17)	Altered (0)
		Normal (17)
Michigan Physical Examination (17)	Initial (17)	Altered (3)
		Normal (14)
	Final (17)	Altered (2)
		Normal (15)

Let us now use Log-Linear models to statistically process the data. To simplify the method we will use 3-way contingency tables. We will calculate the coefficient G^2in each case.

Table 5 contains a summary of these results:

Table 5: G^2 result of the processed models.

Model	G^2
Age*Initial Tactile Sensitivity*Final Tactile Sensitivity	3.376055160243798e-7
Age*Initial Thermal Sensitivity*Final Thermal Sensitivity	3.1509848869970483e-7
Age*Initial Vibration*Final Vibration	2.700844228575832e-7
Age*Initial Michigan Questionnaire*Final Michigan Questionnaire	3.150984931596926e-7
Age*Initial Michigan Physical Examination*Final Michigan Physical Examination	2.700844197327017e-7
Weight*Initial Tactile Sensitivity*Final Tactile Sensitivity	4.95154775293477e-7

Model	G^2
Weight*Initial Thermal Sensitivity*Final Thermal Sensitivity	4.951547730729273e-7
Weight*Initial Vibration*Final Vibration	4.5014069907952255e-7
Weight*Initial Michigan Questionnaire*Final Michigan Questionnaire	4.951547595348044e-7
Weight*Initial Michigan Physical Examination *Final Michigan Physical Examination	4.7264773118348784e-7
Gender*Initial Tactile Sensitivity*Final Tactile Sensitivity	3.3760552898716755e-7
Gender*Initial Thermal Sensitivity*Final Thermal Sensitivity	3.1509849150197596e-7
Gender *Initial Vibration*Final Vibration	2.925914528725019e-7
Gender *Initial Michigan Questionnaire*Final Michigan Questionnaire	3.3760552126440676e-7
Gender *Initial Michigan Physical Examination* Final Michigan Physical Examination	2.925914568035818e-7

Note that, for example, in the first model three vertices were combined to form a SuperVertex, since there is uncertainty and indeterminacy due to the nature of the problem being addressed and that is why statistics are used. See Figure 1, which serves to graphically illustrate this example; for each of the models, there is a similar graphic representation.

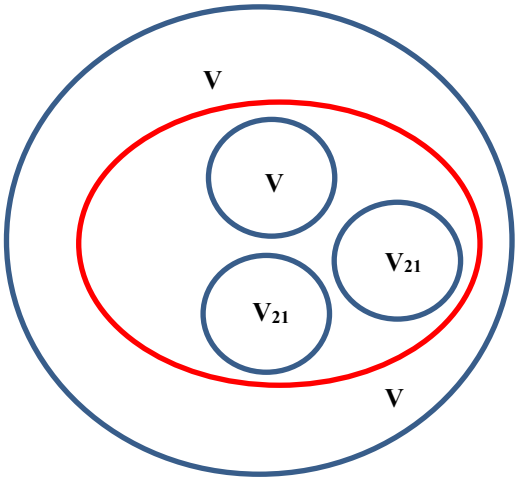

Figure 1: HyperGraph graphically representing the first model in Table 5. The HyperGraph Edge appears in red.

As for dominance, two vertices were always dominant: the initial one (representing the input) and the final one (representing the output). See Figure 2 for representing the plithogenic connector (C_1).

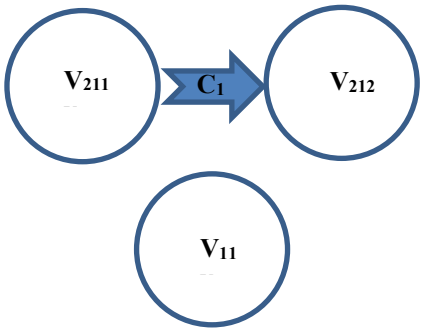

Figure 2: Plithogenic connector (C_1) of the variables represented in the first model in Table 5.

Regarding the statistical interpretation, all values were of a $G^2 < 0.01$ in log-linear nature, so all the obtained log-linear models fit the data well. When the models were analyzed in more detail for all cases, it was concluded that there was significant improvement among the variables analyzed. In other words, the treatment is effective for all patients, regardless of gender, weight, and age.

5. Conclusion

This paper had as its main objective the study of the feasibility of applying the treatment of neural mobilizations in adult patients of the city of Ambato in Ecuador. These patients suffer from Diabetic Peripheral Neuropathy, which is a sequel of diabetes. The results were statistically processed using the log-linear model for multi-way contingency tables. For the representation of the data, plithogenic n-SuperHyperGraphs were used, which allowed to representation of the different combinations of possible variables and the uncertainty and indeterminacy arising from the statistical component of the problem. We use these new tools that emerged within Neutrosophy because they allow us to model complex interactions between variables of different natures, as is the case in which we find ourselves. For the first time, to the knowledge of the authors, the plithogenic tools are combined with SuperHyperGraphs and some traditional statistical methods.

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Antibacterial and Antifungal Properties of Hedyosmum cuatrecazanum Occhioni Essential Oil: A Promising Natural Alternative Studied Using Neutrosophic SuperHyperSoft Sets

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Abstract. Antimicrobial resistance is a global public health issue that, exacerbated by inappropriate antibiotic use, leads to reduced bactericidal effects, multidrug resistance, increased medical costs, prolonged hospital stays, and higher mortality rates. This study evaluates the antibacterial and antifungal activity of the essential oil of *Hedyosmum cuatrecazanum* Occhioni against various clinically relevant bacterial and fungal strains. The essential oil was extracted by hydrodistillation, yielding a 0.26% extract. Its components were characterized by gas chromatography coupled with mass spectrometry (GC-MS). The results demonstrate that the essential oil exhibits significant antibacterial activity, comparable to commercial antibiotics. *Hedyosmum cuatrecazanum* oil showed the highest antibacterial activity against *S. aureus* and antifungal activity, where *C. albicans* and *C. tropicalis* were the most susceptible. For the study, the effective relative values of the chemical components of the essential oil were modeled using Fuzzy_Extension SuperHyperSoft Sets theory, as well as their concentration. Specifically, we model indeterminacy with the use of single-valued triangular neutrosophic numbers. This is because the percentages of each chemical that makes up the essential oil vary in a certain range from one plant to another. This mathematical tool allows us to determine the percentages and concentrations in which the essential oils are most effective in treating bacteria and fungi.

Keywords: Antimicrobial resistance, *Hedyosmum cuatrecazanum Occhioni*, essential oil, antibacterial, antifungal, Neutrosophic SuperHyperSoft Sets, Fuzzy_Extension SuperHyperSoft Sets, single-valued triangular neutrosophic number.

1 Introduction

Drug discovery has its roots in nature since antiquity. Many medicines still in use today have origins in ancestral knowledge related to the properties of medicinal plants. Indeed, several plant extracts and essential oils have been repurposed, significantly expanding their therapeutic scope.

The development of new drugs continuously adapts to the needs of the current healthcare system. This approach has led to the discovery of pharmacological targets with multiple endpoints, especially in the treatment of infectious diseases, where antimicrobial resistance (AMR) is highlighted as a global health issue. AMR occurs when bacteria, viruses, fungi, and parasites no longer respond to current pharmacological treatments, resulting in difficult-to-treat infections and increasing the risk of transmission, which can be lethal.

In the Andean region of South America, the *Chloranthaceae* family is a native, relatively under-

studied family. *Hedyosmum* is the largest genus in the *Chloranthaceae* family, consisting of tree or shrub plants with simple, opposite, serrated, petiolate leaves and pinnate veins. These species are widely distributed in low- and high-mountain rainforests throughout South America, including Ecuador, Peru, Brazil, Bolivia, Mexico, and Central America. Within this family, *Hedyosmum cuatrecazanum, Occhioni* stands out as a tree native to the humid tropical zone, traditionally used as food.

The *Hedyosmum* genus is characterized by its main components, including sesquiterpenes, sesterterpenes, hydroxycinnamic acid derivatives, flavonoids, neolignans, among others. Additionally, this genus is a significant source of essential oils that can be extracted from various plant parts, mainly aerial parts, i.e., leaves, flowers, and fruits. Species within the *Hedyosmum* genus also have diverse traditional uses, such as sources of firewood and construction materials, as well as medicinal preparations, primarily in the form of leaf infusions. The main pharmacological effects associated with members of this genus include neuroprotective, anxiolytic, antidepressant, sedative, analgesic, antinociceptive, anticancer, antibacterial, and antiplasmodial properties.

Previous studies on essential oils from this genus, such as *Hedyosmum luteynii*, identified the most abundant constituents as α-felandrene (32.72%), (Z)-β-pinene (13.20%), (Z)-β-ocimene (10.99%), sylvane (6.51%), bicyclogermacrene (5.05%), 1,8-cineole (4.95%), α-ocimene (3.88%), and germacrene D (3.20%). These constituents differ significantly from those reported for other *Hedyosmum* species in Ecuador, Peru, Bolivia, Colombia, Venezuela, Brazil, and Costa Rica. Additionally, the analysis of the constituents of *Hedyosmum luteynii* essential oil, compared to other species in the same genus, allowed the identification of exclusive compounds such as sylvane (6.51%), cis-muurola 3-5-diene (0.45%), and amorpha-4,9-diene <7.14 -anhydro-> (0.65%).

This article presents the extraction of the volatile oil of *Hedyosmum cuatrecazanum* Occhioni and its characteristics evaluated as its potential antimicrobial activity, accompanied by its respective minimum inhibitory concentration. The oil was tested against clinically relevant strains, including Gram-negative bacteria such as *Klebsiella pneumoniae*, *Escherichia coli*, and *Proteus mirabilis*, as well as Gram-positive bacteria like *Staphylococcus aureus* and *Enterococcus faecalis*, and most problematic fungal strains: *Candida albicans*, *C. tropicalis*, *C. krusei*, and *C. parapsilosis*, to contribute to infection treatment and offer a potential solution in the battle against microbial resistance.

To characterize the chemical composition of the plants studied we use the Fuzzy_Extension SuperHyperSoft Set theory, specifically within the neutrosophic framework. Classical Soft Set theory was introduced by D. Molodtsov, where he defined a function on a set of parameters belonging to an attribute such that its image is the power set of the universe set [1]. More recently F. Smarandache defined HyperSoft Sets where the domain of the function is generalized to the Cartesian product of a set of attribute values, where the number of attributes can be greater than one [2, 3]. Smarandache himself introduced SuperHyperSoft Sets consisting of the Cartesian product of the power sets instead of the sets themselves [4, 5]. Finally, he provided images with evaluations in the form of fuzzy truth values or some of their extensions (intuitionistic fuzzy, neutrosophic, Pythagorean, Fermatean, Plithogenic, etc.) he called this Fuzzy_Extension SuperHyperSoft Set [6,7].

The chemical compounds we study, especially essential oils, are made up of chemical substances in undetermined percentages, which is why they must be represented in an imprecise or indeterminate way. In this article, we characterize the components and proportions that are the basis of the antibacterial and antifungal properties of *Hedyosmum. cuatrecazanum* Occhioni. To this end, we use the Neutrosophic SuperHyperSoft Sets as a variant of Fuzzy_Extension SuperHyperSoft Sets, to determine the components within the essential oil in which the bactericidal and fungicidal properties are most effective. We use single-valued triangular neutrosophic numbers to evaluate the results [8].

To meet the proposed objective, we divide the article into a Preliminaries section where we present the basic notions of Fuzzy_Extension SuperHyperSoft Sets and single-valued triangular neutrosophic numbers. In the following section, we present the details of the study carried out. The article ends with the Conclusions section.

2 Preliminaries

This section serves the purpose of remembering the basic notions of Fuzzy_Extension SuperHyperSoft Sets and neutrosophic theory.

Definition 1 ([1, 9, 10]**).** Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter ε ∈ E, then F(ε) ∈ $\mathcal{P}$(U), where $\mathcal{P}$(U) denotes the power set of U and F(ε) is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set.

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A where we have the following:

F(α) = {x ∈ U | μ_A(x) ≥ α}, α ∈ [0, 1]. Thus, if we know the family F, then we can reconstruct the function μ_A by using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a (F, [0, 1]) soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

(F, A) ∗ (G, B) = (J, A × B), where J(α, β) = F(α) ∗ G(β); α ∈ A, β ∈ B, and A × B is the Cartesian product of the sets A and B.

Definition 2 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}$(U) the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *HyperSoft Set* over U.

Definition 3 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}$(U) the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([4, 5, 13-15])**.** Let U be a universe set, $\mathcal{P}$(U) the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}\left(U\left(x(d^0)\right)\right)$ is called a *Fuzzy_Extension SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc.

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set N* is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^-0$ and 1^+ belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$ and $F_A: U \to [0,1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 7 ([8, 16, 17])**.** The *single-valued triangular neutrosophic number*, $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, and falsity membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 8 ([8, 16, 17])**.** Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Inversion: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$
6. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t- conorm.

3 Results

This section is devoted to presenting the results obtained and mathematically modeling the problem. Some details of the study are given below.

3.1 Details of the experiment

Sample Collection: The extraction of essential oil was carried out using the aerial parts of *Hedyosmum cuatrecazanum* Occhioni. The collection of plant material was authorized by the Ministry of Environment, Water, and Ecological Transition of Ecuador. The aerial parts of *Hedyosmum cuatrecazanum* Occhioni were manually collected between April and August 2023 in the natural forest of Jacarón, located in the Juan de Velasco parish, Colta canton, Chimborazo province, Ecuador. The soils are formed from volcanic ashes, with high organic content, pH between 5.5 and 6.5; classified as dystrandepts within the Inceptisols order, corresponding to young soils with limited development that exhibit high organic matter accumulation and base saturation below 50%.

A specimen sample was deposited at the Herbarium of the "Escuela Superior Politécnica del Chimborazo", Ecuador following botanical identification.

Essential Oil Extraction: The aerial parts of *Hedyosmum cuatrecazanum* (stem, leaves, fresh flowers) were used. After weighing and repeated cycles of washing with distilled water, the samples were ground to increase the contact surface and improve extraction efficiency. Essential oil extraction was performed using the hydrodistillation method. The obtained oil was stored refrigerated at 4 °C in sterile aliquots until analysis.

Bacterial Strains: The bacteria used in this study were obtained from the American Type Culture Collection. The assay included Gram-negative bacteria *Escherichia coli*, *Proteus mirabilis*, and *Klebsiella pneumoniae*, as well as Gram-positive bacteria: *Staphylococcus aureus* and *Enterococcus faecalis*.

Fungal Strains: The fungal strains of the *Candida* genus—*C. albicans*, *C. glabrata*, *C. krusei*, *C. tropicalis*, and *C. parapsilosis*—were provided by the microbiology laboratory collection at the University of Cuenca, Ecuador.

Measurement of Antibacterial Activity: Antibacterial activity was assessed using the disk diffusion method on agar. Strains that showed inhibition zones against the oil were subjected to minimum inhibitory concentration (MIC) determination.

Measurement of Antifungal Activity: A colony of each species (*C. albicans*, *C. tropicalis*, *C. krusei*, and *C. parapsilosis*) was streaked onto Sabouraud agar plates, which were then incubated at 37°C for 24 hours. Subsequently, a sample of the pre-inoculum from each *Candida* species was collected and placed in a tube with 0.89% sterile saline solution.

3.2 Modeling with Neutrosophic SuperHyperSoft Sets

One of the characteristics of the plants studied is that each of them individually has a chemical composition and properties that do not comply with the exact percentages from one plant to another, even when it is the same species. This does not mean that the effectiveness of the antibacterial and fungicidal properties is lost between one plant and another. However, there are values at which effectiveness is lost. On the other hand, it is necessary to determine the concentration of essential oil that is optimally needed for use as a medicine, which cannot be determined exactly with a numerical value. For this end, we study the values of the attributes that essential oils must satisfy to be effective, and for them, these values include the indeterminacy given in the form of a single-valued triangular neutrosophic number, which on the other hand provides accuracy to the proposed values.

Soft Sets are a useful tool for modelling uncertainty, and SuperHyperSoft Sets in particular will allow for a non-exact representation of the essential oil's required characteristics. Additionally, greater possibilities of expression will be obtained if the use of SuperHyperSoft Sets is compared to HyperSoft Sets since with the previous, cases that meet two or more different values of a single attribute can be

considered simultaneously. This is impossible with the former tools. Additionally, each essential oil will be assigned a neutrosophic value of medicinal effectiveness, calculated from its chemical components and concentration.

The attributes to be measured determined in this study are the following as shown in Table 1:

Table 1: Attributes to be measured in the chemical composition of the essential oils studied. The sets of possible values are real numerical values representing the relative values of substances.

Attribute	Name	Relative value (%)
a_1	Eugenol	0-100
a_2	Citronellol	0-100
a_3	1H-Cycloprop[e]azulen-7-ol, decahydro-1,1,7trimethyl-4-methylene-,[1ar(1a α,4a α,7 β,7a β,7b α)]	0-100
a_4	Aromandendrene	0-100
a_5	Cyclohexene, 3-(1,5-dimethyl-4-hexenyl)-6methylene-, [S-(R*, S*)]	0-100
a_6	benzene 2-methoxy-4-methyl-1-(1-methylethyl)-	0-100
a_7	6-Octen-1-ol, 3,7-dimethyl-, acetate	0-100
a_8	2-Acetylcyclopentanone	0-100
a_9	3H-3a,7-Methanoazulene, 2,4,5,6,7,8-hexahydro1,4,9,9-tetramethyl-,[3aR	0-100
a_{10}	Terpineol	0-100
a_{11}	Caryophyllene	0-100
a_{12}	Terpinen-4-ol	0-100
a_{13}	3-Undecen-5-yne, (Z)-	0-100
a_{14}	Phenol, 2-methoxy-4-(2-propenyl)-,acetate	0-100
a_{15}	Linalool	0-100
a_{16}	3-Methyl-4-isopropylphenol	0-100
a_{17}	Cedran-diol, (8S,14)-	0-100
a_{18}	Eucalyptol	0-100
a_{19}	3-Cyclohexen-1-ol, 4-methyl-1-(1-methylethyl)-, (R)-	0-100
a_{20}	Naphthalene, 1,2,3,4,4a,5,6,8a-octahydro-4a,8dimethyl-2-(1-methylethenyl)-, [2R(2 α,4a α,8a β)]	0-100
a_{21}	a1Cyclopropane carboxamide, 2-cyclop ropyl-2methyl-N-(1-cyclopropylethyl)	0-100
a_{22}	Pregnane-3,20-dione	0-100
a_{23}	Pregn-5-en-20-one, 3,21-bis(acetyloxy)-, (3 β)-	0-100
a_{24}	2,6-Octadien-1-ol, 3,7-dimethyl-,acetate	0-100
a_{25}	Benzeneethanol, α-phenyl	0-100
a_{26}	1,6-Octadiene, 3,7-dimethyl-	0-100
a_{27}	Azulene, 1,2,3,3a,4,5,6,7-octahydro-1,4-dimethyl-7(1-methylethenyl)-, [1R(1 α,3a β,4 α,7 β)]	0-100
a_{28}	(+)-2-Bornanone	0-100
a_{29}	Cyclohexene, 4-methylene-1-(1-methylethyl)-	0-100
a_{30}	3,4-Pentadienal, 2,2-dimethyl-	0-100
a_{31}	o-Cymene	0-100
a_{32}	1,6-Octadiene, 3,7-dimethyl-	0-100

Attribute	Name	Relative value (%)
a_{33}	Humulene	0-100
a_{34}	Bicyclo[3.1.0]hex-2-ene, 2-methyl-5-(1methylethyl)	0-100
a_{35}	3,4-Pentadienal, 2,2-dimethyl-	0-100
a_{36}	Geraniol	0-100

Note that the sets A_i with $i \in \{1,2,\cdots,36\}$ corresponding to each a_i are defined as $A_i = [0,100]\%$. Also note that the use of SuperHyperSoft Sets allows these values to be represented as $\{[0,2],\ [4,5]\}$ sets of relative values in the range of 0% to 2% or 4% to 5% for a specific substance. This is not possible with HyperSoft Sets.

Additionally, we have another SuperHyperSoft Set with attributes that represent the concentration of *Hedyosmum essential cuatrecazanum* Occhioni oil, this is summarized in Table 2.

Table 2: Attributes representing the concentration of essential oil necessary to combat nine germs.

Attribute	Concentration of essential oil to combat germs	Name of germ (type)
b_1	$[0.062 \times 10^6, 10^6]$ µg/ml	*Enterococcus faecalis (* Gram-positive Bacteria *)*
b_2	$[0.062 \times 10^6, 10^6]$ µg/ml	*Staphylococcus aureus (* Gram-positive Bacteria *)*
b_3	$[0.062 \times 10^6, 10^6]$ µg/ml	*Klebsiella pneumoniae* (Gram-negative bacteria)
b_4	$[0.062 \times 10^6, 10^6]$ µg/ml	*Proteus mirabilis* (Gram-negative Bacteria)
b_5	$[0.062 \times 10^6, 10^6]$ µg/ml	*Escherichia coli* (Gram-negative Bacteria)
b_6	$[0.062 \times 10^6, 1]$ g/ml	*C. albicans (* Candida Species *)*
b_7	$[0.062 \times 10^6, 1]$ g/ml	*C. tropicalis (* Candida Species *)*
b_8	$[0.062 \times 10^6, 1]$ g/ml	*C. krusei (* Candida Species *)*
b_9	$[0.062 \times 10^6, 1]$ g/ml	*C. parasilopsis (* Candida Species *)*

To summarize, we have two SuperHyperSoft Sets: the first one is formed by the attributes $a_1, a_2, \ldots, a_{36}$ that appear in Table 1, with the values in the sets $A_i = [0,100]\%$. This SuperHyperSoft Set is formed by the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_{36}))$, where:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_{36}) \to \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right).$$

On the other hand, we have the SuperHyperSoft Set $(G, \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_9))$, where:

$$G: \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_9) \to \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right).$$

These are sets for the attributes $b_1, b_2, \ldots, b_9$ where $B_i = [0.062 \times 10^6, 10^6]$ µg/ml for $i \in \{1,2,3,4,5\}$; while $B_i = [0.062, 1]$ g/ml for $i \in \{6,7,8,9\}$.

The elements of the universe set are the $x \in U$, an essential oil extracted from the *Hedyosmum cuatrecazanum* Occhioni with a chemical composition having the substances appearing in Table 1 and in a concentration as shown in Table 2. $\mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right)$ is the neutrosophic evaluation of the elements of the universe set concerning anti-germ effectiveness.

Additionally, we propose the SuperHyperSoft Set $(H, \left(\mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_{36})\right) \times \left(\mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_9)\right))$ as if $x \in U$ with the appurtenance $(T_\alpha, I_\alpha, F_\alpha)$ for the chemical composition parameters and appurtenance $\left(T_\beta, I_\beta, F_\beta\right)$ for the concentration parameters, then $(T_\delta, I_\delta, F_\delta)$ is ob-

tained for the composition of the two $H(\alpha_1, \alpha_2, \cdots, \alpha_{36}, \beta_1, \beta_2, \cdots, \beta_9) = F(\alpha_1, \alpha_2, \cdots, \alpha_{36}) \cap G(\beta_1, \beta_2, \cdots, \beta_9)$. Thus, we have that $T_\delta = min(T_\alpha, T_\beta), I_\delta = max(I_\alpha, I_\beta)$, $F_\delta = max(F_\alpha, F_\beta)$. Note that in turn, $T_\alpha, T_\beta, I_\alpha, I_\beta, F_\alpha, F_\beta$, depend accordingly on the values for the parameters $\alpha_1, \alpha_2, \cdots, \alpha_{36}$ and $\beta_1, \beta_2, \cdots, \beta_9$. Equations 1, 2, and 3 are used for the experimental values shown below in Table 3. These values are maximum when the ideal conditions are reached in which the compound is most effective.

Table 3: Effective antibacterial and antifungal values of the essential oil in its chemical composition.

Attribute	Name	Effective Relative Value (%)
a_1	Eugenol	12.82±1
a_2	Citronellol	11.60±1
a_3	1H-Cycloprop[e]azulen-7-ol, decahydro-1,1,7trimethyl-4-methylene-,[1ar(1a α,4a α,7 β,7a β,7b α)]	10.10±1
a_4	Aromandendrene	7.14±1
a_5	Cyclohexene, 3-(1,5-dimethyl-4-hexenyl)-6methylene-, [S-(R*, S*)]	5.23±1
a_6	benzene 2-methoxy-4-methyl-1-(1-methylethyl)-	4.02±0.5
a_7	6-Octen-1-ol, 3,7-dimethyl-, acetate	3.57±0.5
a_8	2-Acetylcyclopentanone	3.23±0.5
a_9	3H-3a,7-Methanoazulene, 2,4,5,6,7,8-hexahydro1,4,9,9-tetramethyl-,[3aR	2.88±0.5
a_{10}	Terpineol	2.59±0.5
a_{11}	Caryophyllene	2.56±0.5
a_{12}	Terpinen-4-ol	2.47±0.5
a_{13}	3-Undecen-5-yne, (Z)-	2.46±0.5
a_{14}	Phenol, 2-methoxy-4-(2-propenyl)-,acetate	2.36±0.5
a_{15}	Linalool	1.87±0.1
a_{16}	3-Methyl-4-isopropylphenol	1.83±0.1
a_{17}	Cedran-diol, (8S,14)-	1.81±0.1
a_{18}	Eucalyptol	1.63±0.1
a_{19}	3-Cyclohexen-1-ol, 4-methyl-1-(1-methylethyl)-, (R)-	1.58±0.1
a_{20}	Naphthalene, 1,2,3,4,4a,5,6,8a-octahydro-4a,8dimethyl-2-(1-methylethenyl)-, [2R(2 α,4a α,8a β)]	1.57±0.1
a_{21}	a1Cyclopropane carboxamide, 2-cyclop ropyl-2methyl-N-(1-cyclopropylethyl)	1.54±0.1
a_{22}	Pregnane-3,20-dione	1.50±0.1
a_{23}	Pregn-5-en-20-one, 3,21-bis(acetyloxy)-, (3 β)-	1.27±0.1
a_{24}	2,6-Octadien-1-ol, 3,7-dimethyl-,acetate	1.22±0.1
a_{25}	Benzeneethanol, α-phenyl	1.14±0.1
a_{26}	1,6-Octadiene, 3,7-dimethyl-	1.14±0.1
a_{27}	Azulene, 1,2,3,3a,4,5,6,7-octahydro-1,4-dimethyl-7(1-methylethenyl)-, [1R(1 α,3a β,4 α,7 β)]	1.08±0.1
a_{28}	(+)-2-Bornanone	1.05±0.1
a_{29}	Cyclohexene, 4-methylene-1-(1-methylethyl)-	0.97±0.05
a_{30}	3,4-Pentadienal, 2,2-dimethyl-	0.97±0.05
a_{31}	o-Cymene	0.95±0.05

a_{32}	1,6-Octadiene, 3,7-dimethyl-	0.92±0.05
a_{33}	Humulene	0.92±0.05
a_{34}	Bicyclo[3.1.0]hex-2-ene, 2-methyl-5-(1methylethyl)	0.75±0.03
a_{35}	3,4-Pentadienal, 2,2-dimethyl-	0.72±0.03
a_{36}	Geraniol	0.57±0.02

The results in Table 3 indicate the principal values of the single-valued triangular neutrosophic number functions that are used to evaluate any essential oil. For example, eugenol with value 12.82 ±1, means that the single-valued triangular neutrosophic number is formed as $\langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ where $a_2 = 12.82$, $a_1 = 12.82 - 1 = 11.82$ and $a_3 = 12.82 + 1 = 13.82$, for $\alpha_{\tilde{a}} = 1, \beta_{\tilde{a}} = 0, \gamma_{\tilde{a}} = 0$.

This same idea extends to the concentration of essential oil that is effective as a natural medicine. See Table 4.

Table 4: Effective antibacterial and antifungal values of the essential oil in its concentration as a natural medicine.

Attribute	Concentration of essential oil suitable for combating germs	Name of germ (type)
b_1	0.062x10^6 ±10^7	*Enterococcus faecalis (*Gram-positive Bacteria*)*
b_2	0.125x10^6 ±10^7	*Staphylococcus aureus (*Gram-positive Bacteria*)*
b_3	0.125x10^6 ±10^7	*Klebsiella pneumoniae* (Gram-negative bacteria)
b_4	0.125x10^6 ±10^7	*Proteus mirabilis* (Gram-negative Bacteria)
b_5	0.25x10^6 ±10^7	*Escherichia coli* (Gram-negative Bacteria)
b_6	0.25±0.01	*C. albicans (*Candida Species*)*
b_7	0.25±0.01	*C. tropicalis (*Candida Species*)*
b_8	0.50±0.02	*C. krusei (*Candida Species*)*
b_9	1.50±0.07	*C. parasilopsis (*Candida Species*)*

Let us illustrate someway the usefulness of the proposed model. Suppose that the essential oil was extracted from 10 specimens of *Hedyosmum cuatrecazanum* Occhioni in an Ecuadorian laboratory, let us denote them by $x_1, x_2, \cdots, x_{10}$. A study of the chemical composition of each of the 36 compounds summarized in Table 1 is carried out.

These results in the laboratory on the relative values were evaluated in the corresponding single-valued triangular neutrosophic numbers of Equations 1, 2, and 3, with the parameters that appear in Table 3. As in the example we had analyzed, it is used $\langle(11.82,12.82,13.82); 1,0,0\rangle$ to evaluate Eugenol, therefore each essential oil sample will have a value $(\mathrm{T}_{i1}, \mathrm{I}_{i1}, \mathrm{F}_{i1}) \in [0,1]^3$ for each oil sample with $i = 1, 2, \ldots, 10$, where the index 1 means that it is the first compound eugenol. This triple of neutrosophic values means to how degree the essential oil sample satisfies the conditions of effectiveness as a bactericide and fungicide. The lowest x_i neutrosophic value is then obtained, that is, for x_i the values $(\mathrm{T}_{ij}, \mathrm{I}_{ij}, \mathrm{F}_{ij})$ $j = 1,2, \ldots, 36$ are achieved, and from them, the aggregation is calculated which is the value $(min_j\{\mathrm{T}_{ij}\}, max_j\{\mathrm{I}_{ij}\}, max_j\{\mathrm{F}_{ij}\})$. This value results in the single-valued Neutrosophic number meaning the effectiveness of the ith sample of essential oil.

4. Conclusion

Hedyosmum cuatrecazanum Occhioni essential oil is a natural antibacterial and antifungal source. The relative values of 36 chemical substances present in the active ingredient were experimentally determined so that these properties are manifested. The concentration of essential oil necessary for it to be sufficiently effective was also determined, and tested on nine germs between bacteria and *candida*

species. For mathematical modeling, the neutrosophic SuperHyperSoft Sets, allow us to deal with the uncertainty of the present percentages of the different components of the essential raw material. This complete analysis makes it possible to classify the essential oil extracted from any plant and to verify its quality for use as a natural medicine, both for its chemical composition and for its concentration.

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Analysis of the Evolution of Social Competence in Students Through Research Methods Based on Neutrosophic Sets

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Abstract: The present study investigates the evolution of social competence in students through research methods based on set theory and the application of neutrosophic sets. The central question lies in how to accurately assess the development of social skills in educational contexts, given that traditional tools do not always capture the complexity of students' interactions and perceptions. The importance of this topic lies in the need for evaluation methods that faithfully reflect the growth in social competencies, vital for the comprehensive development of students in increasingly diverse and demanding educational environments. Despite the abundance of studies on social competencies, there is a significant gap in the integration of approaches that manage uncertainty and multiple dimensions of social skills. To address this gap, the study uses research methods based on set theory, specifically with neutrosophic sets, which allow for a more flexible and comprehensive assessment of social competencies. The applied methodology includes data collection through surveys and analysis of this data using neutrosophic techniques to capture the complexities and ambiguities inherent in the assessment of social skills. The results show that neutrosophic sets provide a more nuanced and precise view of the development of social competencies compared to traditional methods. The implications of the study are significant: it not only advances theoretical knowledge by introducing an innovative methodology for the assessment of social competencies but also offers practical tools to improve assessment and support students in their social development.

Keywords: Social Competence, Development of Social Skills, Set Theoretical Methods, SQA, Fsqca, Neutrosophic Sets.

1. Introduction

Social competence in students is a crucial component of their comprehensive development and academic and personal success. As educational and social demands evolve, it becomes essential to understand how these competencies develop and change over time. Current research has shown that social skills, such as effective communication, teamwork, and empathy, are critical for academic performance and social adaptation [1]. However, traditional assessment methods often fail to capture the complexity of these competencies, underscoring the need for more advanced and accurate approaches. Historically, the evaluation of social competencies has moved from qualitative approaches, such as direct observations, to more structured and quantitative methodologies. During the 20th century, standardized tests and surveys began to be implemented to measure interpersonal and emotional skills [2]. In recent decades, the integration of technology and new psychological theories has allowed progress toward more dynamic and adaptive models. However, most of these approaches

still face limitations by not adequately considering the uncertainty and ambiguity inherent in social skills assessment [3].

The central problem addressed in this study is the insufficiency of traditional methods to accurately capture the evolution of social competencies in students [4]. In particular, the question arises: how can we effectively assess changes in social competencies using methods that reflect the complexity and variability of these skills? This issue has not been satisfactorily resolved in the existing literature, highlighting a significant gap in the current assessment methodology. To address this gap, the study proposes using research methods based on set theory, specifically applying neutrosophic sets. This methodology allows for a more flexible and complete assessment of social competencies by integrating uncertainty and the multiple dimensions of these skills. Neutrosophic sets, which extend traditional set theory by allowing indeterminacy values, offer an innovative framework for analyzing the evolution of social competencies more precisely. The main objective of this study is to evaluate the effectiveness of neutrosophic sets in capturing the evolution of social competencies in students. This includes analyzing how these methods can better reflect the complexities and variations in social skills compared to traditional approaches. In addition, it seeks to provide practical recommendations based on the findings to improve methods of evaluating social competencies in educational contexts. In summary, the study not only attempts to fill a gap in the literature on social competency assessment but also introduces an innovative methodology that can have a significant impact on the way these skills are understood and valued in the educational environment. Through detailed evaluation and advanced methodology, this work seeks to advance knowledge in the field and offer practical tools to improve the education and social development of students.

2. Preliminaries

2.1. Social Competence

Social competence in students, a crucial dimension of comprehensive development, has become increasingly relevant in the contemporary educational landscape. As societies evolve toward more interconnected and diversified structures, the ability to interact effectively with others becomes an essential determinant of academic and personal success. This phenomenon, far from being a mere matter of soft skills, stands as a fundamental component in preparing students to face real-world challenges [5]. In an educational context that advocates the formation of well-rounded and adaptive individuals, social competence emerges as the axis around which many of the skills necessary for optimal performance in various areas of life revolve. However, despite its indisputable importance, the assessment of social competence has faced multiple challenges and limitations over time. Historically, direct observation and qualitative questionnaires dominated the field, providing valuable but often insufficient data to capture the complexity of social interactions in varied contexts. The evolution of assessment methodologies, from subjective approaches to more structured and quantitative models, has attempted to address these limitations, although with mixed results. Despite advances, traditional methods continue to face criticism for their inability to accurately reflect the dynamics and diversity of social skills [6].

The central problem in the evaluation of social competence lies in the difficulty of capturing its evolution in a precise and dynamic way. Conventional methods tend to simplify social interaction through standardized parameters that, although useful, do not always reflect the complex and multifaceted reality of interpersonal skills. In this context, a fundamental question arises: how can students' social competence be effectively assessed, considering the multiplicity of factors that influence their development? The answer to this question is crucial to improve current approaches and offer a more complete view of students' social growth. To address this issue, it is essential to consider innovative approaches that overcome the limitations of traditional methods. In this sense, models based on set theory and, in particular, neutrosophic sets, offer a fresh and promising perspective. These models allow for a more nuanced assessment by integrating elements of uncertainty and ambiguity, thus providing a more complete view of social competence [7]. By incorporating aspects of

indeterminacy into assessment, neutrosophic sets offer a powerful tool for capturing the complexity inherent in social skills and their evolution. The neutrosophic ensemble-based methodology stands out for its ability to handle and represent ambiguity in data effectively. Unlike traditional approaches, which tend to impose rigid and defined structures, neutrosophic ensembles allow for flexibility that better reflects the dynamic nature of social interactions. This is particularly relevant in the educational context, where social skills are not only developed through direct interactions but also adaptation to various environments and situations.

Analyzing social competence using neutrosophic sets offers several significant benefits. First, it allows for a more accurate representation of social skills by capturing the variability and complexity inherent in human interactions. In addition, it facilitates a deeper understanding of how these skills evolve, providing valuable insights for improving teaching and assessment methods in the educational field. This approach, therefore, not only enriches theory on social competence but also has practical implications for pedagogical practice. In terms of practical application, neutrosophic sets offer a valuable tool for educators and educational program designers. By using these models to assess social competence, more effective intervention strategies can be developed tailored to the individual needs of students. For example, data obtained through this approach could inform the creation of social skills development programs that specifically address areas of difficulty identified in assessments. In this way, the use of neutrosophic sets not only improves evaluation but also contributes to the design of more precise and effective educational interventions.

Social competence in students represents an essential component of comprehensive and academic development, which requires sophisticated and adaptive assessment approaches. The application of set theory-based models, such as neutrosophic sets, offers an innovative perspective to address the limitations of traditional methods. By integrating elements of uncertainty and ambiguity, these models provide a more faithful representation of the dynamics of social skills, enriching both theory and practice in the educational field. This approach not only advances knowledge in the field but also offers valuable tools to improve the education and social development of students.

2.2. Complexity and causality theory and neutrosophic sets

Interactions between variables are not always presented simply; rather, they frequently emerge through intricate, nonlinear patterns, as complexity theory argues. This perspective tells us that the same cause can cause divergent results depending on the context in which it manifests itself. In this theory, three key principles stand out: conjunction, equifinality, and causal asymmetry [8]. The conjunction principle focuses on collaboration between antecedent conditions that act together to produce an outcome, rather than operating independently to explain variability. On the other hand, equifinality states that a system can reach a specific final state through various initial conditions and different trajectories. Causal asymmetry, for its part, suggests that, while certain conditions can lead to the appearance of an outcome, their absence does not guarantee the absence of said outcome [9].

To illustrate this, consider a restaurant famous for its high culinary quality. Although this quality may attract numerous customers, the establishment could face low footfall due to factors such as an unfavorable location or problems with parking [10]. Conversely, a restaurant with average food could still attract many customers if it offers exceptional service, is in a strategic location, or has attractive entertainment options. This shows that the relationship between variables such as food quality, location and service, and the result, that is, the number of customers, is not at all simple or constant. These principles highlight the complexity and lack of stability in the relationship between conditions and outcomes. Furthermore, neutrosophy adds greater depth to the understanding of complex causality by introducing indeterminacy and uncertainty, which are inherent to social phenomena. Neutrosophic set theory, with its ability to handle indeterminacy, provides a more nuanced perspective for understanding these complex and dynamic relationships [11].

2.3. Neutrosophic Liker Scales

Surveys using neutrosophic Likert scales [12, 13, 14] effectively measure the diversity of opinions and their influence on public policies and social discourse, capturing areas of consensus, disagreement, and ambivalence.

Below we present the fundamental definitions and concepts related to neutrosophic sets and single-valued neutrosophic sets.

Definition 1 ([15]). Let U be a discursive universe. N ={ (x , T (x), I (x), F (x)): $x \in U$ } is a neutrosophic set, denoted by a truth membership function, $TN : U \rightarrow]0-, 1+[$; an indeterminacy membership function, $IN : U \rightarrow]0-, 1+[$; and a falsity membership function, $FN : U \rightarrow]0-, 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the universe of discourse U

Definition 2 ([16]). Let U be a discursive universe. A single-valued neutrosophic set is defined as N ={ (x , T (x), I (x), F (x)): $x \in U$ }, which is identified by a truth membership function, $TN : U \rightarrow [0, 1]$; indeterminacy membership function, $IN : U \rightarrow [0, 1]$; and falsity membership function, $FN : U \rightarrow [0, 1]$, with $0 \leq TN(x) + IN(x) + FN(x) \leq 3$

Using neutrosophic scales with single-value neutrosophic sets, responses are classified according to the total of the True, Indeterminate, and False components as follows:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are obtained because, in many cases, the opinions are incomplete or contradictory. This classification is one of the advantages of using neutrosophic methods, as it allows for a more nuanced understanding of the different degrees of truth, indeterminacy, and falsity in responses.

3. Proposed framework

To begin, it is essential to clearly define the desired result: identify and precisely describe the phenomenon, event or condition that you want to explore. This step is essential, as it establishes the focus and framework that will guide the subsequent analysis.

Next, proceed to develop neutrosophic Likert scales. These scales, in contrast to conventional scales that use a fixed range of values (such as 1 to 5), incorporate additional dimensions of truth, indeterminacy, and falsity. Instead of simple numerical scores, neutrosophic scales use a triplet (T, I, F) for each option, where T represents the degree of truth, I the degree of indeterminacy, and F the degree of falsehood. This method allows for a more nuanced and detailed evaluation of participants' responses and perceptions.

Next, collect relevant data on the case studies, using a variety of indicators or measures related to the defined outcome. Data collection must be exhaustive and accurate so that it adequately reflects the variables analyzed. Use Neutrosophic Likert scales in questionnaires and surveys to obtain a more complete data set, which more faithfully captures the complexity of respondents' opinions and attitudes.

This detailed and refined approach ensures a deeper and more accurate interpretation of the results, thus facilitating a comprehensive understanding of the phenomenon in question.

Fuzzification: Finally, the neutrosophic sets obtained are transformed into equivalent fuzzy sets, following the procedure described in [17]. This step is essential for subsequent analysis, allowing you to handle the uncertainty and ambiguity inherent in the collected data. $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$a NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$, where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the similarity equation proposed in,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

Since the range of the similarity measure function is the unit interval [0,1], μA (x) ∈ [0,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [0, 1] and therefore satisfies the property of a membership function of a fuzzy set (FS).

1 **Analysis**: Perform fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. The fsQCA program for Windows is used for data processing [18, 19].

The validity of the configuration is evaluated by measuring the consistency and coverage values. Consistency is the measure of the reliability with which the set of pathways produces the desired result. Coverage refers to the degree to which the result is clarified by this arrangement of pathways [20, 21]:

$$Consistency\ (Y_i \leq\ X_i) = \frac{\sum \min\ (X_i, Y_i)}{\sum Y_i} \quad (2)$$

$$Coverage\ (Y_i \leq\ X_i) = \frac{\sum \min\ (X_i, Y_i)}{\sum X_i} \quad (3)$$

where:

$X_i$is the membership value of case i in the set of causal conditions.

$Y_i$is the membership value of case iii in the result set.

Both are used in comparative analysis to evaluate the relationships established between individual conditions, combinations of conditions, path configurations, and the result. Generally, values greater than 0.8 are considered indicators of a strong relationship [22,23, 24].

4. Results

The perception of the **Evolution of Social Competence (ECS)** constitutes the result that has been established. To capture this perception, a Likert scale is created that is represented through single-value neutrosophic sets. Additionally, the study is not limited to this specific measure but also addresses various additional variables.

◎ Interpersonal **Communication**: Students' ability to communicate effectively, both verbally and non-verbally, plays a crucial role in their social competence. This skill includes the ability to listen actively, express thoughts and emotions clearly, and understand the social cues of others.

◎ Social **Interaction Experiences**: Previous experiences in various social contexts, such as in groups of friends, extracurricular activities, or family environments, significantly affect the development of social competence. Positive and varied interactions can improve social skills, while negative experiences can present additional challenges.

◎ **Self-Esteem and Self-Efficacy**: A student's perception of himself, as well as his belief in his ability to handle social situations, influences his social competence. High self-esteem and self-efficacy are often associated with a greater ability to interact effectively with others and approach social situations with confidence.

A survey was conducted with a group of 20 high school educators with extensive experience as well as educational psychologists from Ecuador (see Table 1).

Table 1. Survey data

Educators	Interpersonal Communication (IC)	Social Interaction Experiences (EIS)	Self-esteem and Self-efficacy (AA)	Competition Evolution (ECS)
1	(0.9, 0.8, 0.1)	(0.6, 1, 0.6)	(0.3, 0.7, 0.3)	(0.8, 0.6, 0.7)
2	(0,6, 0,6,0,6)	(1,1,1)	(0,6,0,1,0,6)	(0,6, 0,6, 0,7)
3	(0,8, 0,7, 0,4)	(0,7, 0,9, 0,6)	(0,8, 0,6, 0,6)	(0,8, 0,6, 0,6)
4	(1,1,0)	(0.8,0.8,0)	(1,0,9,0,3)	(0,7, 1, 0,9)
5	(1,0.6,0)	(1,0.6,1)	(1,0.6,1)	(0,9, 0,6, 0,1)
6	(0,9, 0,9, 0,9)	(0,9, 0,9, 0,9)	(0,9, 0,9, 0,9)	(0,9, 0,9, 0,9)
7	(0,1, 0,6, 0,8)	(1,0,0)	(0,6, 0,6, 0,6)	(0,8, 0,6, 0,1)
8	(1, 0,9, 0,1)	(0,9, 0,9, 0,1)	(0,9, 0,9, 0,1)	(0,9, 0,9, 0,1)
9	(1,1,0)	(0,8, 0,8, 0)	(1,0,0)	(0,9, 0, 0)
10	(0,7, 1, 0,1)	(0,9, 0,4, 0)	(0,6, 0,9, 0,1)	(1,0,0)
11	(0,4, 0,7, 0,1)	(0,3, 0,9, 0,4)	(0,8, 0,4, 0,6)	(0,4, 0,8, 0,3)
12	(0,6, 1, 0,6)	(0,6, 0,6, 0,1)	(0,1, 0,6, 0,7)	(1,0,1)
13	(0,7, 0,8, 0,4)	(0,8, 1, 0,5)	(0,7, 0,5, 0,4)	(0,7, 0,7, 0,5)
14	(0,5, 0,7, 0,2)	(0,6, 0,8, 0,6)	(0,5, 0,6, 0,5)	(0,6, 0,6, 0,4)
15	(0,9, 0,7, 0,3)	(0,7, 1, 0,4)	(0,6, 0,8, 0,4)	(0,8, 0,7, 0,3)
16	(0,8, 0,6, 0,5)	(0,9, 0,7, 0,3)	(0,7, 0,5, 0,6)	(0,8, 0,6, 0,4)
17	(0,6, 0,8, 0,4)	(0,7, 0,8, 0,2)	(0,5, 0,7, 0,5)	(0,7, 0,8, 0,3)
18	(0.7, 0.9, 0.3)	(0.6, 0.9, 0.2)	(0.7, 0.6, 0.6)	(0.8, 0.7, 0.3)
19	(0.6, 0.8, 0.5)	(0.8, 1, 0.3)	(0.6, 0.7, 0.4)	(0.7, 0.8, 0.4)
20	(0.8, 0.7, 0.4)	(0.9, 0.8, 0.2)	(0.7, 0.6, 0.5)	(0.8, 0.7, 0.4)

The fuzzification process is developed using Equation 1. (Table 2)

Table 2. Fuzzy values

Educators	Interpersonal Communication (IC)	Social Interaction Experiences (EIS)	Self-esteem and Self-efficacy (AA)	Competition Evolution (ECS)
1	(0.85)	(0.75)	(0.55)	(0.67)
2	(0.75)	(0.75)	(0.53)	(0.67)
3	(0.78)	(0.73)	(0.63)	(0.67)
4	(0.70)	(0.70)	(0.78)	(0.79)
5	(0.78)	(0.80)	(0.78)	(0.65)
6	(0.89)	(0.89)	(0,89)	(0,89)
7	(0,60)	(0,80)	(0,67)	(0,62)
8	(0,85)	(0,85)	(0,85)	(0,85)
9	(0,78)	(0,75)	(0,50)	(0,75)
10	(0,70)	(0,85)	(0,68)	(0,67)
11	(0,55)	(0,60)	(0,57)	(0,60)
12	(0,75)	(0,70)	(0,54)	(0,67)
13	(0,72)	(0,75)	(0,62)	(0,67)
14	(0,68)	(0,70)	(0,62)	(0,65)

Educators	Interpersonal Communication (IC)	Social Interaction Experiences (EIS)	Self-esteem and Self-efficacy (AA)	Competition Evolution (ECS)
15	(0,80)	(0,75)	(0,68)	(0,67)
16	(0,73)	(0,80)	(0,62)	(0,66)
17	(0,68)	(0,75)	(0,62)	(0,65)
18	(0,72)	(0,75)	(0,67)	(0,65)
19	(0,70)	(0.75)	(0.62)	(0.65)
20	(0.73)	(0.80)	(0.67)	(0.66)

A necessary condition analysis is performed to test consistency and coverage (Table 3).

Table 3. Analysis of necessary conditions

Tested conditions	Consistency	Coverage
Interpersonal Communication (IC)	0.82	0.76
Social Interaction Experiences (SIA)	0.79	0.73
Self-esteem and Self-efficacy (AA)	0.77	0.69
Evolution of Competition (ECS)	0.86	0.91

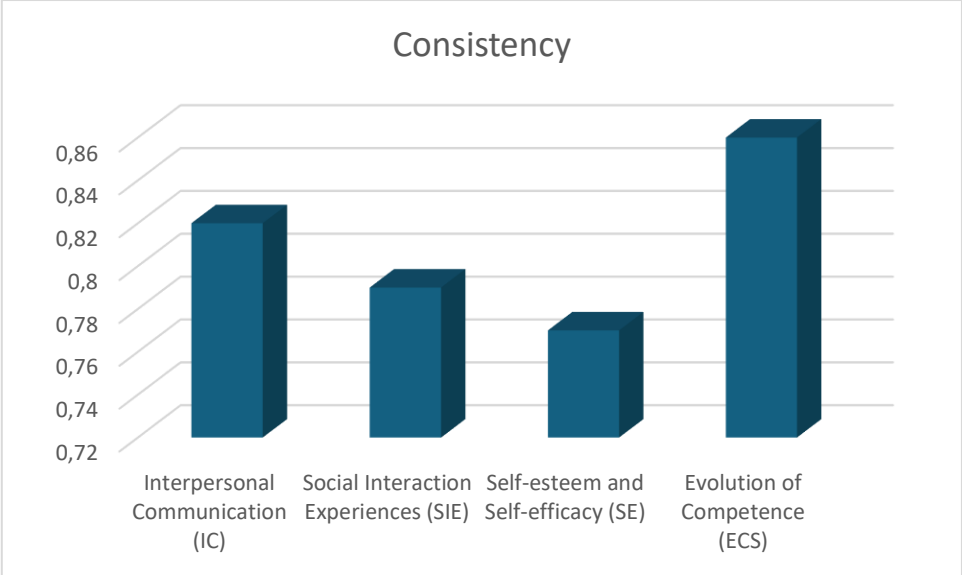

Figure 1. Analysis of necessary conditions (consistency)

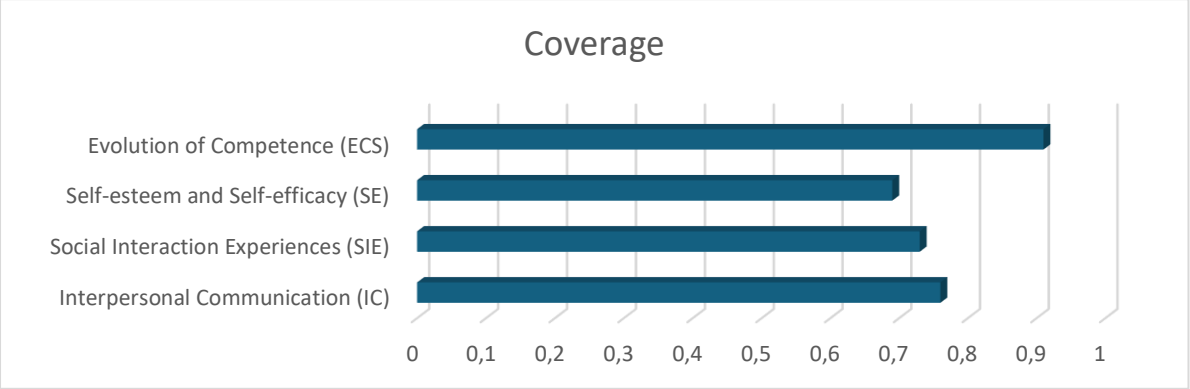

Figure 2. Analysis of necessary conditions (consistency)

In the rigorous exploration of the conditions necessary for the optimization of educational processes, the analysis of consistency and coverage of the different dimensions evaluated offers a critical window into the effectiveness of the factors investigated. Table 3 presents a compendium of revealing data on Interpersonal Communication (IC), Social Interaction Experiences (EIS), Self-esteem and Self-efficacy (AA), and Evolution of Competence (ECS). This analysis unravels the complexity of how each variable correlates with the quality of the educational process, providing a solid foundation for future strategic decisions. Interpersonal Communication, with a consistency of 0.82 and a coverage of 0.76, stands as a significant component in the educational structure. These values indicate that, although IC presents a robust relationship with educational evolution, it does not reach the expected optimal levels of coverage. The relatively high consistency suggests that, in general, instances of interpersonal communication tend to align consistently with desired outcomes. However, the slightly lower coverage points to the need for additional attention to maximize the impact of communication in the educational process.

On the other hand, Social Interaction Experiences, with a consistency of 0.79 and a coverage of 0.73, reveal a less robust pattern compared to Interpersonal Communication. The relatively low consistency indicates that, despite being an important condition, social interaction experiences do not correlate as closely with educational outcomes. The coverage also suggests that, although a positive impact is observed, there is room to improve the influence of these experiences on the educational process. Self-esteem and Self-efficacy, with a consistency of 0.77 and a coverage of 0.69, present the lowest values among the conditions evaluated. This dimension, crucial for personal and professional development, seems to have a less direct correlation with educational results. The low consistency and coverage reflect the inherent difficulty of linking self-perception and self-efficacy with tangible results, suggesting the need for more specific strategies to enhance these aspects in the educational environment. However, it is in the Evolution of Competition (ECS) where an outstanding performance is observed, with a consistency of 0.86 and a coverage of 0.91. These high values underline a very strong relationship between the evolution of competence and educational results. High consistency indicates that the evolution of the competence is closely aligned with the expected results, while high coverage suggests that this condition has a significant and widespread impact on the educational process. The comparative analysis of these metrics provides a deep insight into the relationships between the conditions evaluated and their effects on educational quality. The discrepancy in consistency and coverage values between conditions highlights the variability in the effectiveness of each factor. While the Evolution of Competence proves to be a robust and reliable predictor, other dimensions such as Interpersonal Communication, Social Interaction Experiences, and Self-Esteem and Self-Efficacy require adjustments to improve their impact on educational results.

Furthermore, it is crucial to consider that the high coverage of the Evolution of Competence may also imply that this dimension addresses a wide spectrum of educational aspects, which contributes to its high value. This ability to encompass multiple facets can be a significant advantage in the implementation of educational strategies, offering a more comprehensive and adaptive solution. The consistency and coverage of each condition should not be evaluated in isolation; instead, they must be considered in conjunction with the specific context in which they are applied. The differences in metrics reflect not only the effectiveness of each condition but also the need for continuous adaptation to the changing dynamics of the educational environment. In summary, while the Evolution of Competence is presented as a solid pillar in the educational structure, Interpersonal Communication, Social Interaction Experiences, and Self-esteem and Self-efficacy offer areas of opportunity. Integrating these findings into the planning and execution of educational strategies will allow for more effective optimization, ensuring that all necessary factors are appropriately considered and adjusted to maximize their impact on educational outcomes. This analysis not only highlights the importance of continuous evaluation of each dimension but also highlights the need for strategic implementation that reflects the complexities and variabilities inherent in the educational process. A deep and nuanced understanding of these factors will facilitate the creation of more effective educational environments tailored to the needs of students.

A set matching analysis is performed (see Table 4).

Table 4. Set matching analysis

Conditions	Coincidence
CI, EIS, AA	0.7735
CI, EIS	0.7699
C.I., A.A.	0.7658
EIS, AA	0.7605

The analysis of the agreement between sets of conditions presented in Table 4 offers a fascinating perspective on the interaction between different factors in the evaluated context. Interpreting match values requires not only an understanding of the numbers themselves but also an appreciation of the implications these values have on the relationship between variables.

Let's start with the set of conditions that combines Interpersonal Communication (IC), Social Interaction Experiences (EIS), and Self-Esteem and Self-Efficacy (AA). The match value in this case is 0.7735, which reveals notable robustness in the interrelationship of these three conditions. This high value indicates that, when considered together, CI, EIS, and AA are closely related in the way they influence the phenomenon in question. The high overlap suggests that an increase in any of these variables tends to be associated with increases in the other two, creating an interdependent network of influence. This could imply that programs or strategies that focus on improving one of these aspects could have amplified effects on the others. On the other hand, when we look at the coincidence between CI and EIS, with a value of 0.7699, we notice a strong relationship, although slightly smaller compared to the previous trio. This figure is still indicative of a robust relationship between Interpersonal Communication and Social Interaction Experiences, suggesting that, in the context evaluated, both factors are closely linked. However, the lower agreement compared to the combined value of IQ, EIS, and AA may indicate that the inclusion of Self-Esteem and Self-Efficacy adds a layer of interaction that strengthens the relationship between IQ and EIS. The coincidence value between IQ and AA, which is 0.7658, also shows a significant relationship, although smaller than that found in the combined three-factor analysis. This figure reflects the direct connection between Interpersonal Communication and Self-Esteem and Self-Efficacy, although the decrease in value compared to the three-variable match may

suggest that the influence of IQ on AA is mediated by Social Interaction Experience. The inclusion of EIS in the equation seems to provide additional context that strengthens the connection between IC and AA.

Finally, the match between EIS and AA, which is 0.7605, has the lowest relationship among the pairs considered but still shows a strong match. This suggests that Social Interaction Experiences and Self-Esteem and Self-Efficacy are significantly correlated, but perhaps in a less direct way compared to the other combinations. The minor overlap here could signal that, although these two factors are connected, the relationship is not as strong as that observed between other pairs of conditions. In summary, the coincidence values reflect how different factors are interrelated in the study context, highlighting that the interactions between Interpersonal Communication, Social Interaction Experiences Self-esteem, and Self-efficacy are complex and multifaceted. The data suggest that combining these factors offers a more complete view of their mutual influence, while individual matches, although significant, show how relationships between pairs of conditions can vary in intensity. This nuanced understanding is crucial to developing interventions or strategies that consider the complex interactions between these factors, thus maximizing their impact on the phenomenon evaluated.

The results of the superset analysis are shown in Table 5.

Table 5. Results of the subset/ superset analysis

terms	consistency	coverage	set
CI, EIS, AA	0.773500	0.773500	0.773500
CI, EIS	0.769900	0.769900	0.769900
C.I., A.A.	0.765800	0.765800	0.765800
EIS, AA	0.760500	0.760500	0.760500
IC	0.820000	0.760000	0.790000
OUR	0.790000	0.730000	0.760000
AA	0.770000	0.690000	0.730000

The results of the superset analysis, presented in Table 5, highlight several significant findings on the relationship between **Interpersonal Communication (IC)**, **Social Interaction Experiences (SIA),** and **Self-Esteem and Self-Efficacy (AA)**. In particular, the subset that includes all variables (CI, EIS, AA) shows a consistency and coverage of 0.773500, the highest among all the evaluated subsets. This finding indicates that the combination of these three conditions provides a robust and coherent interrelationship, suggesting a strong connection between these factors in the context of the study. The interpretation of these results suggests that the simultaneous presence of CI, EIS, and AA contributes significantly to a stronger and more homogeneous relationship between the variables. The high consistency and coverage in this subset reflect an effective and coherent integration of these dimensions, supporting the hypothesis that these variables when combined, generate a more robust set of conditions. The gradual decrease in consistency and coverage values when analyzing subsets with fewer variables (CI, EIS; CI, AA; EIS, AA) and individual variables indicates that each additional factor contributes to improving the cohesion and strength of the interrelationships.

Comparing these findings with previous studies, it is observed that the results are consistent with research that highlights the importance of the integration of multiple variables to obtain a more complete and accurate view of the interactions between psychological factors. For example, previous studies have shown that combining psychological and social variables often results in a more robust and meaningful analysis. However, the results also contrast with research that has found weaker relationships between individual factors or limited combinations, suggesting that the context or methodological approach may influence the results obtained. The study has several limitations that must be considered. First, the methodology of superset and subset analysis, although useful, may not

capture all the complexities of the interactions between variables. Additionally, the lack of diversification in the data sample could limit the generalizability of the results to broader contexts. The variability in coverage and consistency between different subsets also suggests that there could be additional factors that influence the relationship between the variables studied, which have not been considered in this analysis. The implications for future research are broad. The results suggest that the inclusion of a greater number of variables in the analysis can provide a deeper understanding of the interrelationships between psychological and social factors. It is recommended to explore other subsets and additional variables that could influence the results to obtain a more complete view. Furthermore, it would be beneficial to apply these methods in different contexts and populations to validate and extend these findings.

Some anomalous results, such as the lower consistency and coverage observed in the CI, AA, and EIS, AA subsets, require special attention. The explanation for these discrepancies could be related to the intrinsic variability of the variables or the lack of inclusion of specific contextual factors in the analysis. These anomalous results underscore the need for additional studies to better understand the conditions under which these combinations of variables have a significant impact. In summary, this discussion reflects an advance in the understanding of how different combinations of psychological and social variables interact with each other. The findings underscore the importance of considering multiple factors to obtain a more complete and robust picture of interrelationships in similar studies. This study not only contributes to the existing body of knowledge but also establishes a solid foundation for future research in this area.

5. Conclusions

The present study has unraveled the complex web of relationships between **Interpersonal Communication (IC)**, **Social Interaction Experiences (SIA),** and **Self-Esteem and Self-Efficacy (AA)**, revealing an intricate interdependence that provides a more complete view of how these factors influence each other. The results of the superset analysis show that the highest agreement is found in the CI, EIS, and AA set, with a value of 0.7735. This finding highlights the importance of considering these factors together since the interrelationship between them is remarkably strong and coherent. In comparison, the overlaps between pairs of conditions, although still significant, are smaller, reflecting the additive and potentially amplified influence of including multiple factors in the analysis. The practical importance of these findings is considerable. Understanding how CI, EIS, and AA interact provides a solid foundation for developing interventions that address multiple dimensions simultaneously. For example, in the design of personal or educational development programs, integrating strategies that improve interpersonal communication, social experiences, and self-esteem can maximize results and enhance the impact of interventions. This not only optimizes resources but also ensures that programs are more effective by addressing multiple areas of influence simultaneously.

The contributions of this study are significant to the field of psychology and social sciences. By identifying and quantifying the interrelationship between these key variables, the study provides a richer and more nuanced understanding of how these dimensions are intertwined. This contribution is crucial for the advancement of knowledge in the area, allowing researchers and professionals to appreciate the complexity and interdependence of psychological and social factors in specific contexts. However, the study has certain limitations. The main one is the possible influence of variables not considered in the analysis, which could affect the observed coincidences. Additionally, the specific context of the sample may limit the generalizability of the results to other populations or settings. Limitations in the variability of the conditions evaluated may also influence the ability of the study to capture all the nuances of the relationships between the variables. For future research, it is recommended to explore additional factors that could influence the dynamics between CI, EIS, and AA. The incorporation of contextual variables and the expansion of the sample to different populations could offer a more complete and generalizable vision. Furthermore, the application of alternative methods and advanced techniques could provide additional insights and validate the findings obtained

in this study. Anomalous results, such as the lower agreement observed between EIS and AA, should be further investigated to better understand the underlying reasons. These discrepancies could point to the need for a more detailed analysis of how social interaction experiences and self-esteem interrelate, possibly suggesting areas for further study. In summary, this study provides a solid foundation for understanding the complex interactions between CI, EIS, and AA. The findings highlight the importance of considering multiple factors comprehensively to capture the richness of the interrelationships between them. This holistic approach not only advances theoretical knowledge in the field, but also offers practical tools for designing more effective and well-informed interventions.

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Gender Gap in SME Ownership: An Analysis of Female Entrepreneurship in Santa Elena Canton, Ecuador, Using TreeSoft Sets

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Abstract. The paper we present here is an approach to the problem of gender inequality in the artisanal production association called ASOPROMAHER, located in Santa Elena Canton, Ecuador. It is a pretext to address the issue and extrapolate it to the current situation of the entire country. Without a doubt, gender inequality persists in Ecuador today, including an important sphere of economic development in the country such as entrepreneurship in SMEs. There are multiple issues to address in this situation, ranging from psychological, economic, social, cultural, and political issues. Each of these spheres is related to each other in a complex way. In this work, we propose to study the situation regarding female entrepreneurship in ASOPROMAHER. To meet this objective we use the tool known as TreeSoft Set, which allows the representation in the form of trees of relationships among attributes, sub-attributes, sub-sub-attributes, and so on. We propose an algorithm where we use the evaluation of ten experts on the situation of women in this association, as well as analyze other important issues.

Keywords: Gender gap, SME, entrepreneurship, female growth, TreeSoft Set.

1 Introduction

Over time, social, political, cultural, and economic changes in Latin America have shown that women are responsible for the care of their children, which allows them to play a leading role in society. Therefore, the quality and efficiency of their work are essential to complement the family environment, benefiting its economic growth, whether through saving or entrepreneurship. Likewise, throughout the world, the promotion of entrepreneurship has become a public policy measure. In countries, it is a business activity stimulated by educational systems and programs of public or private entities, which are intended to create new companies and introduce innovations in existing small and medium-sized companies.

In the last two decades, women's participation in the labor market and business activity has increased significantly, contributing to the strengthening of local economies. Currently, the value of the growing participation of women in the development of companies has been highlighted, which has sparked the interest of research. Likewise, entrepreneurship has a significant impact on job creation and the economic growth of a state, but to all this, the values that women represent are added.

It is for this reason that activities related to female entrepreneurship contribute to improving their quality of life since they currently represent an important means of adding constant value to the performance of their various roles of change. Nationally, the current government has developed programs at all levels to educate, train, and inspire all people to generate future business ideas and to provide them with financial means for the initial investment.

Internationally, entrepreneurship is one of the income-generating activities for all countries that support it. Ecuador is no exception, as it is known as one of the most entrepreneurial countries in the

world, especially in its urban and rural areas. The country has also attracted the international interest of many tourists, in fact, travel agents can create low-cost packages to generate income and these tourists, in turn, invest in rest and entertainment.

However, the institutional environment in Latin America is not particularly favorable for women entrepreneurs, since there is a negative influence of informal institutions on formal institutions, that is, cultural and social influences are immersed in the development of government policies in different countries.

Locally, the province of Santa Elena stands out for having several ways to generate income, including entrepreneurship, which is currently a non-oil source of income in the country. It has been studied that 29% of the population aged 18 to 64 has started a business. This demonstrates the strong entrepreneurial spirit of Ecuadorians. From this we can ask ourselves if institutional factors are immersed in the development of some type of entrepreneurship.

The issue is based on the fact that entrepreneurship is gaining a high significance for the residents of the Canton of Santa Elena. However, there are factors and some areas that have not been widely recognized within it. Such is the case of our object of study, which is located in the Dos Mangas commune of the Canton of Santa Elena, Manglaralto Parish, and communes located within the Colonche Parish. The first one is the main commune where the headquarters of the association is located. This has approximately 950 residents, whose main source of income is crafts and livestock, for this reason, they seek new ways to improve their development through entrepreneurship where there are great disadvantages. Several institutional factors prevent adequate development and prevent providing stable and effective results, which manage to exploit the entrepreneurial progress of the community, especially of the members of the association of artisanal production ASOPROMAHER, of the Canton of Santa Elena.

The purpose of this study is to identify the formal and informal institutional factors in the entrepreneurial process developed by the ASOPROMAHER Association, in the Canton of Santa Elena, and to determine which factors have the greatest impact on entrepreneurship, to consequently analyze them and then propose the corrective actions that the research entails.

So, the objectives of this article are:

- Identify the institutional factors that promote or limit the artisanal production association ASOPROMAHER, through an internal and external analysis.
- To analyze the institutional factors that affect the entrepreneurship of women from the ASOPROMAHER association, in the Canton of Santa Elena.

This paper compiles data on ASOPROMAHER. To do so, we hired ten experts who evaluated different aspects to measure the problems, challenges, and achievements of entrepreneurship in the association. In addition, we identified criteria, sub-criteria, and sub-sub-criterion to measure to carry out the study. The criteria, sub-criteria, and sub-sub-criteria are represented using the tool created by F. Smarandache called TreeSoft Set, which is a tree where this model is represented simply.

TreeSoft Sets are the result of a series of mathematical tools that start from Molodtsov's Soft Sets, as well as HyperSoft Sets, and SuperHyperSoft Sets, among others by F. Smarandache [1-5]. These theories are generalized in SuperHyperStructures, where relations are extended to the field of power sets of power sets, where the empty set may be present, indicating indeterminacy and uncertainty [6].

For the evaluation of the results, we propose an algorithm that we introduce in this paper, where we rely on the criteria of several experts to evaluate the collected data and in this way determine what remains to be done and what has been done regarding the subject of study.

This paper contains a section of Preliminaries, where we present the most important concepts within SuperHyperStructures, especially TreeSoft Sets as an alternative to the theory of HyperSoft Sets and SuperHyperSoft Sets [7-11]. The Study and Results section is dedicated to the presentation of the proposed algorithm and to showing the results of the study. The last section is Conclusions.

2 Preliminaries

This section contains the basic elements of the TreeSoft Sets theory developed by F. Smarandache. We start from the formal definition given by him on TreeSoft Sets ([7-17]).

Let $\mathcal{U}$ be the universe of discourse, and H is a nonempty set, a subset of $\mathcal{U}$, where $\mathcal{P}(H)$ is the power set of H. Let A be the set of attributes (parameters, factors, etc.), $A = \{A_1, A_2, \dots, A_n\}$, for an integer $n \geq 1$, where $A_1, A_2, \dots, A_n$ the *first-level attributes* are considered, which is indicated by the single-number indices.

Each attribute A_i $1 \leq i \leq n$ is made up of sub-attributes:

$A_1 = \{A_{1,1}, A_{1,2}, \dots\}$

$A_2 = \{A_{2,1}, A_{2,2}, \dots\}$

…….....

$A_n = \{A_{n,1}, A_{n,2}, \dots\}$

Where $A_{i,j}$ are sub-attributes or *second-level attributes* (since they have two-number indexes).

Again, each sub-attribute $A_{i,j}$ is made up of sub-sub-attributes or *third-level attributes*:

$A_{i,j,k}$

So on, when we have sub-sub-…-sub-attributes or *m-level attributes* (has m numbers as indices):

$A_{i_1,i_2,\dots,i_m}$.

This is how the graph tree is formed, which we denote by $Tree(A)$, whose root is A (which is considered level zero), then the nodes of level 1, level 2, up to level m are located.

We call *the leaves* of the graph tree all the terminal nodes (nodes that have no descendants).

So the TreeSoft Set is:

$F: \mathcal{P}(Tree(A)) \to \mathcal{P}(H)$

$Tree(A)$ is the set of all nodes and leaves (from level 1 to level m) of the graph tree, and $\mathcal{P}(Tree(A))$ is the power set of $Tree(A)$.

All node sets of the TreeSoft Set of level m are:

$Tree(A) = \{A_{i_1} | i_1 = 1,2, \dots\}$

The first set consists of nodes of level 1, the second one are nodes of level 2, the third ones are nodes of level 3, and so on, such that the last set consists of nodes of level m. If the graph tree has only two levels, then the TreeSoft Set is reduced to a MultiSoft Set [18, 19].

Example 1: This is an example that appears in [13]. See Figure 1.

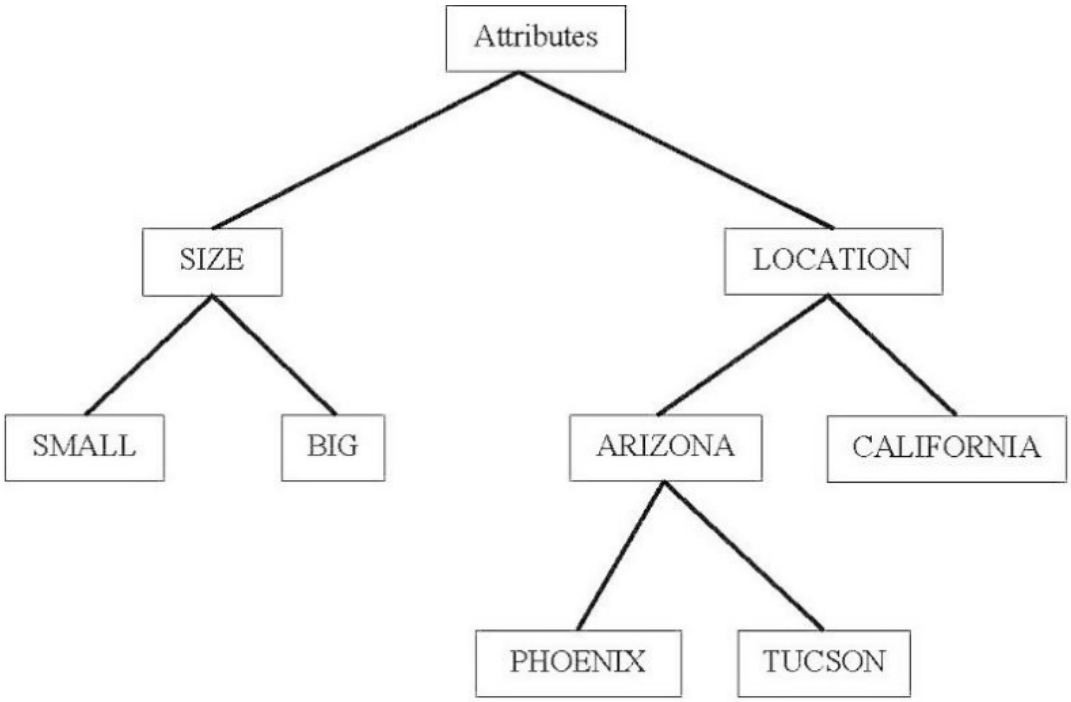

Figure 1: A Level 3 TreeSoft Set. Source: [13].

This is a tree such that:

Level 0 or the root is the node called "Attributes";
Level 1 consists of the node's Size, Location;
Level 2 consists of the Small, Big, Arizona, and California nodes;
Level 3 consists of the Phoenix and Tucson nodes.
Let $H = \{h_1, h_2, \ldots, h_{10}\}$ be a set of houses and the power set $\mathcal{P}(H)$ of the set H.
In addition, we have the set of attributes: $A = \{A_1, A_2\}$ where A_1 = Size, A_2 = Location.
So $A_1 = \{A_{11}, A_{12}\} = \{Small, Big\}$, $A_2 = \{A_{21}, A_{22}\} = \{Arizona, California\}$ as American states.
More precisely, $A_{22} = \{A_{211}, A_{212}\} = \{Phoenix, and\ Tucson\}$ as cities in Arizona.
Let us assume that the function F takes the following values:
$F(Big, Arizona, Phoenix) = \{h_9, h_{10}\}$
$F(Big, Arizona, Tucson) = \{h_1, h_2, h_3, h_4\}$
$F(Big, Arizona) =$ all big houses from both cities, Phoenix and Tucson $= F(Big, Arizona, Phoenix) \cup F(Big, Arizona, Tucson) = \{h_1, h_2, h_3, h_4, h_9, h_{10}\}$.

3 Study and results

In this section, we start with some fundamental concepts on which we base our determination of the degree of participation of women in the artisanal production association ASOPROMAHER.

Context (A_1): Context refers to the idea of an organization's capabilities to meet its goals and objectives, as well as the circumstances, time, place, people, or events involved.

Institutional Context (A_{11}): The institutional context plays an extremely important role in the internal and external development of the company. It can influence the focus of the organization, understood as the set of agents, activities, and mechanisms that regulate and support the activities and companies.

Entrepreneurship (A_2): It is an attempt to create new businesses or new companies, self-employment, new business organization, or expansion of an existing business, by an individual, a team, or an established business. Therefore entrepreneurship not only helps to generate more profits but also to create products that are different from the competition that cover satisfaction and a new form of added value.

Female entrepreneurship (A_{21}): refers to business activities in which women effectively participate in business innovation. In addition, entrepreneurs are considered to be people who create and manage companies for profit, development, and growth purposes. They are characterized by innovative and integrative behavior, based on the use of strategy in business organizations.

Decision (A_3): It is an information process developed by individuals or groups to solve problems and exploit organizational opportunities. Therefore it is vitally important to know that in this process the members select among alternatives those that meet the purpose, needs, and goals of the organization.

Skills (A_4): Skills have been considered habits or capacities; as morphological, structural, or mental properties associated with action and manipulation that require conscious control, manifested both in theory and in practice, subordinated to a purpose, and formed in actions that lead to the solution of problems to change reality. On the other hand, they can be grouped into one or different areas of aptitudes, dexterity, and intelligence.

Knowledge (A_{41}): It is one of the most important human skills because it allows us to judge the nature, relationships, and properties of the things around us. It is also essential to develop critical thinking, problem-solving, and creativity. Therefore it is important for every human being to always know because it will be useful when creating or undertaking some idea in the course of their life.

Autonomy (A_{42}): It is a specific human ability, which allows adjusting the behavior of an individual or a social being. However, it is made very clear that autonomy allows a person to act within certain limits that are attributed to norms and regulations. In addition, it is the imperative capacity that any business has to satisfy its operational and financial needs.

Proactivity (A_{43}): It refers to a collection of different behaviors that are motivated by change and directed at the environment to achieve specific individual, group, or organizational objectives. Therefore being proactive takes time because it involves weighing several options, and possible alternatives and making our own decisions to achieve our objectives.

Initiative (A_{44}): Involves paying attention to a dynamic process in which individuals recognize opportunities for innovation and act by converting ideas into practical activities within a social, cultural, and economic environment in which they can cultivate their entrepreneurial spirit.

Anticipation (A_{45}): Refers to the ability to foresee possible future events and act before they occur, alerting about current events. The importance of this is to try to explore and forecast the future to identify potential risks and opportunities that we may face, helping us to make decisions, plan in advance, and carry out operations so that we can achieve our planned goals and safeguard the interests of the organization.

Freedom (A_{46}): This means the ability to make decisions and act without external pressures. This is based on the knowledge of the needs of nature, control over oneself and external nature; while for organizations, freedom lies in forming or dissolving, without being committed to the limitations derived from social functions that must be considered an integral part of their configuration.

To evaluate (A_5): Ultimately, it is the center of the entire system, whose theory and practice is based on valuation, that is, assigning value to things, phenomena, or people. Similarly, corporate values are the characteristics that define a company and serve as the axis of its growth.

Normative (A_{51}): They are the degree to which the inhabitants of a given country evaluate and value both entrepreneurial activity and the creative and innovative mindset of people. On the other hand, some norms fit a prescriptive, evaluative, and mandatory dimension in social life. Understanding how laws and regulations can affect the economic and social development of a nation is crucial in terms of their influence on entrepreneurial and innovative culture. It is a crucial factor in political decision-making and the formulation of policies that foster innovation and entrepreneurship.

Innovation (A_{511}): It is a key process for companies since it can create competitive advantages by introducing new or improved products and services to the market. In addition, innovation is based on production and organizational efficiency, specifically on improvements within production processes, basically in production and delivery. Perhaps the most promising part of the innovation process is turning ideas into concepts.

Regulation (A_{52}): They are the different general rules or specific actions applied by entities that are part of the Public Administration. These interfere directly in the market price allocation mechanism, and indirectly in the supply and demand decisions of consumers and producers. In addition, these rules preserve social, economic, political, and technological aspects related to the public interest.

Cognitive (A_6): Includes the processes of acquiring, transforming, organizing, storing, retrieving, and using information. Agents actively acquire information from the environment, process it, and use it to acquire new knowledge and then act. They are also closely related to intelligence, learning, and experience through which people can grow and learn to perform complex tasks.

Creativity (A_{61})**:** It is a human potential, composed of cognitive, emotional, intellectual, and will components, which manifests itself in a creative environment, to create new products of great social value and communicate them at certain moments in the social and historical context of life. It is worth mentioning that it is also the capacity of employees to generate ideas develop new channels, technologies, and products, and achieve visual perception.

Laws (A_7): Laws are legal measures that organize the facts and regulate the behavior of people and situations that live within the same environment, and also establish the operation of institutions such as schools, hospitals, civil registry offices, and the powers of state and government institutions.

Legal basis (A_{71}): A legislative framework is taken into account that supports the growth of companies or organizations; which includes relevant aspects such as the creation of solidarity investment funds, a set of policies, the promotion of social innovation, and the exploitation of popular and solidarity economies.

Policies (A_8): These are the different processes of actions and decisions that the state takes to change or modify a specific situation, to respond to different segments of the population. Also, policies are the responses of the state to certain interest groups in the same way they give solvency to different matters of institutional or social order. In addition, they help to develop individual capacities and guarantee the regulation of an environment.

Advantage (A_9): It can be interpreted as the implementation of strategies, that is, implementing a tactic or technique that creates value through the development of internal and external competencies that are competitive and unattainable for competitors, so that through this the company can become a leader in its managed industry, thus achieving high profitability.

New products (A_{10}): The development of a new product or service is a systemic task that requires constant engineering, marketing, and commercialization skills. Therefore, all members of the organization must be involved in the production of other elements that modify an existing product or create completely new and original products; to compete with other companies based on innovation; but this requires a proactive business culture.

Figure 2 contains the graph tree representing this TreeSoft set.

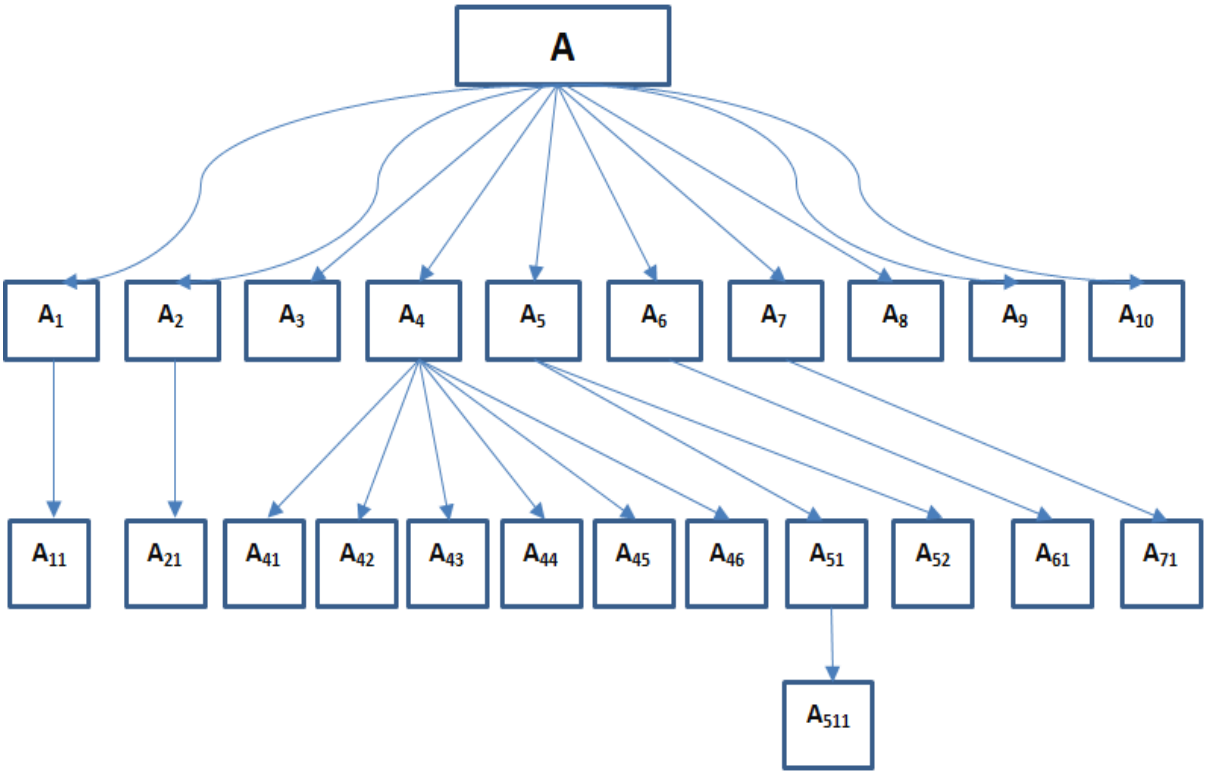

Figure 2: TreeSoft Set of the study carried out.

To perform the calculations we designed the following algorithm:

1. There were ten experts, let us denote them by the set: $E = \{e_1, e_2, \cdots, e_{10}\}$.
2. Each of them evaluates the 125 members (companies) of the artisanal production association ASOPROMAHER, according to those that meet each of the criteria, sub-criteria, and sub-sub-criterion that appears in each branch of the graph tree. That is, if $M = \{m_1, m_2, \cdots, m_{10}\}$ denotes the set of members of the association, then those that satisfy them are selected:

   $F_i(A_1, A_{11}) \subset M$, $F_i(A_2, A_{21}) \subset M$, $F_i(A_3) \subset M$, $F_i(A_4, A_{41}) \subset M$, $F_i(A_4, A_{42}) \subset M$, …, $F_i(A_4, A_{46}) \subset M$, $F_i(A_5, A_{51}, A_{511}) \subset M$, $F_i(A_5, A_{52}) \subset M$, $F_i(A_6, A_{61}) \subset M$, $F_i(A_7, A_{71}) \subset M$, $F_i(A_8) \subset M$, $F_i(A_9) \subset M$, and $F_i(A_{10}) \subset M$.

   Here the ith subscript denotes the evaluation of that expert, with $i = \{1,2,\cdots,10\}$.
3. The intersection sets of each of the expert sets are calculated and this is the final value which is denoted by F, for example, $F(A_1, A_{11}) = \cap_i F_i(A_1, A_{11})$, $F(A_2, A_{21}) = \cap_i F_i(A_2, A_{21})$, and so on.
4. The cardinality of each of the above sets is calculated and divided by 125. In this way, each

element of the lowest level within the tree has an associated number between 0 and 1, where 0 denotes that no company satisfies that combination of criterion, sub-criterion, and sub-sub-criterion, according to each case; while the value 1 means that all of them satisfy it for all experts.

If we denote by $S = \{p_1, p_2, \cdots, p_{16}\}$ each of the combinations evaluated, then the results were the following as shown in Table 1:

Table 1. Results of the evaluations of the 12 combinations carried out by the experts.

Combination	Algorithm result
p_1	0.64
p_2	0.336
P_3	0.216
p_4	0.6
p_5	0.112
p_6	0.56
p_7	0.184
p_8	0.44
p_9	0.616
p_{10}	0.296
p_{11}	0.448
p_{12}	0.704
p_{13}	0.536
p_{14}	0.336
p_{15}	0.288
p_{16}	0.888

As can be seen, the worst results are for p_2, p_3, p_5, p_7, p_8, p_{10}, p_{11}, p_{13}, p_{14}, and p_{15}, which are lower than 0.56. This means that Female Entrepreneurship, Autonomy, Decision, Initiative, Anticipation, Innovation, Regulation, Policies, and Advantage are the elements of the companies in the association in the worst conditions.

4. Conclusion

Female entrepreneurship in Ecuador is limited, as is the limited capacity of women to develop in a predominantly male and patriarchal environment. Above all, women's entry into the business world is still frowned upon, as this is a male-dominated space par excellence. This article consists of a study of this situation in the 125 member companies of the artisanal production association ASOPROMAHER. To do so, we use the tool known as TreeSoft Set, which is used to create tree-shaped representations when there are cases that need to be evaluated according to criteria, sub-criteria, sub-sub-criteria, and so on. Additionally, we propose an algorithm that is useful for determining to what extent each of the possible combinations is fulfilled, where the nodes of the tree that cross all levels intervene, according

to the evaluation of 10 experts on the subject.

The result was that there are problems in Female Entrepreneurship, Autonomy, Decision, Initiative, Anticipation, Innovation, Regulation, Policies, and Advantage, with less than 56% of companies evaluated as successful by the 10 experts. On the other hand, Institutional Context, Knowledge, Proactivity, Freedom, Creativity, Legal basis, and New products obtained results of more than 56%. Note that female entrepreneurship is below what is desired.

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Impact of Irrigation Water Technification on Seven Directories of the San Juan-Patoa River Using Plithogenic n-SuperHyperGraphs Based on Environmental Indicators in the Canton of Pujilí, 2021

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Abstract. The study analyzed interactions among economic, environmental, and sociocultural factors in agricultural irrigation systems to evaluate and promote integral sustainability, using the application of plithogenic n-SuperHyperGraphs. Significant interactions between vertex attributes were identified during the study's development. Results highlighted that optimizing economic profitability, implementing ecological pest management, and fostering social cohesion are essential for improving system sustainability. It is concluded that an interdisciplinary approach is critical for achieving comprehensive agricultural sustainability, proposing an evaluative model adaptable to local settings to strengthen sector resilience. Additionally, the application of plithogenic n-SuperHyperGraphs provides a useful tool across various study areas for management and decision-making in diverse agricultural environments.

Keywords: Agricultural sustainability, irrigation systems, plithogenic n-SuperHyperGraphs, food sovereignty

1 Introduction

The Pujilí Canton, located in the Cotopaxi Province, has a predominantly agricultural economy and a population of 458,581 inhabitants. In this region, land use is primarily determined by altitude, which influences the predominant crops in each area [1]. At lower altitudes (2200-2400 m.a.s.l. (meters above sea level)), sugar cane, fruit trees, and tomatoes are grown, while at higher altitudes (2400-3200 m.a.s.l.), maize, beans, wheat, barley, and tubers are the main crops [2]. Over the past two decades, livestock farming has expanded, primarily affecting the Andean forest areas. As a result, 42.88% of agricultural land has been allocated to pastures, 38.26% to short-cycle crops, and 18.86% to permanent crops, such as temperate fruits, sugar cane, and coffee [3].

Despite the agricultural importance of the region, there is a significant lack of previous studies on irrigation systems in the La Matriz parish, where the seven directories of the San Juan-Patoa River are located [4]. This gap limits the knowledge of producers regarding the potentials and deficiencies of their irrigation systems, hindering the continuous improvement of agricultural practices [5]. Hence, there is an urgent need to apply advanced tools for decision-making in the management of water resources and the sustainability of irrigation systems [6].

On the other hand, the low agricultural productivity in the region is attributed to the limited application of cultivation techniques and the lack of strategies to incorporate ecological technologies into the production processes. Additionally, the lack of awareness of environmental factors, such as the deterioration of local biodiversity, negatively affects productivity by contributing to land abandonment and degradation [7]. Users of the San Juan-Patoa River irrigation system need to understand the current state of agricultural management in the region to assess the sustainability of their practices and make

informed investments [8].

To address this issue, the research proposes the application of plithogenic n-SuperHyperGraphs, an advanced mathematical tool that models complex interactions between social, economic, and environmental factors. This approach incorporates and considers the uncertainties and indeterminacies inherent to the agricultural system in the region. N-SuperHyperGraphs allow for the combination of vertices to form SuperVertices, organizing information into categories of known, indeterminate, and unknown, thus facilitating the representation of complex and contradictory relationships within the irrigation system [9]. This approach is essential due to the uncertain nature of the problem being addressed and the need to use statistics to make informed decisions.

Moreover, the use of Plithogeny, developed by Smarandache, provides the appropriate theoretical framework to analyze irrigation systems by integrating multiple perspectives of uncertainty. By combining n-SuperHyperGraphs with log-linear statistical models applied to contingency tables [10], the research aims to identify patterns and relationships that affect the sustainability of irrigation systems in the region [11]. This approach enables a more precise evaluation of the viability of agricultural practices such as crop rotation, soil conservation, and the adoption of environmentally responsible ecological technologies.

Therefore, the main objective of this research is to analyze the interaction of economic, environmental, and sociocultural vertices in agricultural irrigation systems to evaluate and promote integral sustainability [12] [13], through the application of plithogenic n-SuperHyperGraphs. This comprehensive analysis is essential to provide farmers with the necessary tools to improve productivity, promote cooperation, and access new markets, thereby contributing to the sustainable development of the region.

2 Materials and Methods

This section contains two sub-sections, the first one is dedicated to explaining the basic notions of Plithogenic n-SuperHyperGraphs defined in[14]. Then, subsection 2.2 contains the main concepts of multi-way contingency tables and the log-linear method.

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in[15] [16]

First, an n-SuperHyperGraph is defined as follows:

Given $V = \{V_1, V_2, \cdots, V_{\mathrm{m}}\}$, where $1 \leq \mathrm{m} \leq \infty$ is a set of vertices, containing *Single Vertices* that are classical, *Indeterminate Vertices* that are unclear, vague, partially known, and the *Null Vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq \mathrm{n} \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq \mathrm{n} \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1) - SuperVertex,* and the others can be *r- SuperVertex* for $r \leq \mathrm{n}$.

E_n contains the following types of edges:

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n-SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

Within the framework of the theory of Plithogenic n-SuperHyperGraphs, there are the following concepts:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervention,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights.

2.2 Multi-Way Contingency Tables

A multi-way contingency table is a contingency table defined for two or more cross-ratio classification variables. Two-dimensional tables are generally called contingency tables, while the term multi- is applied when the number of variables is at least three [17].

A *Generic Multi-Way Table* is defined using $I = I_1 \times I_2 \cdots \times I_q$ as the set of indices for each variable to be studied $X_1, X_2, \cdots, X_q$, such that I_j is the set of indices corresponding to the possible classifications of the jth variable. Thus, $n_{i_1 i_2 \cdots i_q}$ is the frequency of occurrence of the classifications $i_1, i_2, \cdots, i_q$ for each of the corresponding variables.

The *Partial/Conditional Tables* involve fixing the category of one of the variables. Fixed variables are denoted in parentheses. For example, the XZ and YZ-partial tables are denoted by $n_{i(j)k}$ and $n_{(i)jk}$, respectively. Also, the *Partial/Conditional Probabilities* are calculated by $\pi_{ij(k)} = \pi_{\frac{ij}{k}} = Prob\left(X = i, Y = \frac{j}{Z} = k\right)$. *Partial/Conditional Proportions* are defined by $p_{ij(k)} = p_{\frac{ij}{k}} = \frac{\pi_{ijk}}{\pi_{++k}}$ for $k = 1, 2, \ldots, K$. Where π_{++k} is the frequency for i and j setting k, for more information see.

Below, there is a brief explanation of what log-linear models consist of. To simplify the exposition, the case of the three-way contingency table is chosen. If X, Y, and Z are the variables, then the following possible models are obtained [16, 17]:

- Model (X, Y, Z): All variables are considered independent, the model is as follows:

$$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z \quad (1)$$

- Model (X, YZ): Only the YZ association is considered, while X is independent of the other two variables.
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{jk}^{YZ} \quad (2)$$
- Model (XY, YZ): X and Z are independent for each value of Y:
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ} \quad (3)$$
- Model (XY, YZ, XZ): There is a pair-wise association between all variables, but there is no joint association between the three.
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} \quad (4)$$
- Model (XYZ): If the above model does not fit the data well, then the association between the three variables should be considered:
  $$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ} \quad (5)$$

To contrast two different models, the statistic called *likelihood ratio is used*, which is calculated as:

$$G^2 = 2\sum f ln\left(\frac{f}{F}\right) \quad (6)$$

Where f is the observed frequency and F is the expected frequency according to the model. This statistic is distributed according to a chi-square under the hypothesis that the model is correct, with degrees of freedom depending on the parameters used to fit the model.

3 The Study

Sample: For the selection of the sample, a proportional stratified random sampling method was used, ensuring adequate representation of users under different irrigation implementation conditions. The sample consisted of 60 agricultural production units (UPA), selected from a total of 569 UPAs in the target population. In this calculation, the following parameters were assumed: the probability of success (P) was set at 0.5, and the probability of error (Q) was also set at 0.5, ensuring an optimal confidence level for the representativeness of the sample.

Weighting and consensus of the results: The weighting of the parameters was carried out through a consensus process, in which experts in the field were consulted. They assigned a value to each attribute based on the relative relevance of the parameters considered in the selected indicators (see Tables 1 and 2).

Table 1: Assessment elements. Source: Own elaboration.

Scale	Assessment	Level of sustainability
0	Very critical or extreme level of sustainability.	Extreme
1	Low or critical level of sustainability of production units.	Critical
2	Minimum sustainability threshold of production units.	Weak
3	Medium level of sustainability.	Medium
4	Maximum threshold at a high level of sustainability of production units.	High

Table 2: Sustainability levels. Source: Own elaboration.

Level of sustainability	Very critical	Low sustainability	Minimum sustainability	Medium sustainability	High sustainability
Decision criteria on a scale of 0 to 4	< a 1	1 to 2.4	2.41 to 2.9	2.91 to 3.4	3.41 to 3.9

Vertex, vertex attributes, vertex sub-attributes, and vertex sub sub-attributes: In this study, the vertices (V) are represented by the different dimensions influencing the irrigation systems in the Pujilí region, covering the economic, environmental, and socio-cultural areas. The dominant envelope of this problem is related to the attributes and sub-attributes of the following categories: economic, environmental, and socio-cultural, as well as their respective frequencies (see Table 3).

V_1 = Economic dimension

V_2 = Environmental dimension

V_3 = Socio-cultural dimension

Attribute sets = {Economic dimension, Environmental dimension, Socio-cultural dimension}.

Table 3: Absolute frequency obtained for each vertex, vertex attributes, and sub-attributes. Source: Own elaboration.

Vertex	Vertex attributes	Vertex sub-attributes	Vertex sub-sub-attributes	Frequency
Economic (V_1)	Food sovereignty (V_{11})	Extreme		3
		Critical		7
		Weak		25
		Medium		25
		High		0
	Economic profitability (V_{12})	Extreme		0
		Critical		3
		Weak		12
		Medium		30
		High		15
	Economic risk (V_{13})	Extreme		13
		Critical		14
		Weak		12
		Medium		15
		High		6
Environmental (V_2)	Conservation of soil life (V_{21})	Ground cover (V_{211})	Extreme	2
			Critical	2
			Weak	8
			Medium	35
			High	13
		Crop residue management (V_{212})	Extreme	0
			Critical	5
			Weak	11
			Medium	15
			High	29
		Crop diversification (V_{213})	Extreme	6
			Critical	11
			Weak	18
			Medium	15
			High	10
		Water supply (V_{214})	Extreme	2
			Critical	1
			Weak	18
			Medium	19

Vertex	Vertex attributes	Vertex sub-attributes	Vertex sub-sub-attributes	Frequency
			High	20
			Extreme	0
			Critical	13
		Air pollution level (V_{215})	Weak	17
			Medium	30
			High	0
			Extreme	1
			Critical	13
		Predominant slope (V_{221})	Weak	23
			Medium	15
			High	8
			Extreme	7
			Critical	13
		Vegetation cover (V_{222})	Weak	25
			Medium	12
	Risk of erosion (V_{22})		High	3
			Extreme	1
			Critical	2
		Soil conservation works (V_{223})	Weak	17
			Medium	25
			High	15
			Extreme	1
			Critical	11
		Soil typology (V_{224})	Weak	22
			Medium	10
			High	16
			Extreme	2
			Critical	12
		Spatial biodiversity (Biodiversity and crop use) (V_{231})	Weak	24
			Medium	16
			High	6
			Extreme	1
			Critical	11
		Temporary biodiversity (Use of agroforestry) (V_{232})	Weak	16
			Medium	16
	Biodiversity management (V_{23})		High	16
			Extreme	1
			Critical	1
		Ecological management of pests and diseases (V_{233})	Weak	2
			Medium	15
			High	41
			Extreme	3
			Critical	3
		Local or improved seed diversity (V_{234})	Weak	7
			Medium	10
			High	37
			Extreme	4

Vertex	Vertex attributes	Vertex sub-attributes	Vertex sub-sub-attributes	Frequency
Social-cultural (V_3)	Satisfaction of basic needs (V_{31})	Property Succession Management (V_{235})	Critical	13
			Weak	24
			Medium	17
			High	2
		Access to health and health coverage (V_{311})	Low coverage	7
			Medium coverage	28
			High coverage	25
		Housing (V_{312})	Critical conditions	5
			Poor conditions	30
			Optimal conditions	25
	Acceptability of the production system (V_{32})	Low acceptability		0
		Moderate acceptability		0
		High Acceptability		60
	Integration into organizational systems (V_{33})	Low		41
		Average		14
		High		5
	Ecological knowledge and awareness ()V_{34}	Low		41
		Average		10
		High		9
	Tourism potential ()V_{35}	Low		14
		Average		31
		High		15

Table 3 presents the sustainability of the agricultural production system in three main vertices: economic, environmental, and socio-cultural. In the economic vertex, sustainability shows strengths in profitability, while economic risk is concentrated at critical levels, suggesting financial vulnerabilities that require attention. Meanwhile, in the environmental vertex, good practices in soil conservation and waste management are observed, but risks of erosion and variability in water supply persist, as these factors compromise long-term sustainability. However, in the socio-cultural vertex, the high acceptability of the production system is an advantage, although low ecological awareness and weak organizational integration represent areas for improvement. These results indicate the need for specific interventions to strengthen sustainability in critical areas and maintain existing strengths, thus achieving a comprehensive balance that favors system resilience.

To simplify the method, three-way contingency tables were used, allowing the examination of how different attributes relate within the social, economic, and environmental vertices. In each case, the G^2 coefficient was calculated to evaluate the intensity of interactions between the vertex attributes and their impact on the sustainability of the irrigation systems. A summary of the results obtained from this analysis is presented below (see Table 4).

Table 4: G^2 result of the processed models. Source: Own elaboration.

Model	G^2	Model	G^2	Model	G^2
v11* v21* v211	0.53387767	v12* v21* v211	0.07178898	v13* v21* v211	0.51354438
v11* v21* v212	0.88022916	v12* v21* v212	0.3611417	v13* v21* v212	0.8633287
v11* v21* v213	0.32416714	v12* v21* v213	0.25033138	v13* v21* v213	0.14454299
v11* v21* v214	0.58257722	v12* v21* v214	0.21431717	v13* v21* v214	0.68953039

v11*	v21*	v215	0.13292064	v12*	v21*	v215	0.12674366	v13*	v21*	v214	0.40037058
v11*	v22*	v221	0.27624909	v12*	v22*	v221	0.18219062	v13*	v22*	v215	0.29821965
v11*	v22*	v222	0.20498465	v12*	v22*	v222	0.37478081	v13*	v22*	v221	0.07388026
Model			G^2	**Model**			G^2	**Model**			G^2
v11*	v22*	v223	0.43860016	v12*	v22*	v223	0.04848691	v13*	v22*	v222	0.60701032
v11*	v22*	v224	0.51594351	v12*	v22*	v224	0.34520369	v13*	v22*	v223	0.47576924
v11*	v23*	v231	0.19771223	v12*	v23*	v231	0.17077222	v13*	v23*	v224	0.22130034
v11*	v23*	v232	0.46845231	v12*	v23*	v232	0.20274568	v13*	v23*	v231	0.41933184
v11*	v23*	v233	1.4774075	v12*	v23*	v233	0.77877943	v13*	v23*	v232	1.32022421
v11*	v23*	v234	1.17201392	v12*	v23*	v234	0.70106735	v13*	v23*	v233	0.94952424
v11*	v23*	v235	0.10495879	v12*	v23*	v235	0.23693938	v13*	v23*	v234	0.10806506
v11*	v31*	v311	1.22739172	v12*	v31*	v311	0.49178911	v13*	v31*	v235	1.08216903
v11*	v31*	v312	1.23772781	v12*	v31*	v312	0.47163422	v13*	v31*	v311	1.13491694
v11*	v32		2.77258872	v12*	v32		1.97972803	v13*	v32		2.44333893
v11*	v33		1.25925443	v12*	v33		0.91251033	v13*	v33		0.79308002
v11*	v34		1.40994678	v12*	v34		1.01691775	v13*	v34		0.87986628
v11*	v35		1.04388895	v12*	v35		0.40624927	v13*	v35		0.85068087

The analysis of Table 4, based on the G^2 coefficient, shows key interactions between economic, environmental, and socio-cultural factors in agricultural irrigation systems. These interactions highlight that strengthening biological pest management and biodiversity can optimize both food sovereignty and economic profitability. Improving biodiversity and succession planning reduces economic risk and promotes system resilience. Additionally, the high acceptability of the production system by the community emphasizes the need for sustainable practices that foster long-term social cohesion.

Regarding the sustainability vertices, the socio-cultural vertex achieves the highest sustainability index with a value of 2.70, highlighting social acceptance and access to essential services as pillars of sustainability in the agricultural system. It is followed by the environmental vertex with 2.63, reflecting progress in soil and biodiversity management, though with room for improvement. Finally, the economic vertex shows the lowest value (2.31), indicating the need to strengthen profitability and mitigate economic risks. Therefore, the vertices are within the sustainability acceptance zone.

Regarding the dominant attributes in each vertex, these include economic profitability in the economic vertex (2.95), ecological pest and disease management in the environmental vertex (3.57), and the acceptability of the production system in the socio-cultural vertex (4). These results indicate that, for integral sustainability, it is essential to balance the economic and environmental dimensions while strengthening social cohesion.

Measures to enhance sustainability in the San Juan-Patoa River sectors are:

- Economic: Promote the use of organic inputs and strengthen agroecological producer associations to optimize marketing and improve prices. Additionally, it is essential to strengthen institutions that support the agricultural sector and offer training in the economic-financial field.
- Environmental: Implement irrigation technology to prevent the loss of vegetation cover and train producers in environmental protection practices and agro-ecological production, while improving crop rotation to prevent soil erosion.
- Socio-cultural: Encourage community integration into organizational systems through the support of governmental institutions. Additionally, promote entrepreneurial projects to open new markets, create employment, and strengthen social and economic ties.

4. Conclusion

The study has demonstrated that in agricultural irrigation systems, the interaction of economic,

environmental, and socio-cultural factors is key to achieving sustainability. Through the use of plithogenic n-SuperHyperGraphs that combine main vertices, dominant attributes in each dimension were identified. The acceptability of the production system and ecological pest management, as well as biodiversity, were highlighted for their significant contribution to sustainable balance. The results have shown that, although economic profitability is fundamental, its effect is limited when evaluated without considering other factors, thus emphasizing the priority of an interdisciplinary approach in agricultural management. The developed methodology, represents an advance in the comprehensive evaluation of sustainability, providing a replicable and adaptable model to different local environments, and facilitating agricultural resilience in diverse study areas.

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Tree Tobacco Extract (Nicotiana glauca) as a Plithogenic Bioinsecticide Alternative for Controlling Fruit Fly (Drosophila immigrans) using n-SuperHyperGraphs

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Abstract. Concern regarding the negative effects of synthetic pesticides has driven the development of bioinsecticides as sustainable alternatives. Consequently, *Nicotiana glauca* has gained prominence for its insecticidal properties due to phytochemical compounds effective against pests such as *Drosophila immigrans*. This study, therefore, evaluated the efficacy of *Nicotiana glauca* extract as a bioinsecticide by optimizing extraction parameters and analyzing its insecticidal action under laboratory conditions across eight combined treatments. For this purpose, multiple experimental combinations were analyzed through Plithogenic n-SuperHyperGraphs, allowing a multidimensional evaluation of the attributes and properties of each treatment. Results indicated that treatment with 96% ethanol, 48 hours of maceration, and fresh leaves maximized insecticidal metabolites and pest mortality. It is concluded that *Nicotiana glauca* represents a promising bioinsecticide alternative for sustainable agriculture, with optimal treatments for pest control.

Keywords: treatment efficacy, tree tobacco, bioinsecticide, plithogenic n-SuperHyperGraphs.

1 Introduction

In recent years, concern has grown regarding the effects of synthetic insecticides on the health of farmers and the safety of both human and animal food consumers [1]. This has driven the development of naturally-derived toxins for pest control, offering effective botanical alternatives for pests that have developed resistance to synthetic compounds [2]. The repeated use of synthetic agrochemicals not only increases producers' costs but also poses a public health risk. Consequently, the search for more sustainable and affordable alternatives for producers has intensified, promoting environmentally friendly options.

Among new alternatives, natural compounds have proven effective in reducing contamination and improving the health, productivity, and economic and social sustainability of farmers [3]. Plants synthesize chemical compounds as a defense mechanism against pests, and over 2,000 plant species with insecticidal properties have been identified. In the Americas, wild tobacco (*Nicotiana glauca*) stands out for its tolerance to drought and soil salinity and contains anabasine, an alkaloid toxic to insects, presenting a viable option for the production of natural insecticides [4][5]. This compound, with an action similar to organophosphate and carbamate insecticides, is highly toxic due to rapid dermal absorption, thus its use is recommended in open spaces [6].

Generally, plant extracts are obtained using polar solvents such as water and ethanol, which are suitable for extracting secondary metabolites and ensure a low toxicity level in bioinsecticide bioassays [7]. Extracting these compounds is particularly relevant in fruit farming, where pest control—such as for fruit flies of the *Anastrepha* genus—is one of the primary productivity challenges [8]. This pest causes

crop losses that can reach between 30% and 50% in Latin America and up to 100% in some regions of Ecuador.

The fruit fly represents a global threat due to its adaptability and prolific life cycle, which can reach up to 12 generations per year under favorable conditions [9]. The larvae of this pest cause physical damage to the pulp, allowing microorganisms to enter, affecting post-harvest quality, and impeding fruit export due to phytosanitary restrictions [10]. These losses impact both local and international markets due to fruit decomposition [11], as fruits become colonized by fungi and bacteria after infestation.

Therefore, the present study focuses on evaluating the effectiveness of tree tobacco (*Nicotiana glauca*) extract as a bioinsecticide by optimizing extraction parameters and analyzing its composition and insecticidal action against *Drosophila immigrans*. To this purpose, eight treatments derived from combining the extract characteristics under laboratory conditions are proposed, along with the selection of the best treatment using Plithogenic n-SuperHyperGraphs to facilitate multidimensional evaluation based on the attributes and sub-attributes defined at each proposed vertex.

2 Materials and Methods

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in [12] [13]

First, an n-SuperHyperGraph is defined as follows:

Given $V = \{V_1, V_2, \cdots, V_{\mathrm{m}}\}$, where $1 \leq \mathrm{m} \leq \infty$ is a set of vertices, containing *Single Vertices* that are classical, *Indeterminate Vertices* that are unclear, vague, partially known, and the *Null Vertices* that are empty or completely unknown [14].

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq \mathrm{n} \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq \mathrm{n} \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1) - SuperVertex,* and the others can be *r- SuperVertex* for $r \leq \mathrm{n} - 1$.

E_n contains the following types of edges:

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),

- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

Within the framework of the theory of Plithogenic n-SuperHyperGraphs, there are the following concepts:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervene,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights.

3 The Study

Case study: This study evaluated the effectiveness of *Nicotiana glauca* (false tobacco) extracts for pest control by using fresh and dried leaves macerated in 96% ethanol and distilled water for 24 and 48 hours. The analysis of treatments was conducted using the plithogenic n-SuperHyperGraphs method, identifying the optimal combination to achieve the highest concentration of insecticidal metabolites. For this purpose, fly infestation was monitored in specific areas using traps and fruit sampling. Larvae were collected from infested guavas, and their development was managed under suitable conditions until reaching the adult stage.

Weighting and consensus of the results were achieved through the application of the Plithogenic n-SuperHyperGraphs method, which facilitates the multidimensional evaluation of treatments in complex studies. This method allowed for the integration and weighting of evaluations of multiple parameters, considering both treatment variability and the relative importance of each variable in the insecticidal effect (see Table 1).

Table 1: Plithogenic evaluation criteria. Source: Own elaboration.

Vertex Sub Sub-Attributes	Description	Plithogenic criterion (PC)
NO	The state is deficient, not meeting the basic requirements established.	Non-optimal
LO	State slightly below an acceptable standard, with evident issues.	Low
ST	Meets minimum quality standards, though still susceptible to variations.	Stable
GO	State above minimum, with stable and acceptable quality.	Good
OP	The highest level of quality, achieving ideal parameters without variation.	Optimal

Additionally, component analyses were conducted to identify and weigh the most significant variables, enabling greater precision in selecting the optimal treatment. Feedback from participating experts was incorporated through a consensus aggregation process to ensure the applicability of results in bioinsecticide applications.

In this study, the Plithogenic n-SuperHyperGraphs methodology allows for a detailed analysis of

the combinations of *Nicotiana glauca* extract characteristics to optimize its effectiveness as a bioinsecticide (see Table 2). The n-SuperVertices organize each main attribute (type of solvent, maceration time, and leaf condition) into SuperVertices and Vertices that specify experimental values (such as distilled water or 96% ethanol). The SuperVertex defines the treatments (T1 to T8) by representing specific interactions among these values; for example, T1 includes distilled water, 24 hours of maceration, and fresh leaves (see Table 3). This structure enables an analysis of variable interactions and facilitates the selection of treatment configurations, demonstrating potential for bioinsecticide use.

Table 2: Extract characteristics. Source: Own elaboration.

n-SuperVertex (NSV)	SuperVertex (SV)	Vertex (V)	Vertex Attributes (V)
Extract characteristics (V_1)	Solvent type ($SV_{1,2}$)	Distilled water (V_1)	Purity, pH, conductivity, temperature, and volume.
		Ethanol 96% (V_2)	pH, polarity, temperature, and volume
	Maceration time ($SV_{3,4}$)	24 hours (V_3)	Standard or non-standard conditions
		48 hours (V_4)	Standard or non-standard conditions
	Leaf status ($SV_{5,6}$)	Fresh (V_5)	Leaf moisture and integrity
		Dry (V_6)	Leaf moisture and integrity

Table 3: Proposed treatments derived from the grouping of Vertex into SuperVertex. Source: Own elaboration.

Treatments	HyperEdge (HE)	SuperVertex (SV)	Vertices
T1	$(HE_{1,3,5})$	$(SV_{1,3,5})$	$\{V_1, V_3, V_5\}$
T2	$(HE_{1,3,6})$	$(SV_{1,3,6})$	$\{V_1, V_3, V_6\}$
T3	$(HE_{1,4,5})$	$(SV_{1,4,5})$	$\{V_1, V_4, V_5\}$
T4	$(HE_{1,4,6})$	$(SV_{1,4,6})$	$\{V_1, V_4, V_6\}$
T5	$(HE_{2,3,5})$	$(SV_{2,3,5})$	$\{V_2, V_3, V_5\}$
T6	$(HE_{2,3,6})$	$(SV_{2,3,6})$	$\{V_2, V_3, V_6\}$
T7	$(HE_{2,4,5})$	$(SV_{2,4,5})$	$\{V_2, V_4, V_5\}$
T8	$(HE_{2,4,6})$	$(SV_{2,4,6})$	$\{V_2, V_4, V_6\}$

Interaction Analysis: The statistical results in Table 4 indicate a significant difference in the SuperVertex ($SV_{1,2}$) and in the interaction of the SuperVertices ($SV_{1,2} \leftrightarrow SV_{5,6}$). Meanwhile, the following SuperVertices (SV_{34}, $SV_{5,6}$) do not show significant differences, nor do the interactions between ($SV_{1,2} \boxleftrightarrow SV_{3,4}$) and ($SV_{34} \leftrightarrow SV_{5,6}$). Additionally, the interaction among the SuperVertices comprising the treatments ($SV_{1,2} \boxleftrightarrow SV_{3,4} \leftrightarrow SV_{5,6} \leftrightarrow SV_{1,2}$) shows no significant differences. Thus, a 2.40% coefficient of variation is obtained, indicating a low error rate in the study and demonstrating effective handling of the experimental data.

Table 4: ADEVA from the tobacco extracting tree (Nicotiana glauca). Source: Own elaboration.

VF	SC	GL	CENTIMETER	F	p Value	
$SV_{1,2}$	6905.08	1	6905.08	95703.84	<0.0001	**
SV_{34}	0.39	1	0.39	5.41	0.0233	ns
$SV_{5,6}$	0.01	1	0.01	0.09	0.7598	ns
$SV_{1,2} \leftrightarrow SV_{3,4}$	0.13	1	0.13	1.85	0.1787	ns
$SV_{1,2} \leftrightarrow SV_{5,6}$	4.55	1	4.55	63.06	<0.0001	**
$SV_{34} \leftrightarrow SV_{5,6}$	0.01	1	0.01	0.16	0.6943	ns
$SV_{1,2} \leftrightarrow SV_{3,4} \leftrightarrow SV_{5,6} \leftrightarrow SV_{1,2}$	0.39	1	0.39	5.41	0.0233	ns
REPLICA	1.72	2	0.86	11.95	<0.0001	**
ERROR	4.47	62	0.07			
TOTAL	6916.76	71				
% CV		2.40				

VF	SC	GL	CENTIMETER	F	p Value
** significant with p≤0.001, ns: not significant, CV: Coefficient of variation.					

Once the treatments have been defined, the most effective bioinsecticide is determined by measuring the new properties obtained and the connection through HyperEdges (HE). The following table shows the vertices, attributes, and sub-attributes to measure the effectiveness of the treatments (see Table 5). In addition, it can be observed that the sub-attributes V_{721}, V_{811}, and V_{821} are input envelope vertices, while V_{723}, V_{812}, $and\ V_{822}$ are the output envelope vertices. As for the rest of the envelope vertices, they are considered as intermediate.

Table 5: Vertices, attributes, and sub-attributes to measure the efficacy and resistance of the bioinsecticide. Source: Own elaboration.

Vertex	Vertex Attributes	Vertex Sub Attributes	Vertex Sub Sub-Attributes
Dosage and duration of treatment (V_7)	Extract concentration (dilutions in bioassays) (V_{71})	5% (V_{711})	• Plithogenic criterion (PC) (V_{7111})
		10% (V_{712})	• CP (V_{7121})
		20% (V_{713})	• CP (V_{7131})
	Application time (duration in minutes) (V_{72})	Initial phase: 3 to 10 minutes (V_{721})	• CP (V_{7211})
		Intermediate phase: 10 to 20 minutes (V_{722})	• CP (V_{7221})
		Advanced phase: 20 to 30 minutes (V_{723})	• CP (V_{7231})
Performance and resistance to bioinsecticide (V_8)	Pest mortality (V_{81})	Initial Phase (V_{811})	• Low (V_{8111}) • Moderate (V_{8112}) • High (V_{8113})
		Final Phase (V_{812})	• Low (V_{8121}) • Moderate (V_{8122}) • High (V_{8123})
	Tolerance levels (V_{82})	Initial Phase (V_{821})	• Low tolerance (V_{8211}) • Moderate tolerance (V_{8212}) • High tolerance (V_{8213})
		Final Phase (V_{822})	• Low tolerance (V_{8221}) • Moderate tolerance (V_{8222}) • High tolerance (V_{8223})

3.1 Analysis between treatments and n-SuperVertex V_7 and V_8.

The results in Table 6 indicate that treatment T8 presents the highest Brix concentration, making it the most suitable for application. The extract consists of 96% ethanol with a 48-hour maceration. Meanwhile, using fresh leaves, treatment T7, which also contains 96% ethanol with a 48-hour maceration, follows closely. The organic solvent ethanol is polar and shows a greater interaction with polar organic compounds compared to water.

Table 6: Tukey test at 5% extract. Source: Own elaboration.

Treatments	Mean	Ranks
T1	0.98	A
T2	1.33	AB
T3	1.62	BC
T4	1.73	C
T5	20.64	D

Treatments	Mean	Ranks
T6	20.88	DE
T7	21.19	EF
T8	21.30	F

Therefore, the particularity of the two mentioned treatments lies in the use of fresh leaves, which prove to be more effective than dried ones. The progressive drying of the plant removes the bitter taste and, in many cases, also the secondary metabolites. The analysis of the variance of dead flies is presented next in Table 7.

Table 7: ADEVA dead flies. Source: Own elaboration.

VF	SC	GL	CENTIMETER	F	p Value	
DOSE	135.85	2	67.92	28.89	0.0002	**
TIME	424.28	9	47.14	20.05	0.0001	**
DOSE*TIME	46.39	11	4.22	1.79	0.2082	ns
REPETITION	6.19	2	3.10	1.32	0.3202	ns
ERROR	18.81	8	2.35			
TOTAL	631.52	32				
% CV		19.46				
**significant with p≤0.001, ns: not significant, CV: Coefficient of variation.						

According to Table 8, there is a significant difference in "dose" and "time," while the interaction of "dose-time" is not significant. This demonstrates that the different doses of the tree tobacco extract do not affect the fly mortality time variable (Drosophila immigrans). The coefficient of variation is 19.46%, confirming that the development and process of the bioinsecticide regarding the "dose" and "time" parameters were correctly adapted to the study. A Tukey test at 5% was then performed for the dose variable, as shown below in Table 8.

Table 8: Tukey test at 5% for the dose. Source: Own elaboration.

Dose	Mean	Ranks
5%	4.67	C
10%	8.50	BC
20%	9.67	A

Table 8 shows three ranges of significance, where the 20% concentration yields the best results in controlling the attack of the fly (Drosophila immigrans), followed by the 10% dose, and the statistically lowest range corresponds to the 5% concentration. The study determined that higher concentration results in greater effectiveness of the bioinsecticide. Previous studies mention that, after 24 hours of aqueous application of tobacco (N. tabacum) at a 10% concentration, 50% of the flies are killed. This indicates that the increase in concentration is directly proportional to fly mortality.

Within the results of Table 9, 10 different "application times" of the tree tobacco bioinsecticide extract are presented, showing 4 ranges of significance, with the most significant being 20 minutes (A). The times of 2, 9, and 15 minutes show no difference among them, and shorter times exhibit a low probability of mortality.

Table 9: Tukey test at 5% for time. Source: Own elaboration.

Time	Mean	Ranks
3	3.50	D
4	5.00	D

Time	Mean	Ranks
5	5.20	D
6	5.50	D
8	5.60	D
12	6.75	CD
9	9.00	ABC
15	11.67	BC
19	13.00	AB
20	17.50	A

Once the relationships and interactions of each vertex represented in Table 5 have been defined, the evaluations for each treatment are represented. These treatments are conceptualized as the union of vertices that form SuperVertices and n-SuperVertices within the Plithogenic n-SuperHyperGraphs, allowing each treatment to be represented as a complex system. This system groups attributes and sub-attributes, such as extract concentration, application time, pest mortality, and tolerance levels, facilitating the analysis of the interactions between these elements and their contribution to overall performance. The treatments are configured as composite entities, whose performance depends on the interaction of their specific qualities. Through the union of several SuperVertices, n-SuperVertices are formed, adding complexity to the structure and allowing exploration of how different treatment configurations complement or compete according to the evaluated criteria (see Table 10).

Table 10: Treatment evaluation based on the connection between vertices, attributes, and sub-attributes. Source: Own elaboration.

Vertex Attributes	Vertex Sub Attributes	Vertex Sub Sub-Attributes	T1	T2	T3	T4	T5	T6	T7	T8
Extract concentration (V_{71})	5% (V_{711})	CP (V_{7111})	NO	NO	NO	NO	NO	NO	BA	BA
	10% (V_{712})	CP (V_{7121})	ES	ES	ES	ES	ES	ES	ES	ES
	20% (V_{713})	CP (V_{7131})	BU	BU	BU	ES	ES	ES	BU	OP
Application time (V_{72})	Initial phase: 3 to 10 minutes (V_{721})	CP (V_{7211})	BA	BA	ES	ES	ES	ES	OP	BU
	Intermediate phase: 10 to 20 minutes (V_{722})	CP (V_{7221})	ES	ES	ES	BU	BU	BU	OP	OP
	Advanced phase: 20 to 30 minutes (V_{723})	CP (V_{7231})	OP	OP	OP	OP	OP	OP	OP	OP
Pest mortality (V_{81})	Initial Phase (V_{811})	Low (V_{8111})	BA	BA	BA	BA	BA	BA	ES	ES
		Moderate (V_{8112})	ES	ES	ES	ES	ES	ES	ES	BU
		High (V_{8113})	BU	BU	BU	BU	BU	BU	OP	OP
	Final Phase (V_{812})	Low (V_{8121})	BA	BA	BA	BA	BA	BA	OP	BU
		Moderate (V_{8122})	ES	ES	ES	ES	ES	ES	OP	BU
		High (V_{8123})	BU	BU	BU	BU	OP	OP	OP	OP
Tolerance levels (V_{82})	Initial Phase (V_{821})	Low tolerance (V_{8211})	B	B	B	B				
		Moderate tolerance (V_{8212})					M	M	M	M
		High tolerance (V_{8213})								
	Final Phase (V_{822})	Low tolerance (V_{8221})								
		Moderate tolerance (V_{8222})	M	M		M				
		High tolerance (V_{8223})			A		A		A	A

The connections between SuperVertices and n-SuperVertices are established based on the affinity of their attributes, revealing patterns of efficacy or compatibility between treatments. This approach facilitates the identification of treatment groupings with high effectiveness under specific conditions. Mean-

while, the plithogenic connections, highlighting the extract concentration, application time, pest mortality, and tolerance, emphasize that T7 is the optimal treatment, followed by T8. Both treatments stand out due to the optimization of quality and effectiveness criteria, supporting their selection as the most suitable based on their specific interactions and overall performance.

Plant extracts, such as the ethanolic extract of Nicotiana glauca (T7), present key physicochemical characteristics for evaluating their quality. In this case, the total solids reach 20.25%, the pH is 5.92, the acidity is 0.05%, and the density is 0.8467 g/mL, meeting the established parameters. The phytochemical profile highlights the predominant presence of alkaloids and phenolic compounds, with pharmacological and antioxidant properties, suggesting their potential as a natural insecticide and for health purposes. Factors such as genetics, environmental conditions, and the extraction process influence the quality of bioactive compounds. Thus, the need to control these aspects to obtain high-quality extracts, essential for various industrial applications, is emphasized. Regarding maceration time, this should be considered in future research, as appropriate maceration for a bioinsecticide is generally done over a period of six days, during which the plant's secondary metabolites remain unchanged. Furthermore, although maceration is an efficient process for extracting active principles from plants, its effectiveness decreases significantly over time.

4. Conclusion

This study demonstrated the effectiveness of Nicotiana glauca extracts as bioinsecticides, highlighting that the combination of 96% ethanol, 48 hours of maceration, and fresh leaves optimized the concentration of insecticidal metabolites, thereby maximizing pest mortality. Similarly, the use of the Plithogenic n-SuperHyperGraphs method allowed for precise analysis of the interactions between experimental variables and the selection of optimal configurations, confirming that the solvent and maceration time are dominant enveloping vertices in the efficacy of the extracts. These results emphasize the potential of Nicotiana glauca as a sustainable and effective alternative in biological pest control. Furthermore, it is suggested that future research evaluate its applicability under field conditions and for other pest species, promoting its integration into sustainable agricultural strategies.

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Analysis of the Impact of Soft Skills on Organizational Competitiveness in Cotopaxi through Modeling of the Plithogenic IADOV

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Abstract. Training plays an important role in the development of skills and organizational competitiveness. In Cotopaxi, a misalignment was identified between employees and organizational objectives concerning organizational performance and human development. Consequently, this study analyzed the influence of soft skills, the elimination of skill gaps, adherence to standards, and organizational alignment on job satisfaction and performance, aiming to enhance organizational competitiveness. The plithogenic IADOV method was applied to evaluate satisfaction levels and the interrelationships among these dimensions. Findings indicated significant dissatisfaction among employees across all assessed dimensions, underscoring the urgent need for training programs focused on soft skills. It was concluded that closing existing skill gaps and fostering the development of soft skills are essential steps toward reducing training needs and enhancing both organizational performance and competitiveness.

Keywords: job performance, skill development, job competencies, talent management, plithogenic IADOV.

1 Introduction

In today's business environment, characterized by high competitiveness and constant change [1], human talent training is considered a critical factor for maintaining competitiveness in productive sectors [2] [3]. Organizations recognize that success significantly depends on the development of their collaborators, who must acquire new skills and competencies to meet the dynamic challenges of the work environment [4]. Training contributes not only to employees' professional growth but also supports career planning and performance evaluation, making it a fundamental process in human resource management.

Training, as a cornerstone of organizational development, seeks to continuously enhance individual competencies, positively impacting the organization's overall performance [5]. However, this process entails inherent challenges due to the need to balance organizational goals, departmental objectives, and individual aspirations. This complexity underscores the need to align training programs with organizational objectives to maximize relevance and effectiveness. In this way, the integral growth of the organization is promoted, and productivity is increased.

In productive sectors, an adequate identification of training needs enables companies to improve work quality, increase job satisfaction, and retain talent, while employees acquire updated and relevant skills, enhancing their employability. Despite these benefits, many organizations lack a deep understanding of their collaborators' development needs. This gap hinders the design of programs that truly address skill and competency gaps, thus limiting the effectiveness of training efforts [6].

Employees' perception of their own training needs is crucial for a successful organizational development strategy [7]. In Ecuador, although training is recognized as an essential tool, individual employee perceptions are sometimes overlooked, resulting in centrally designed training programs that are disconnected from the specific needs of each worker. Identifying these needs, therefore, is an essential step to ensure that companies and their employees are well-equipped to face the challenges of a continuously evolving work environment.

The objective of identifying training needs is to increase productivity through human capital development by enabling training programs to meet the requirements demanded by each job position. This identification occurs at two levels: first, assessing the minimum knowledge and skills required for performing a specific function, and second, comparing these aspects with job requirements over various timeframes. Alignment between training needs and employees' perception of their skill gaps is crucial in designing effective programs [8].

Employees, being at the forefront of implementation, face the challenges of meeting organizational objectives, bridging skill gaps, and complying with external regulations. Their perception of these gaps and difficulties provides invaluable insight for identifying areas where training would be particularly beneficial. Additionally, each employee's professional aspirations should be considered to adapt training programs and ensure alignment with both individual growth objectives and organizational goals [9].

To optimize resources, the approach to identifying training needs must be systematic. This involves carefully planning training activities and conducting a comprehensive analysis of the areas requiring development. In this way, unnecessary training sessions are avoided, and key areas with a direct impact on organizational outcomes are addressed. A rigorous analysis allows for the prioritization of training efforts and maximizes their impact on organizational efficiency and performance [10].

Consequently, the objective of this research is to analyze how soft skills [11], the elimination of skill gaps, compliance with standards, and alignment with organizational goals influence employee satisfaction and job performance in the productive sectors of the Cotopaxi province. Additionally, this study aims to identify areas for improvement in training programs and to strengthen organizational competitiveness in the productive sector [12]. To achieve this, the plithogenic IADOV method was applied to evaluate the relationships between various dimensions and their influence on work outcomes.

2 Materials and Methods

2.1 Plithogenic IADOV

The plithogenic IADOV method is an evaluation method used to measure respondents' satisfaction. By extending this technique to plithogenic analysis and employing a neutrosophic scale [13], the ability to measure indeterminate or inaccessible aspects through conventional methods is introduced. Mathematical modeling through plithogenic logic enables the inclusion of indeterminacy and contradiction in the evaluation of sets and systems. To understand this extended method [14], it is essential to define what constitutes a plithogenic set:

Plithogenic set [15]: An extension of classical, fuzzy, intuitionistic fuzzy, and neutrosophic sets. A plithogenic set (P,a,V,d,c):

- Where "P" represents a set, "a" is an attribute (generally multidimensional), "V" is the range of values for the attribute, and "d" is the degree of membership of each element x's attribute value within set P according to certain criteria. The term "d" is defined as a fuzzy membership degree d_F, an intuitionistic fuzzy membership d_{IF}, or a neutrosophic membership degree d_N, respectively, for an element x in the plithogenic set P; $x \in P$.
- "c" is defined as the contradiction degree function c_F of a fuzzy attribute value, the contradiction degree function c_{IF} of an intuitionistic fuzzy attribute value, or the contradiction degree function c_N of a neutrosophic attribute value, respectively [16].
- The functions are defined according to the applications that experts need to address. $d(\cdot,\cdot)$

and c(·,·) then use the following notation: x(d(x,V)), where d(x,V) = {d(x,v), for all v ∈ V}, for all x ∈ P. The contradiction degree function for an attribute value is calculated between each attribute value with respect to the dominant attribute value (denoted by v_D) in particular, as well as for other attribute values v.

Thus, the Plithogenic IADOV method allows for addressing the complexity of respondents' perceptions. This requires a linguistic evaluation system adapted to the plithogenic model to accurately capture expert opinions (see Table 1). This system and its equivalents on a plithogenic scale are defined as the T-score function of a neutrosophic number, according to Smarandache's proposed formula, as shown in equation (1) [17].

$$F(T) = \frac{T + F - I}{2} \tag{1}$$

Table 1: Evaluation system for experts. Source: Own elaboration.

Linguistic term of the plithogenic IADOV	SVNN	$\mathcal{S}([T, I, F])$	w_j	IADOV
Extremely Satisfied (ES)	(1,0,0)	1.00	1.00	Clearly Satisfied
Very Very satisfied (VVS)	(0.95,0.15,0.1)	0.90	-	
Very satisfied (MS)	(0.85,0.25,0.2)	0.80	-	
Satisfied (S)	(0.75,0.35,0.3)	0.70	0.70	More satisfied than dissatisfied
Moderately satisfied (MDS)	(0.65,0.45,0.4)	0.60	-	-
Undefined (U)	(0.55,0.55,0.5)	0.50	0.5	Not defined (ND)
Moderately Dissatisfied (MDD)	(0.45,0.65,0.6)	0.40	-	-
Dissatisfied (D)	(0.35,0.75,0.7)	0.30	0.30	More dissatisfied than satisfied
Very Dissatisfied (VD)	(0.25,0.85,0.8)	0.20	-	
Very Very Dissatisfied (VVD)	(0,0.95,1)	0.05	-1.00	Clearly dissatisfied
Contradictory (C)	(1,0,1)	0	0	Contradictory

Another component specific to the method is the IADOV logic table, where the term "I" in neutrosophy is interpreted as a unit of indeterminacy. This table assigns numerical values to the closed-ended questions posed to experts. Additionally, if necessary, open-ended questions are included in the surveys. Among the questions used in this study are:

- Are you satisfied with the level of alignment between organizational objectives and your daily work within the company?
- Are you satisfied with the actions taken by your company over the past year to reduce skill gaps among employees?
- Are you satisfied with the level of compliance with standards within your organization and the support provided to follow them adequately?
- Are you satisfied with the development of soft skills within your organization and its impact on the work environment and performance?
- From your perspective, what specific actions could be taken to improve soft skill development, better align organizational objectives, optimize compliance with standards, and eliminate performance gaps within the company?

To calculate the Neutrosophic Plithogenic Global Satisfaction Index (NPGSI) of the respondents, the aggregation operator H_N^P was used, taking into account the evaluations of each element X in the plithogenic set P; $x \in Pd_F\, d_{IF}\, d_N$. In this way, the NPGSI is obtained from the sum of the elements analyzed within the evaluated plithogenic subset, where L_r^P represents the frequency, and w_j is the assigned weight (see Equation 2).

$$H_N^P\left(S_{1,}^P S_2^P, \ldots, S_n^P\right) = \sum_{i=1}^{n} \left[w_j\, , L_r^P\right] \tag{2}$$

3 The Study

This study employed a plithogenic scale using the plithogenic IADOV method to assess training needs in companies in Cotopaxi, focusing on employees in administrative roles. The questionnaire underwent internal validation through a pilot test (45 participants), yielding a global Cronbach's alpha coefficient of 0.854, indicating high reliability. Data adequacy was evaluated using the KMO index, which resulted in a value of 0.774, and Bartlett's sphericity test, with a p-value of 0.000, confirming the feasibility of the analysis. This analysis revealed commonalities above 0.5, validating the questionnaire dimensions as appropriate. Table 2 presents the characteristics of the analyzed plithogenic set.

Table 2: Characteristics of the Plithogenic set. Source: Own elaboration.

Plithogenic set:	**Organizational performance and human development,** $\forall P_{S_n} = \{S_{V_1}, S_{V_2}, \dots, S_{V_n}\}$
Plithogenic subsets (Dimensions)	• Organizational objectives (S_1). • Gap elimination (S_2). • Compliance with standards (S_3). • Soft skills (S_4).
Attributes:	Acceptance dimension, $\forall S_{V_{1n}} = \{S_{V_1}, S_{V_2}, S_{V_3}, S_{V_4}\}$
Variable:	Level of satisfaction of each plithogenic dimension.
Measuring scale	Plithogenic linguistic term (See Table 1).

The results of the group satisfaction analysis indicate participants' perceptions in relation to the four evaluated dimensions: organizational objectives, gap elimination, compliance with standards, and soft skills. The Group Satisfaction Index (GSI), calculated for each dimension and compared with the plithogenic IADOV evaluation system, provides an accurate view of overall satisfaction.

Among these, organizational objectives obtained a GSI of 0.75, which falls within the "Satisfied" category, situated between the levels of "more satisfied than dissatisfied" and "very satisfied (VS)" (see Table 3). This score reflects a generally positive perception regarding the alignment and fulfillment of organizational objectives with employees' expectations.

Table 3: Calculation of the GSI of the Organizational Objectives Dimension (S_1). Source: Own elaboration.

Linguistic term	**SVNN**	**Punctuation** (w_j)	**Frequency** (L_p^r)	$W_j \cdot L_p^r$	$\frac{W_j \cdot L_p^r}{n}$
Clearly satisfied (CS)	(1,0,0)	1.00	21	21	0.47
More satisfied than dissatisfied (MSD)	(0.75,0.35,0.3)	0.70	15	10.5	0.23
Undefined (U)	I	0.50	4	2	0.04
More dissatisfied than satisfied (MDS)	(0.35,0.85,0.7)	-0.30	3	-0.9	-0.02
Clearly dissatisfied (CD)	(0,0,1)	-1.00	1	-1	-0.02
Contradictory (C)	(1,0,1)	0.00	1	0	0.00
Group Satisfaction Index					**0.75**

Meanwhile, gap elimination reached a GSI of 0.68, also classified as "satisfied," but approaching the category of "moderately satisfied (MS)" (see Table 4). This suggests that, although progress is perceived in reducing skill and opportunity gaps, some employees believe there are still areas for improvement to promote greater equality within the organization.

Table 4: Calculating the GSI for the Gap Elimination Dimension (S_2). Source: Own elaboration.

Linguistic term	**SVNN**	**Punctuation** (w_j)	**Frequency** (L_p^r)	$W_j \cdot L_p^r$	$\frac{W_j \cdot L_p^r}{n}$
Clearly satisfied (CS)	(1,0,0)	17	17	0.38	17

Linguistic term	SVNN	Punctuation (w_j)	Frequency (L_p^r)	$W_j \cdot L_p^r$	$\frac{W_j \cdot L_p^r}{n}$
More satisfied than dissatisfied (MSD)	(0.75,0.35,0.3)	20	14	0.31	20
Undefined (U)	I	2	1	0.02	2
More dissatisfied than satisfied (MDS)	(0.35,0.85,0.7)	1	-0.3	-0.01	1
Clearly dissatisfied (CD)	(0,1,1)	1	-1	-0.02	1
Contradictory (C)	(1,0,1)	4	0	0.00	4
Group Satisfaction Index					**0.68**

On the other hand, compliance with standards recorded a GSI of 0.70, placing it in the "satisfied" category and within the range of "more satisfied than dissatisfied" (see Table 5). This result indicates a favorable perception regarding compliance policies, although it may suggest the need for improvements in training and clarity in communicating standards to increase overall employee compliance.

Table 5: Calculating the GSI of the compliance dimension (S_3). Source: Own elaboration.

Linguistic term	SVNN	Punctuation (w_j)	Frequency (L_p^r)	$W_j \cdot L_p^r$	$\frac{W_j \cdot L_p^r}{n}$
Clearly satisfied (CS)	(1,0,0)	1.00	21	21	0.47
More satisfied than dissatisfied (MSD)	(0.75,0.35,0.3)	0.90	15	10.5	0.23
Undefined (U)	I	0.50	4	2	0.04
More dissatisfied than satisfied (MDS)	(0.35,0.85,0.7)	-0.10	3	-0.9	-0.02
Clearly dissatisfied (CD)	(0,1,1)	-1.00	1	-1	-0.02
Contradictory (C)	(1,0,1)	0.00	1	0	0.00
Group Satisfaction Index					**0.70**

Finally, soft skills received the lowest GSI, 0.64, placing it in the "satisfied" category but close to the "moderately satisfied (MDS)" range (see Table 6). This result suggests that while employees value the development of soft skills, this dimension requires attention to achieve greater alignment with the company's strategic objectives and improve satisfaction in this specific aspect.

Table 6: Calculation of the GSI of the soft skills dimension (S_4). Source: Own elaboration.

Linguistic term	SVNN	Punctuation (w_j)	Frequency (L_p^r)	$W_j \cdot L_p^r$	$\frac{W_j \cdot L_p^r}{n}$
Clearly satisfied (CS)	(1,0,0)	1.00	18	18	0.40
More satisfied than dissatisfied (MSD)	(0.75,0.35,0.3)	0.90	17	11.9	0.26
Undefined (U)	I	0.50	2	1	0.02
More dissatisfied than satisfied (MDS)	(0.35,0.85,0.7)	-0.10	1	-0.3	-0.01
Clearly dissatisfied (CD)	(0,1,1)	-1.00	2	-2	-0.04
Contradictory (C)	(1,0,1)	0.00	5	0	0.00
Group Satisfaction Index					**0.64**

In addition to the satisfaction levels, the analysis revealed frequencies in the undefined (U) and contradictory (C) response categories. The undefined category (U), with a value of 0.50, represents cases where participants have no clear position, potentially indicating a lack of information or understanding of certain organizational aspects. On the other hand, contradictory responses (C), with a neutral value of 0, reflect opposing or inconsistent opinions, which may suggest varied experiences or tensions in the perception of policies and practices within the organization. This latter category, therefore, shows a greater need for training, which aligns with the results obtained. The four dimensions present a moderately high satisfaction level, where their GSIs indicate differentiated priorities for the organization (see Figure 1).

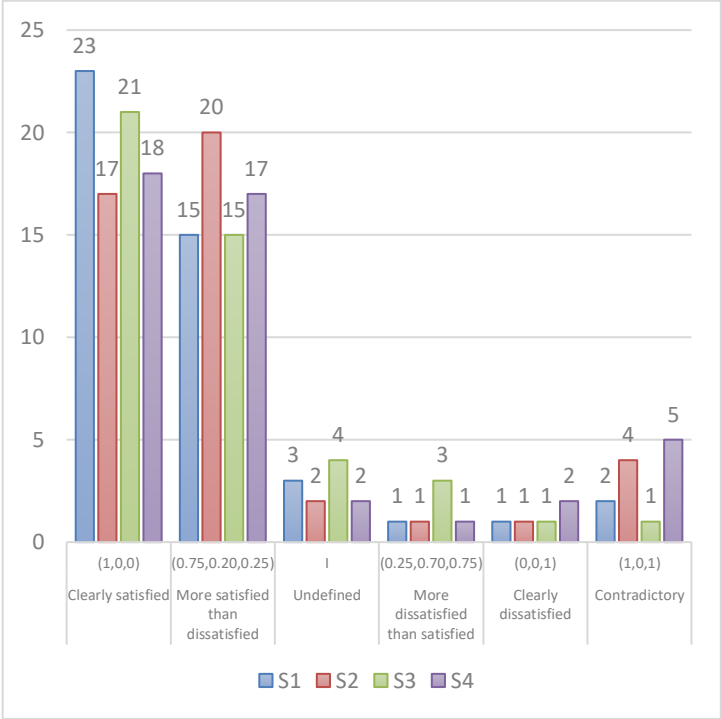

Figure 1: Comprehensive evaluation of S_1, S_2, S_3 and S_4 Dimensions. Source: Own elaboration.

On the other hand, the analysis of the plithogenic areas, based on employees' perceptions of the evaluated dimensions (organizational objectives, elimination of gaps, compliance with standards, and soft skills), reveals a majority distribution of positive opinions. 43.89% of participants are grouped in the "clearly satisfied (CS)" category, while 37.22% position themselves as "more satisfied than dissatisfied (MSD)." This suggests that a significant proportion of employees perceive a positive alignment with the evaluated organizational aspects. However, 6.11% of participants are in the "undefined (U)" category, indicating ambiguity or a lack of clarity about their satisfaction. Meanwhile, 6.67% fall into the "contradictory (C)" area, reflecting opposing perceptions about the studied dimensions.

In contrast, the areas of "more dissatisfied than satisfied (MDS)" and "clearly dissatisfied (CD)" are minimally represented, with 3.33% and 2.78%, respectively, signaling that a small proportion of employees feel dissatisfied or very dissatisfied with the evaluated organizational aspects. In terms of proportion, the "clearly satisfied (CS)" and "more satisfied than dissatisfied (MSD)" areas dominate the analysis, representing more than 80% of the responses, suggesting an acceptable satisfaction level (see Figure 2). However, the "contradictory" and "undefined" areas indicate areas of uncertainty or lack of clarity that require attention. Overall, these results suggest that, although most workers perceive the evaluated dimensions positively, there are sectors that need greater clarity in communication and efforts to address existing dissatisfaction and contradictions within the organization.

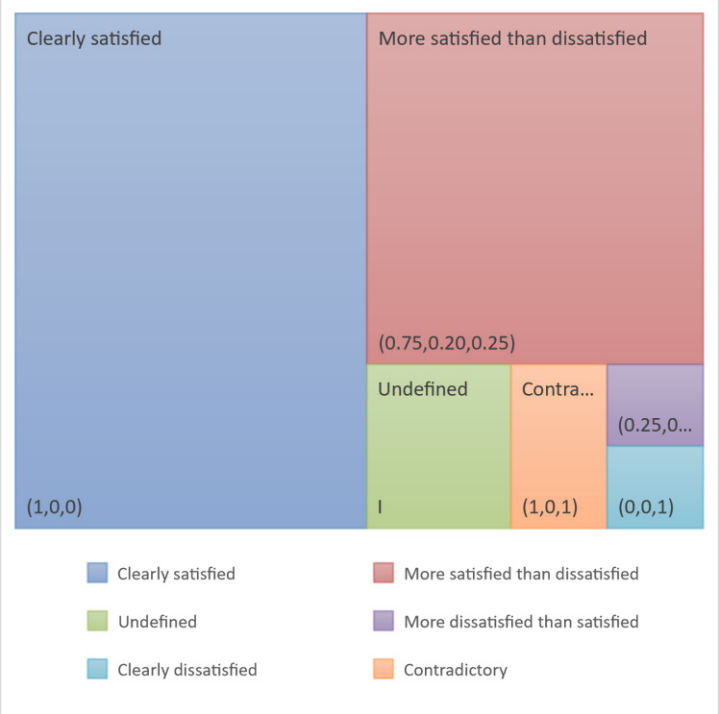

Figure 2: Plithogenic areas within organizational performance and human development. Source: Own elaboration.

Plithogenic Interrelationship: The relationships between the dimensions suggest that they do not operate independently but are closely interconnected. Any variability or change in one dimension impacts the others, reflecting the interdependence within the organizational system. The relationships between the dimensions are moderate, indicating that although each dimension has its specific function, variability in one causes chain effects that affect the behavior of the other dimensions.

Plithogenic Intersections: By segmenting workers into groups with similar perceptions of the evaluated dimensions, the existence of subgroups (arising from the intersection of subsets or dimensions) within the organizational set is reflected. For example, a significant group of employees shows high levels of satisfaction with all dimensions, indicating consistent alignment between workers' perceptions and organizational objectives. On the other hand, a smaller group of employees expresses dissatisfaction with the evaluated dimensions, highlighting the existence of misalignments that require attention to improve integration within the organizational system.

Plithogenic Links: The hierarchical analysis of the dimensions reveals how they cluster based on similarity, indicating varying degrees of interdependence within the organizational set. Generally, some dimensions, such as gap elimination and soft skills, show a high correlation and cluster together, suggesting that they share common and functional dimensions within the organizational structure. In contrast, other dimensions, such as compliance with standards, show a weaker relationship with the others, indicating that this dimension requires more intervention or improvement to strengthen its integration within the system.

The plithogenic analysis of satisfaction dimensions within organizations reveals an interconnected and interdisciplinary system, which must be considered when implementing improvement actions. Even organizations showing high levels of satisfaction in several key areas suggest the need for soft skills training for employees in Cotopaxi, as they demonstrate a positive impact on work performance, productivity, and organizational efficiency. Furthermore, the development of skills such as communication and teamwork is essential to improve adaptability and reduce competence gaps. Additionally, these skills are key to ensuring compliance with standards and aligning with organizational objectives,

by promoting a collaborative and results-oriented environment. They also contribute to employee satisfaction and retention, strengthening organizational stability and overall business success.

4. Conclusion

The results obtained through the analysis conducted with the plithogenic IADOV method have revealed contradictions and indeterminacies in the data regarding training and its impact on organizational competitiveness. While the need for soft skills and employee development training has been acknowledged, a significant portion of the workers has expressed dissatisfaction, particularly concerning compliance with standards. The plithogenic analysis has facilitated the identification of gaps between organizational expectations and the needs perceived by employees. This suggests that the training strategies implemented thus far have not achieved the expected results. Consequently, the study emphasizes the need to redirect training programs towards strengthening soft skills, to reduce existing gaps, optimizing job satisfaction and retention, and thus increasing organizational competitiveness.

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Neutrosophic statistical analysis of cardiovascular disease prevention from a physical culture in urban communities

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Abstract. The research presented here considers the educational potential of physical activities for disease prevention. Its objective is to elaborate a physical-educational intervention strategy for the prevention of cardiovascular diseases in urban communities in the department of Cundinamarca, Colombia. The research was carried out in the municipalities of Funza, Fusagasugá Albán, and Supatá in the department of Cundinamarca in Colombia, based on the theoretical foundations of prophylactic and therapeutic physical culture and related sciences for disease prevention. Theoretical, empirical, and mathematical-statistical scientific methods were applied. In addition, neutrosophic techniques were used to validate the study. The results indicate an adequate level of statistical significance. Future work should focus on expanding the use of neutrosophic methods to enhance cardiovascular disease prevention efforts across diverse and vulnerable populations, improving decision-making and adaptability in community health programs.

Keywords: neutrosophic, neutrosophic statistics, prevention, cardiovascular diseases, urban communities.

1 Introduction

The World Health Organization (WHO) ¸ [1] estimated in 2011 that 6 out of 10 deaths and 70% of disability-adjusted life years are due to noncommunicable diseases. It is also projected that by 2030 more than three quarters of deaths will be due to NCDs. Today, in the Americas region, approximately 77% of deaths and 69% of the burden of disease are due to NCDs. On the other hand, WHO [1] also considers that the lack of physical activity is a considerable risk factor for this type of disease, where sedentary behavior is typical of the way of living, consuming, and working in advanced societies.

NCDs are of long duration and slow progression. More than 80% of deaths are due to cardiovascular diseases, type II diabetes, and about one-third of cancers. The latter can be prevented by eliminating the risk factors they share, such as tobacco use, poor dietary habits, and physical inactivity, all of which are preventable [2].

Accordingly, this study focuses on cardiovascular diseases (CVD), as they have the highest incidence and prevalence in the world population and South America in particular. These diseases are currently responsible for 60% of all deaths worldwide, and 80% of these occur in low- and middle-income countries such as Colombia [3].

Concerning this, the Executive Committee of the World Health Organization stated that cardiovascular disease; has reached enormous proportions and affects younger and younger people. In the coming years, if this trend does not change through research to identify its causes and how to prevent it, it will become the greatest epidemic of mankind¸[1].

To go deeper into the subject, the author of the research conducted an initial bibliographic search

where Alvarez [4] stands out, whose research is aimed at the treatment of hypertensive patients and offers an improvement strategy for Physical Education teachers, aimed at the treatment of obesity. It also stands out [5], [6] those who carry out the protocol of disease prevention in primary health care; however, it should be noted that it is only aimed at guiding the patient in what to do, without reflecting the methodological guidelines for its practical implementation.

The research developed by [2], who elaborated a physical exercise program for the prevention of cardiovascular diseases in patients in the Republic of Congo, stands out; the actuality and novelty of this research are recognized, however, to achieve its implementation in the current context, it requires the sensitization of the actors and the formation of healthy habits and lifestyles; which shows that the educational perspective is a viable tool.

On the other hand, the program of the therapeutic areas of physical culture and the work for Physical Activity teachers in the community are reflected in the normative documents of Inder, but they do not contain educational actions that promote the prevention of these diseases.

In the particular case of Colombia, the Ten-Year Public Health Plan in the years 2012 - 2021, outlines the pillars for the prevention of these diseases, where the standards and dimensions for a healthy habitat are defined, as described in the Department of Cundinamarca, in Colombia [7]. Based on the above, actions and strategies should be established to promote their contextualization, based on the requirements and characteristics of each of the states of the country.

Specifically, in the department of Cundinamarca, in Colombia, the main causes of mortality in the period 2005-2018 were, in the first place, non-communicable diseases and cardiovascular diseases (CVD) which represented 35%, in second place, those of group of other causes, with 26%; in third place, neoplasms, which represent 16% of the mortality of the department, according to the Ten-Year Public Health Plan, [7].

In a diagnosis carried out in the urban communities of the Department of Cundinamarca of Colombia, where interviews, surveys, a documentary review, and the study of the initial bibliographic sources were applied; these instruments evidenced the main results, which are enunciated below:

- The existence of an adequate integration of the main actors of the urban community to enhance the prevention of cardiovascular diseases from the physical-educational point of view.
- Urban dwellers have insufficient knowledge of the benefits of community physical culture for the prevention of cardiovascular diseases.
- Few tools integrate physical education in the process of prevention of cardiovascular diseases in urban communities in Colombia.

In correspondence with the above, the following scientific problem is stated: How to assess the prevention of cardiovascular diseases in the process of physical culture for urban communities in the department of Cundinamarca, Colombia?

Following the above, it is recognized as a research objective: to apply neutrosophic statistics to assess the prevention of cardiovascular diseases in the process of physical culture for urban communities in the department of Cundinamarca, Colombia.

2 Materials and methods

A set of theoretical, empirical, and mathematical-statistical methods and techniques were taken into account for this research. These were found in correspondence with the stages of the work:

Methods of the theoretical level

The historical-logical one, allowed the study of the main antecedents of the educational intervention in the prevention of diseases in urban communities, in addition to the chronological and logical of the events presented.

The analytical-synthetic approach made it possible to break down the object of study (the preventive process of physical culture in cardiovascular diseases) into its different parts, to establish the system

of essential relationships between the psychosocial, pedagogical, biological, and medical foundations; it was also used for the presentation of the results, the partial and conclusions.

The inductive-deductive approach allowed the contextualization of knowledge related to the educational intervention in the prevention of cardiovascular diseases in urban communities, to increase the knowledge of urban dwellers regarding the risk factors of these diseases.

The system approach provided the general orientation of the educational intervention, based on the hierarchical structuring of each structure of the elaborated strategy (introduction, foundation, diagnosis, strategic planning, instrumentation, and evaluation), by establishing the relationships of subordination or coordination among its constituent elements; in which the components, structure, functional relationships and hierarchy of it are denoted.

Empirical level methods

Non-participant scientific observation. It was also carried out in the exploratory study stage, in the diagnosis and its follow-up to know the behavior of the implementation of the educational intervention in the prevention of cardiovascular diseases, in the urban communities of the department of Cundinamarca in Colombia.

The survey made it possible to deepen particular aspects of the educational intervention in the prevention of cardiovascular diseases in the urban communities of the Department of Cundinamarca in Colombia; and also to know the opinions of the different actors and decision-makers about the object of the research and the verification of the feasibility of the proposal.

The documentary analysis allowed the study of official documents such as the Health Situation Analysis of Cundinamarca, the Integral Health Routes, and the Disease Prevention Strategy of the Government of Cundinamarca.

Methodological triangulation was applied to the different research methods and techniques to evidence and verify similarities and differences between the data and information. Different instruments were used to ratify the conclusions or assessments. These include the survey, interview, and documentary review.

Pedagogical experiment pre-experimental design: with pretest and post-test for one group. The research fieldwork was conducted in the period from February 2021 to February 2022.

Mathematical statistical methods

From descriptive statistics, empirical distributions of absolute and relative frequencies were used, these expressed in percentages, their graphic representations, and numerical measures such as mode, mean, and standard deviation.

From the inferential statistics, the Mc Nemar nonparametric test for related samples was used to test the significance of the changes before and after the intervention. This is a before-and-after hypothesis test with the particularity that it is only applicable to dichotomous variables; it allows us to know whether a YES or NO response varies from one moment to the next.

2.1 Population and sample

A neutrosophic sample is a set of elements that are characterized by having properties that cannot be determined with certainty, i.e., that can have degrees of truth, falsity, and indeterminacy. In other words, a neutrosophic sample is composed of elements for which the truth, falsity, or indeterminacy of certain properties are not known precisely, [7].

In neutrosophic set theory, a neutrosophic sample is represented by a three-dimensional vector (T, I, F), where:

T represents the degree of truth membership indicating the membership of an element to a specific property.

I represent the degree of indeterminacy membership, which reflects the uncertainty or ambiguity about the property.

F represents the degree of falsehood membership, which indicates the non-membership of an element to the property.

Each of these values ranges from 0 to 1, where 0 means that the property is completely false, 1 means that it is completely true and 0.5 denotes that it is completely indeterminate.

Neutrosophic set theory and neutrosophic samples are applied in fields such as artificial intelligence, medicine, and others, for decision-making, fuzzy logic, and imprecise reasoning, where uncertainty and imprecision in data and information need to be handled. These tools are useful when facing problems where the available information is vague or incomplete, allowing to represent and process knowledge more broadly and flexibly, [8].

Based on the above, the following procedure is presented for the selection of the sample under study in this research:

The desired confidence level (95%) is defined.

The acceptable margin of error is established (±5%).

The formula used to calculate the required sample size is the formula for finite populations:

N = population.

Z = critical value based on the confidence level (e.g., for a confidence level of 95%, Z ≈ 1.96).

p = estimated proportion (0.5 was used if you do not have a prior estimate).

q = 1 - p.

E = margin of error as a proportion (0.05).

$$n = \frac{(Z^2 * p * q)}{\frac{E^2}{(N-1)} + Z^2 * p * q} \quad (1)$$

Applying these values in the formula:

n = (1.96^2 * 0.5 * 0.5) / (0.05^2 / (2625 - 1) + 1.96^2 * 0.5 * 0.5)

n ≈ 130.78

The sample studied consisted of a total of 125 urban dwellers, distributed by strata of 4 municipalities of the department, these were chosen at random, in addition to stratified random sampling, according to the procedure and spreadsheet of the University of Murcia Spain and the sample was structured as follows: (Funza (n = 30), Fusagasugá (n = 30), Albán (n = 30) and Supatá (n = 35). Overall, 56 males and 69 females, aged between 22 and 51 years), randomly selected, using the letter selection procedure.

Table 1. Distribution of behavior by sex and age of the selected sample.

Municipality sex Age	Male sex	Female sex	(mean± SD)
Funza (30)	15	17	31.1±13.6
Fusagasugá (30)	11	19	46.7 ± 10.2
Albán (30)	13	17	39,6 ± 10.1
Supatá (35)	17	16	44.5 ± 9.7
TOTAL (125)	56	69	44.3 ± 10.8

The study followed the postulates of the Declaration of Helsinki, where all subjects gave their consent and received the information required for the study.

2.2 Neutrosophic method

In correspondence with the methods and approaches assumed in the research, single-valued neutrosophic sets (SVNS) [7] [8], [9] were used from the neutrosophic perspective, which allow the use of linguistic variables that increase the interoperability in the recommendation models and the use of indeterminacy [10], [11], [12], [13], [14].

Neutrosophic sets are a generalization of a fuzzy set (spatially of an intuitionistic fuzzy set) [13]. Let U, a universe of discourse, and M be a set included in U. An element to x of U is noted in respect of the set M as x (T, I, F)and belongs to M in the following way [16, 17]:

Statistically T, I, F are subsets, but dynamically T, I, F are functions or operations dependent on many unknown or known parameters [14]. Let X be a universe of discourse. An SVNSA over X is an object of the form:

$$A = \{ \left(x, u_{A(x)}, r_{A(x)}, v_{A(x)} \right) : x \in X \} d \quad (2)$$

For work with neutrosophic numbers, neutrosophic statistics have also been defined [10], [11]. The confidence interval represents an application domain of applied neutrosophic statistics [12]. The population sample neutrosophic confidence interval can be defined in the same way as the classical large sample confidence interval for the population proportion π [14].

$$p \pm (\text{Critical value Z}) * sqrt\left(\frac{(p*(1-p))}{n}\right) \quad (3)$$

For a confidence level of ≥ 5.

Derived from the above and to perform neutrosophic statistical analysis, the following methodological algorithm was taken into account, which illustrates the developed logic (Figure 1). Each of its phases is a step of progress in the research developed, where the problem arises from a valuation need that is represented.

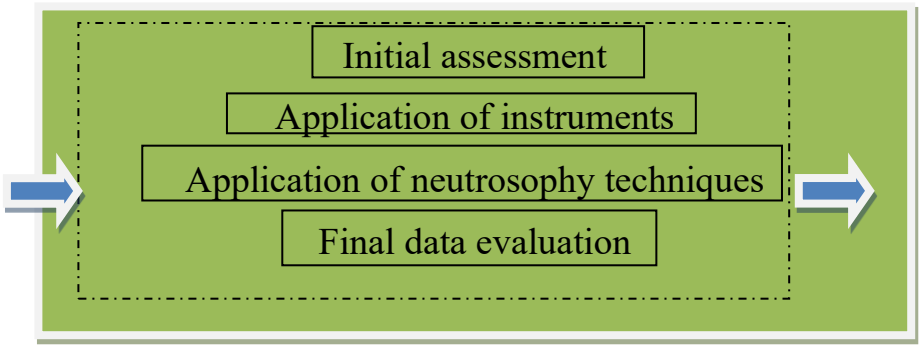

Figure 1. Schematic representation of the neutrosophic logic followed in the research.

3 Results and discussion

In this section we present the results obtained for this pre-experimental design was developed as follows:

- Initial evaluation (before)
- Application of an educational intervention strategy.
- Final evaluation (after application).

Indicator knowledge of risk factors

When answering question 1 of the survey, it becomes evident that before applying the strategy, the majority did not know the risk factors for cardiovascular diseases. A total of 96, 80% did not know at this time of the investigation what the risk factors for the disease under investigation were. However, once the educational intervention strategy was applied, this situation was transformed, since 113 (94.1%) revealed that they did know. It is important to note that most of them argued that they had learned about it during the period of the research, a result that shows the influence of the research on the urban dwellers who participated in the research.

The following are two examples of what was reflected by the residents concerning the contents of this indicator. An example of this is that, when arguing their response, they applied the strategy that there was a group of people who used tobacco or cigarettes (88 for 73.3%), in the case of the latter up to more than 5 per day. After the educational intervention, the number of people who consumed tobacco or cigarettes was reduced to 58 (48.3%), although this is a high figure for how harmful it is to the organism; however, the effect of the strategy on this risk factor for cardiovascular diseases is evident.

On the other hand, at the initial moments, there were 76 inhabitants (63.6%) who consumed alcohol three or more times in high proportions, which greatly affected the human organism. After receiving the educational actions, this risk factor was reduced, since after the strategy was applied, it dropped to 49 for 40.8%, which makes it evident that when educational actions are carried out systematically and with a scientific basis, it is possible to sensitize and educate people about the harmfulness of this substance for the human organism.

Table 2. Results of question 1 of the survey.

Answer	Before	After
Yes	29 (20%)	113 (94,1%)
No	96 (80%)	12 (9,6%)

Indicator knowledge of proper nutrition

The results of question 2 of the survey applied to the members of the sample, where 11 of the inhabitants, for 9.1% of the sample under study, stated that they had a balanced diet and exemplified the amount of food intake per day, with the nutrients in their diet. However, the majority (84.1%) reported eating street food such as hamburgers, hot dogs, pizzas and KFT chicken and other dishes with high fat content, as well as not having a stable schedule to do so, since they depend on their work schedule. On the other hand, 8 inhabitants (6.8%) left the survey blank.

A very different result was observed after the educational intervention strategy was applied. A total of 111 villagers, or 92.5% of the sample, reported consuming healthy and balanced nutrients. In addition, most of them explained how important the research was to change their eating style. This shows the effectiveness of the proposal after its application.

The exposed results make evident that the population involved in the research, after interacting with the actions and activities of the intervention strategy, were educated about the measures for the prevention of cardiovascular diseases. This is significant since they contribute to a healthy and active lifestyle.

Table 3. Results of question 2 of the survey.

Answer	Before	After
Yes	11 (9,1%)	111 (92,5%)
No	114 (91,2%)	14 (11,2%)

Knowledge of the benefits of physical activity indicator

The results of this indicator belong to question 3 of the survey and show that at the beginning of the study, before applying the strategy, there was a group of people who did not know the benefits of systematic physical activity, as reflected in the fact that 99 (99 for 82.5% of the sample under study) indicated the option no. The rest, only 21 for 17.5%, said yes. While the rest, only 21, or 17.5%, said yes. In the answer that they were asked to argue, the majority stated that physical activity is performed by athletes, an issue that makes evident the lack of knowledge that they present about this content.

After the educational actions of the strategy were applied, there was an increase, since 117 for 97.5% of the sample stated that they knew the benefits of physical activity. In addition, 100% of those involved in the research stated that during the intervention process, they had changed their opinion about physical activity and now understand that it is very beneficial for health in general and for cardiovascular diseases in particular.

Table 4. Results of question 3 of the survey.

Answer	Before	After
Yes	99 (82,5%)	117 (97,5%)
No	21 (17,5%)	8 (6,5%)

By way of summary, this dimension shows that urban dwellers show an adequate level of knowledge on the prevention of cardiovascular diseases in urban communities, based on physical culture, which was acquired from the media and scientific and specialized literature, which served as a basis for the actions and activities of the strategy presented for this purpose.

The statistical comparison using the nonparametric test of Mc. Nemar, processed by the statistical package SPSS version 20.0 for Windows, corroborated the existence of very significant changes in each one of the indicators of the dimensions of the study variable by obtaining in all cases that the calculated probability was lower than the prefixed probability ($p= 0.00 < 0.05$). This result evidences that the prevention of cardiovascular diseases in the process of physical culture for the urban communities of the department of Cundinamarca of Colombia in the selected sample changes significantly in the second moment of the research.

Discussion

In agreement with the idea Tuomilihto et al. [15] point out that it has been demonstrated that physical exercise, together with proper diet, can prevent up to 58% of the incidence of these diseases, in addition, they benefit especially, with a regular program of aerobic exercise. On these arguments, the results through orientation workshops individually prepared the actors involved to carry out the activity at home.

The results of the research by Delgado and Acuña [16] reveal that it is not precisely defined what the duration of the exercise session should be to reduce cardiovascular risk; although time does not seem to have an influence. However, in this research, it was shown that in one year with systematic physical activity and with three weekly frequencies some cardiovascular indexes are improved.

According to the criteria of Planas [17] in the relationship of the effects of exercise on cardiovascular risk factors, poor physical fitness seems to be associated with a determining factor in these diseases. Individuals with poor physical fitness were 3-6 times more likely to develop these cardiovascular risk factors than those with good physical fitness. That is why in the present study we worked three times a week and the results are evident in the level of significance derived from this research.

According to the study conducted by Dupotey et al. [18] where they reflected on which exercises to apply for the prevention of cardiovascular diseases, they evaluated the local and general effects of physical activity on this system. This study was the basis for the results presented today.

The work offered urban dwellers and social managers a tool to contribute to the awareness of all the members of the community, from different interventions approved in the normative documents of Colombia; to achieve an adequate prevention of cardiovascular diseases.

4. Conclusion

The systematization developed pieces of evidence of insufficiencies related to the integration of the promotion and awareness of the actors on healthy lifestyles, the creation of healthy environments in different community spaces, the development of educational physical activities, and the prevention of cardiovascular diseases in the urban communities of the department of Cundinamarca, Colombia.

The analysis and interpretation of the results, through expert consultation, confirmed the theoretical assessment, while application in practice, demonstrated the effectiveness of the educational intervention strategy for the prevention of cardiovascular diseases from physical culture for urban communities in the department of Cundinamarca, Colombia. Based on the results of the application of neutrosophy, it is necessary the timely intervention in the student population with tools that promote the prevention of cardiovascular diseases from the physical culture for urban communities in the department of Cundinamarca, Colombia.

For future work, it is suggested to deepen the application of neutrosophic methods to assess and address the complexity and uncertainties involved in promoting cardiovascular disease prevention. This approach could be extended to other demographic groups within Cundinamarca, including vulnerable populations, to enhance the understanding of different variables affecting health outcomes. Furthermore, incorporating neutrosophic tools into community health programs could improve decision-making processes, allowing for more adaptable and inclusive educational strategies in the promotion of healthy lifestyles.

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English for specific purposes in the medical sciences to strengthen the professional profile of the higher education medicine student: a knowledge representation using SuperHyperSoft Sets

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Abstract. The English language is an important added value for the professional profile of students in all higher education courses due to the demands of the current century, in which mastering more than one language is undoubtedly one of the preponderant skills to access better and more opportunities within the workplace, especially in careers such as medicine. Therefore, the curricular design and the implementation of a program focused on the development of the linguistic skills of the future health professional is pertinent and necessary to strengthen their professional profile; not only in the discourse at the level of talks, congresses, training, and international participation where English is the official communicative language; but also at the level of research as one of the fundamental pillars of this career because most of the high-impact medical bibliography is developed and available in English. The present study proposes the strengthening of English for specific purposes in medical students through the implementation of a lexicographic material that provides them with the definition and pronunciation in English of the most prevalent medical vocabulary in this field, as well as its translation into Spanish, which would significantly increase the professional skills of future doctors. The present research proposal is also justified by the fact that many medical students did not have advanced preparation in the English language at school, which is why the proposed lexicographic material would represent an added complementary value to the curricular programming of this subject, which would help to make the learning of English viable in a much more specific and contextualized way in this branch of science, an important aspect given the substantial need to be able to communicate both locally and internationally. This paper proposes a model where medical terms are organized in English and Spanish using the theory of SuperHyperSoft Sets. The importance of the model is that it allows this knowledge to be automated so that students can access it easily. Additionally, it deals with ambiguity in medical definitions, as well.

Keywords: English for specific purposes, lexicographic material, medical lexicon, SuperHyperSoft Sets.

1 Introduction

English is the language par excellence that should be required of all medical science professionals. Virtually all the most relevant literature on the subject is written in English. Scientific articles with the most important discoveries and experiments are written in English for greater scientific dissemination. On the other hand, doctoral, master, or undergraduate theses are recommended to have an English version for dissemination. The exchange of knowledge is easier when communicating in English while two specialists cannot communicate in one of their mother tongues. Generally, English is the language that should be mastered by both of them and in this way, communication is possible [1, 2].

For all these reasons, effective teaching of technical English in medical terms is a challenge. To do so, some key concepts used for teaching English in this branch of knowledge must be established.

In particular, we will focus on Spanish-speaking specialists from Latin America.

The first important term is "English for Specific Purposes," which differentiates English from "English for General Purposes." English for Specific Purposes is subdivided into "English for Academic Purposes" (EAP) and "English for Vocational Purposes" (EVP).

"English for academic purposes" is further subdivided into "English for general academic purposes" and "English for specific academic purposes". Within medicine, the term "English for medical purposes" (EMP) is used. EMP is usually subdivided into EVP, which is related to professionals working in the health sector, and English for specific academic purposes, which is mainly related to university studies.

Within the field of medicine, there are some particular characteristics of the teaching of this science; it is students of this specialty who, for the most part, end up being doctors, which is why it is very clear what needs to be taught to students of the specialty. This does not happen with students of other careers, such as engineering, where future engineers do not know what jobs they will be placed in once they graduate. The distinction between English for General Academic Purposes (EGAP) and ESAP is that the former involves the development of skills for the study of academic subjects in English, while the latter is studied to meet the specific needs of students, based on their immediate and real needs.

Physicians must communicate with patients in more or less colloquial language. Meanwhile, they must use the medical language in their presentations with other professionals in academic and scientific contexts, which are highly technical. For this reason, physicians and health science professionals, such as nurses, laboratory technicians, etc., must be able to master both technical terms in Spanish and English.

The aim of this article is to formalize English terms and their definitions, translation, pronunciation, and relationship with other terms. This will allow for easier automation of the terms and will therefore serve as support for the learning of this language by professionals who are studying it. In addition, it will personalize knowledge through better structuring.

To achieve this goal, we have selected the theory of SuperHyperSoft Sets [3-6]. This theory, introduced by F. Smarandache, allows us to represent complex situations typical of the real world. This theory further extends Professor Molodtsov's Soft Set theories where a function is defined on the values of a single parameter [7-13]. HyperSoft Sets contain a function on the Cartesian product of several sets, each containing the possible values of several parameters [14-18]. SuperHyperSoft Sets are defined as the previous one on the Cartesian product but in this case on power sets. Fuzzy-extended SuperHyperSoft Sets have as their destination set the fuzzy set or some of its extensions of the universe set. Where "fuzzy-extended" means fuzzy, intuitionistic fuzzy, interval-valued fuzzy, Pythagorean, Fermatean, neutrosophic, plithogenic, etc. [3, 19]

We use this tool as a model to represent medical terms translated from English to Spanish because in some cases these terms cannot be clearly defined. For example, blood vessels, including veins and arteries, are part of the circulatory system and also the cardiovascular system. On the other hand, fever is a "sign or symptom" that is part of several infectious diseases, each of them of different origin. The proposed model can serve as a basis for automated learning of English terms by medical students.

This article is divided into a Materials and Methods section, where the main concepts of SuperHyperSoft Sets are recalled. The following section contains the details of the proposed model. The article finishes with the conclusions.

2 Materials and Methods

Definition 1 ([7])**.** Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where

$\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set.

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A where we have the following:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0,1]$. Thus, if we know the family F, then we can reconstruct the function μ_A by using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0,1])$ soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 2 ([14])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *HyperSoft Set* over U.

Definition 3 ([3])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([3, 19])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n))$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}\left(U\left(x(d^0)\right)\right)$ is called a *Fuzzy-Extension-SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, and so on.

3 The linguistic Model

The model we propose in this article organizes the different terms that a doctor must know in English. SuperHyperSoft Sets are used to deal with the indeterminacy due to ambiguity that exists in the classification of terms. Some terms are classified in at least two different ways because the human body physiologically behaves as a whole and that is why there are diseases that must be treated by two or more different specialists. Let us explain the details below:

1. We have the universe set $U = set\ of\ \{Term\ in\ English, definition, pronunciation, Translation\ into\ Spanish\}$. That is, the universe set contains as a basic component the set of four terms formed by the name of the term in English, the definition in English, the pronunciation, and its translation into Spanish.
2. The parameters to be measured are as follows:
   2.1. a_1: Letter of the alphabet with which the term begins in English $A_1 = \{a, b, c, \cdots, z\}$.
   2.2. a_2: Letter of the alphabet with which the term begins in Spanish $A_2 = \{a, b, c, \cdots, z\}$.
   2.3. a_3: Classification of the term.

A_3 = {{Disease, Signs and Symptoms},
{Design, Organ System, Organ},
{Disease, Bacterial origin},
{Disease, Virus origin},
{Disease, Other},
{Medical Procedures, Examinations},
{Medical Procedures, Hospital Staff},
{Hospital Vocabulary, Treatments},
{Hospital Vocabulary, Hospital Departments},
{Medical Specialty},
}

3. The SuperHyperSoft Set is defined by the pair $(T, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3))$, where:

$T: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3) \to \mathcal{P}(\mathrm{U})$.

Remark

1. {Disease, Organ System, Organ} is a generic way; it is assumed that we have several sets of this type, for example {Disease, Cardiovascular System, Heart}, etc.
2. {Disease, Other} includes a classification that is not part of the previous disease classifications.

Below we demonstrate in general what the model is like based on more specific cases, where we show a small database with medical terms:

MEDICAL HEALTH LEXICOGRAPHICAL MATERIAL

Diseases

• **Fever**
Definition: a very high-temperature condition.
Pronunciation : /ˈ fiːvər /
Translation: Fiebre

• **Cough**

Definition: a sudden expulsion of air from the lungs.
Pronunciation : / kɒf /
Translation: Tos

• **Allergy**

Definition: a response from the immune system (red eyes, runny nose, skin irritation, among others), usually as a reaction to certain food, medicine, material, or smell.
Pronunciation: /ˈ æl.ə.dʒi /
Translation: Alergia

• **Infection**

Definition: invasion and multiplication of microorganisms such as bacteria, viruses, etc.
Pronunciation: /ɪnˈfek.ʃən/
Translation: Infección

• Asthma: a respiratory condition that causes difficulty in breathing.

- Diabetes: a chronic condition that affects the body's ability to use sugar.
- Migraine: a type of severe headache.
- Anxiety: a mental health disorder that causes a lot of fear and worry.
- Depression: a mental health condition characterized by a constant feeling of sadness and disinterest.
- Arthritis: a condition that causes pain and stiffness in the body's joints.
- Fracture: a break or crack in a bone usually requiring medical attention.
- Flu: influenza; an infectious disease caused by a virus.
- Indigestion: discomfort in the stomach, often after eating.
- Hypertension: also known as high blood pressure.

Medical Procedures
Examinations

- Check-up: a routine examination by a doctor or dentist.
- X-ray: a type of imaging test using radiation to view inside the body.
- Blood test: laboratory analysis of a blood sample.
- Biopsy: the removal and examination of body cells, liquid, or tissue to diagnose a condition.
- MRI scan: an imaging test using magnetic fields to create detailed images of the body.
- Ultrasound: a diagnostic tool using sound waves to view internal organs.
- CT scan: a type of X-ray providing cross-sectional images of the body.
- ECG: a test that records heart activity and can detect heart problems.

Treatments

- Surgery: a medical procedure used to repair or remove parts of the body.
- Therapy: treatment to relieve or heal a disorder.
- Medication: a substance used to treat an illness or condition.
- Vaccination: administration of a vaccine to help the body develop immunity to a disease.
- Chemotherapy: a cancer treatment using drugs to kill or slow the growth of cancer cells.
- Radiation therapy: a cancer treatment using high-energy radiation to kill cancer cells.
- Dialysis: a treatment for kidney failure to remove waste products from the blood.
- Insulin therapy: a treatment for diabetes to control blood sugar levels.
- Antibiotics: medications that fight bacterial infections.
- Immunotherapy: a treatment that boosts the immune system often used for cancer patients.
- Transplant: replacing a failing organ with a healthy one from a donor.
- Physical therapy/rehabilitation: a program to restore strength and mobility after injury or surgery.
- Inhaler: a device delivering medication to the lungs, often used for asthma.
- Acupuncture: therapy using needles to stimulate specific points and treat pain.

Hospital Vocabulary
Hospital Staff

- Doctor: a medical professional.
- Nurse: a healthcare provider trained to assist doctors and care for patients.
- Surgeon: a doctor specialized in performing surgeries; operates on patients.
- Pharmacist: a professional who dispenses prescribed medications.
- Therapist: a professional who provides therapy.
- Radiologist: a doctor specialized in imaging.
- Anesthesiologist: a doctor who administers medicines to prevent pain during surgeries and

procedures.

- Pathologist: a doctor who analyzes body tissues and fluids.
- Psychologist: a mental health professional who provides counseling and therapy.
- Paramedic: a healthcare professional who provides emergency medical care outside the hospital.
- Dietitian: creates eating plans for patients.

Hospital Departments

- Emergency Room (ER): part of the hospital providing immediate treatment for urgent conditions.
- Operating Room (OR): a sterile environment where surgical procedures are performed.
- Ward: a hospital unit where patients with similar conditions are treated.
- Maternity ward: a section of the hospital specifically for childbirth and newborn care.
- Pediatric ward: a section of the hospital dedicated to treating children.
- Pharmacy: a place where medications are stored and dispensed.
- Clinic: a healthcare facility providing regular or specialized outpatient care.
- Intensive Care Unit (ICU): a special hospital department for critically ill patients.
- Laboratory: a place where medical tests are conducted.
- Rehabilitation center: a facility that helps patients recover after surgeries or injuries.
- Waiting room: an area where patients and their guests wait before, during, or after appointments or treatments.
- Radiology department: the part of the hospital where imaging tests are performed.
- Outpatient department: where patients receive treatments without being admitted overnight.

Let us take the above terms as a small illustrative medical database. So, we have the following:

Suppose we have a database containing terms such as {$Fever$, "a very high-temperature condition", "/ˈfiːvər/", fiebre}, {Cough, "a sudden expulsion of air from the lungs", "/kɒf/", tos}, among many others. These are the elements of the universe set U.

The model we propose, as it is based on SuperHyperSoft Sets rather than HyperSoft Sets or Soft Sets, allows flexibility in the representation of elements. For example:

$T(\{a, b, c, \cdots, z\}, \{a, b, c, \cdots, z\}, \{\text{Disease, Signs and Symptoms}\})$ part of the image of $T$are the elements $\{Fever, "a\ very\ high\text{-}temperature\ condition",\ "/ˈfiːvər/", fiebre\}$, $\{Cough,$ $"a\ sudden\ expulsion\ of\ air\ from\ the\ lungs", "/kɒf/", tos\}$,

$\{Allergy, "a\ response\ from\ the\ immune\ system\ (red\ eyes, runny\ nose, skin\ irritation, among\ others),$ $usually\ as\ a\ reaction\ to\ certain\ food, medicine, material, or\ smell", "/ˈæl.ə.dʒi/", Alergia\}$, among others.

Note that there are elements of the universe set that begin with any letter in both Spanish and English and that are signs and symptoms of diseases.

If on the other hand, it is taken $T(c, \{a, b, c, \cdots, z\}, \{\text{Disease, Signs and Symptoms}\})$ then neither Fever nor Allergy is included, but it is included $\{Cough,\ "a\ sudden\ expulsion\ of\ air\ from\ the\ lungs", "/kɒf/", tos\}$, because "c" is the first letter of cough.

On the other hand, it can be included $T(c, \{a, b, c, \cdots, z\}, \{\{\text{Disease, Signs and Symptoms}\}, \{\text{Disease, Bacterial origin}\}\})$, where there are signs and symptoms or bacterial diseases that begin with c in English. This includes $\{Cough,$ $"a\ sudden\ expulsion\ of\ air\ from\ the\ lungs", "/kɒf/", tos\}$, and also $\{Cholera,$ $"an\ acute\ diarrhoeal\ disease\ that\ can\ kill\ within\ hours\ if\ left\ untreated", "/ˈkäləɾə/", cólera\}$, among others, see the Venn diagram in Figure 1.

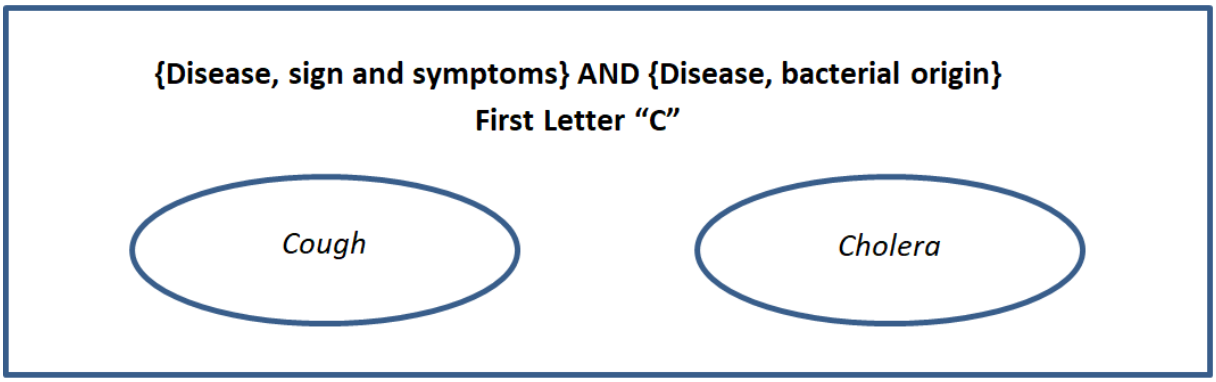

Figure 1: Venn diagram for the example above. *Cough* and *cholera* are a sign and symptom, and a bacterial disease, respectively; both starting by "c".

Another demonstration of the flexibility of the proposed model is the following:

$T(\{a,b,c,\cdots,z\},\{a,b,c,\cdots,z\},\{Disease,\text{Cardiovascular system, Blood vessels}\}) =$ $\{Ictus,$ *"A stroke or seizure", "/ik·tuhs/", ictus*$\}$ but also $T(\{a,b,c,\cdots,z\},\{a,b,c,\cdots,z\},\{Disease,\text{Nervous system, Brain}\}) = \{Ictus,$ *"A stroke or seizure", "/ik·tuhs/", ictus*$\}$.

Since ictus has the ambiguity of being a disease of the cardiovascular system that supplies blood to the brain, it, therefore, has the affectation of blood vessels as a cause and neurological sequelae as a consequence, see Figure 2.

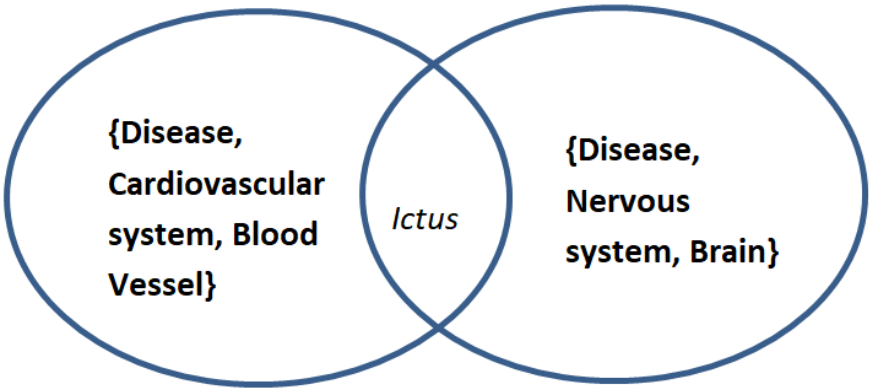

Figure 2: Venn diagram for ictus, demonstrating the ambiguity supported by the model for classification.

4. Conclusion

The study of the English language is essential for any professional today in any branch of science and beyond. Medical personnel are no exception. This paper proposes a way to help organize the knowledge of medical terms in English, and the relationship between each of them, including definition, pronunciation, and translation into Spanish. To do this, we use the theory of SuperHyperSoft Sets that allows combining medical terms in English, and Spanish, their pronunciation and definition, which help to classify the elements being studied. This article takes into account the indeterminacy and uncertainty to accurately classify the medical terms. This fact is due to the interconnection between the biological components of the human organism. So, the model allows to structure of language that is usually un-structured. Also, the model is feasible to be automated very easily since it allows the organization of knowledge in databases. This will contribute to the development of students in a self-taught way.

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Plithogenic Statistical Analysis of Labor Demand and its Impact on Stability in the Floricultural Sector

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Abstract. The floricultural sector faces significant challenges concerning job stability, primarily due to fluctuations in demand, policies, and recruitment capacity. This study aims to analyze the relationship between various factors influencing employability and labor sustainability within the economically active population in this sector. A plithogenic statistical analysis was conducted, utilizing observed and expected frequencies to examine the interrelationships among the variables. The results revealed that labor demand directly impacts job stability, contributing to increased unemployment and underemployment. Additionally, recruitment and retention policies were found to have a limited effect, largely due to the high dependency on agricultural labor. It was concluded that adjustments to recruitment strategies and the diversification of labor opportunities are essential to enhance job stability and sustainability in the floricultural sector.

Keywords: Recruitment policies, job stability, labor sustainability, labor market, plithogenic statistics.

1 Introduction

The analysis of labor demand in key productive sectors is essential to understanding the economic and social dynamics that affect the communities involved [1] [2] [3]. Within this context, the floricultural sector in the parish of José Guango Bajo serves as a significant case study due to its economic impact and its integration with local, national, and international markets. Specifically, the company AGRINAG, dedicated to the cultivation, harvesting, and commercialization of flowers, provides an appropriate environment to examine how fluctuations in labor demand influence job stability and the well-being of the economically active population.

Labor demand in the floricultural sector is closely linked to the seasonal variations of the market, as flower production is cyclical [4] [5]. During certain times of the year, a greater amount of labor is required, while at other times, demand decreases significantly. This pattern of high and low demand poses challenges in terms of hiring and job stability, often resulting in elevated levels of underemployment and unemployment during periods of lower demand [6]. Furthermore, the lack of clear policies for training, hiring, and retaining personnel, combined with the costs associated with hiring and dismissing workers, leads to an excess of labor during crises and a precarious employment situation [7].

Additionally, the constant fluctuation in labor demand in the floricultural sector affects workers' job stability. The uncertainty generated by these seasonal changes negatively impacts the emotional and psychological well-being of the workforce [8] [9]. This accentuates the need to implement recruitment strategies that ensure long-term job stability [10].

Another relevant aspect of analyzing labor demand in the José Guango Bajo region is the structure of the economically active population. In many rural areas, the lack of access to formal education limits workers' skills and restricts their opportunities to obtain stable and well-paying jobs. This phenomenon is exacerbated by the scarcity of continuous training programs that would allow workers to update their skills and adapt to changing market demands [11]. The limited availability of jobs in other productive sectors also forces many workers to accept precarious employment, which in turn contributes to underemployment and instability in the floricultural sector.

Underemployment, therefore, emerges as one of the main challenges for the floricultural sector,

manifesting in two primary forms: underemployment due to insufficient working hours and underemployment due to skill mismatches [12]. In both cases, workers are unable to secure jobs that match their skill levels or full-time schedules, thereby limiting both their income and professional development [6]. This issue is particularly pronounced in rural areas like José Guango Bajo, where the lack of alternative job opportunities compels workers to accept jobs that do not fully meet their capacities or expectations, thereby restricting their well-being and personal growth [13].

This study focuses on analyzing the relationship between the various variables that influence employability and labor sustainability among the economically active population in the floricultural sector, employing plithogenic statistical analysis. The goal is to enhance the stability and sustainable development of the sector, implement actions that promote job stability, improve workers' well-being, and ensure a proper alignment between the skills of the labor force and market demands.

2 Materials and Methods

2.1 Plithogenic Statistics

Plithogenic statistics is applied to analyze complex data, providing an understanding of the interactions among these factors and how they simultaneously affect the analyzed environment. To implement this method, it is necessary to define the plithogenic dynamics [14], which consist of various types of opposites, and/or their neutrals, and/or non-opposites, as well as their organic fusion. Plithogeny generalizes dialectics (the dynamics of one type of opposites: <A> and <antiA>) and neutrosophy (the dynamics of one type of opposites and their neutrals: <A>, <antiA>, and <neutA>), as Plithogeny studies the dynamics of many types of opposites, their neutrals, and non-opposites (<A>, <antiA>, <neutA>; <B>, <antiB>, <neutB>, etc.), and many non-opposites (<C>, <D>, etc.), all together.

A specific application and derived case is the plithogenic set, which extends the classical set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set [15], and has numerous scientific applications [16].

So, it is called a plithogenic set (P, a, V, d, c):

- Where "P" is a set, "a" is an attribute (generally multidimensional), "V" is the range of attribute values, "d" represents the degree of membership of the attribute value for each element xxx to the set PPP under given criteria, and "d" can correspond to d_F, d_{IF}, or d_N, indicating a fuzzy membership degree, an intuitionistic fuzzy membership degree, or a neutrosophic membership degree, respectively, for an element xxx in the plithogenic set P ($x \in P$);
- Similarly, "c" can correspond to c_F, c_{IF}, or c_N, representing the degree of contradiction function for fuzzy attribute values, intuitionistic fuzzy attribute values, or neutrosophic attribute values, respectively [17].

The functions are defined according to the applications that experts need to address. $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are used with the following notation: $x(d(x,V))$, where $d(x,V) = \{d(x,v) \text{ for all } v \in V\}$, for all $x \in P$.

Thus, plithogenic statistical analysis enables addressing the complexity of the perceptions within the analyzed sample. This requires a linguistic evaluation system adapted to the plithogenic model to accurately capture the opinions of the experts (see Table 1).

Table 1: Plithogenic measurement scales. Source: Own elaboration.

Scale	Plithogenic scale	S([T,I,F])	Plithogenic evaluation criterion or category
6	(0.95, 0.05, 0.15)	0.90	Very High (VH)
5	(0.75, 0.25, 0.35)	0.75	High (H)
4	(0.50, 0.55, 0.55)	0.50	Medium or indeterminate (M)
3	(0.35, 0.65, 0.75)	0.32	Low (L)
2	(0.10, 0.75, 0.95)	0.13	Very Low (VL)
1	(1,0,1)	0	Contradictory (C)

Consequently, the data set is evaluated, which is formed entirely or partially by data with some degree of indeterminacy and contradiction. To do this, the plithogenic statistical method is used, which allows for the interpretation and organization of the data to reveal underlying patterns. For plithogenic statistical modeling, the methodology found in the analyzed reference materials has been reviewed [18].

On the other hand, the calculation of the chi-square statistic (χ^2) is a tool used to determine whether

there is a significant relationship between two categorical variables. It is particularly useful in association studies and allows for assessing whether the differences observed in a contingency table are due to chance or indicate a real relationship. For this, the calculation of χ^2 is carried out according to equation (1).

$$X^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \quad (1)$$

Where:

- O_{ij} is the observed frequency in cell i, j,
- E_{ij} is the expected frequency in the cell i, j.

To determine the critical value in a chi-square (χ^2) test, a significance level is first selected, typically set at 0.05 or 0.01. This significance level (α) defines the probability threshold used to evaluate the null hypothesis, indicating the maximum allowable error probability when rejecting it. Additionally, the degrees of freedom (df) are calculated, which allows the test to be adapted to the dimensions of the data using equation (2).

$$gl = (r - 1) \cdot (c - 1) \quad (2)$$

Where r represents the number of rows and c represents the number of columns in the contingency table. Once α and the degrees of freedom are determined, the chi-square critical values table is consulted to obtain the value that defines the threshold for accepting or rejecting the null hypothesis within the context of the test.

3 The Study

3.1 Analysis of the behavior of the frequencies of each variable.

The floriculture sector has emerged as one of the key industries in several countries, particularly in the floriculture sector of the parish of José Guango Bajo. However, despite its economic relevance, this sector faces significant challenges related to job stability, which is affected by various factors, including labor demand, hiring and retention policies, and the recruitment capacity of companies. These variables influence employability and job sustainability for the economically active population, making their analysis crucial for the formulation of effective strategies.

Table 2 presents the relationships between the independent and dependent variables in the floriculture sector. These relationships allow for the analysis of how variations between these variables impact job stability and employability within the sector. For the development of the statistical analysis, a sample of 120 respondents from three areas {administrative (4), post-harvest (36), and cultivation (80)} has been proposed.

Table 2: Relationships between the study variables. Source: Own elaboration.

	Independent variable	Dependent variable
Relationships between variables	Labor demand (VI-1)	Job stability (VD-2)
	Hiring and retention policies (VI-3)	
	Recruitment capacity (VI-4)	Economically active population (employed in the sector) (VD-5)

Previous studies have indicated that in seasonal sectors like floriculture, labor demand increases labor turnover, affecting the stability of workers. Table 3 shows the observed frequencies between labor demand and job stability, using plithogenic categories to classify the impact in different labor areas (see Table 1). Among these, the prevalence of indeterminate relationships is observed, caused by external factors with an impact on these variables.

Table 3: Frequency observed between the relationship VI-1 and VD-2. Source: Own elaboration.

Job demand/job stability	C	VH	H	M	L	VL	Total
Administrative (A)	1	0	1	2	0	0	**4**
Post-Harvest (PH)	9	3	5	13	4	2	**36**
Cultivation (C)	5	5	10	42	15	3	**80**
Total	**15**	**8**	**16**	**57**	**19**	**5**	**120**

Table 4 visualizes how policies affect stability in different labor areas of the floriculture sector. It also shows, in the contradictory zone, factors influencing an inverse relationship between hiring and retention policies, impacting labor turnover. Finally, Table 5 presents the observed frequencies between recruitment capacity and the economically active population in the sector, mostly classified as (VH).

Table 4: Frequency observed between the relationship VI-3 and VD-2. Source: Own elaboration.

Hiring and retention policies/job stability	**C**	**VH**	**H**	**M**	**L**	**VL**	**Total**
Administrative (A)	2	0	1	1	0	0	**4**
Post-Harvest (PH)	18	2	4	4	0	8	**36**
Cultivation (C)	25	4	11	30	8	2	**80**
Total	**45**	**6**	**16**	**35**	**8**	**10**	**120**

Table 5: Frequency observed between the relationship VI-4 and VD-5. Source: Own elaboration.

Recruitment capacity / economically active population	**C**	**VH**	**H**	**M**	**L**	**VL**	**Total**
Administrative (A)	0	3	1	0	0	0	**4**
Post-Harvest (PH)	9	1	7	10	6	3	**36**
Cultivation (C)	7	38	1	24	9	1	**80**
Total	**16**	**42**	**9**	**34**	**15**	**4**	**120**

To evaluate the relationship between the variables, hypotheses must be defined and expected frequencies calculated. Therefore, the chi-square test requires grouping similar categories to increase the expected frequencies and ensure the validity of the analysis. For example, instead of analyzing the categories "Very High" and "High" separately, they are grouped as (H+VH), and "Low" and "Very Low" are grouped as (L+VL). Similarly, the respondent groups "Administrative (A)" and "Post-Harvest (PH)" are combined into a single multigroup (see Figure 1), as they share similar characteristics, referred to as (A+PH).

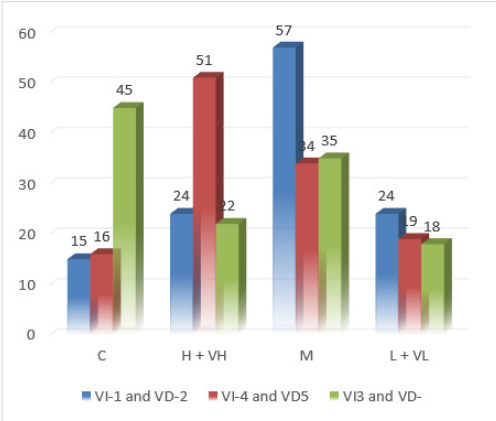

Figure 1: Variable frequencies using plithogenic multigroups. Source: Own elaboration.

In Figure 1, three relationships are observed, with high-value frequencies highlighted. Among them, the relationship (IV-1) and (DV-2) point to a plithogenic zone of indeterminacy, while the relationship (IV-4) and (DV-5) are located in a high-importance area. In contrast, the relationship (IV-3) and (DV-2) are found in a contradictory zone. These important zones are analyzed in the following sections (3.2, 3.3, and 3.4).

3.2 Relationship: Labor demand (VI-1) and job stability (VD-2).

Formulation of the plithogenic hypotheses:

- Null hypothesis (Ho): There is no relationship between labor demand and job stability in

the floriculture sector (false hypothesis).

- Alternative hypothesis (H1): Job stability in the floriculture sector increases directly with the increase in labor demand (true hypothesis).
- Alternative hypothesis (H2): The relationship between labor demand and job stability is unpredictable; even though demand increases, other factors influence the stable relationship between these variables (indeterminate hypothesis).
- Alternative hypothesis (H3): An increase in labor demand reduces job stability (contradictory hypothesis).

Once the hypotheses are defined, the observed and expected structured frequencies are calculated (see Tables 6 and 7). Finally, the chi-square test will be calculated to assess the discrepancy between the observed and expected values.

Table 6: Structured observed frequencies. Source: Own elaboration.

Job demand/job stability	**C**	**(H + VH)**	**Medium (M)**	**(L + VL)**	**Total**
Administrative + Post-Harvest (A+PH)	10	9	15	6	**40**
Cultivation (C)	5	15	42	18	**80**
Total	**15**	**24**	**57**	**24**	**120**

Table 7: Expected frequencies. Source: Own elaboration.

Job demand/job stability	**C**	**(H + VH)**	**Medium (M)**	**(L + VL)**	**Total**
Administrative + Post Harvest (A+PH)	5.00	8.00	19.00	8.00	**40**
Cultivation (C)	10.00	16.00	38.00	16.00	**80**
Total	**15.00**	**24.00**	**57.00**	**24.00**	**120**

To calculate the degrees of freedom (df), formula (2) is used, where for r=2 and c=4, df=3. Therefore, with a significance level of α=0.05, a critical value of 7.815 is reached. Since $X^2 = 9.701 > 7.815$, the null hypothesis is rejected and the alternative hypothesis is accepted.

In this case, the alternative H2 hypothesis, or indeterminate plithogenic hypothesis, is selected, as it has a weight of 47.50% {Medium (M)} over the other plithogenic categories in the analyzed sample (see Figure 1). Additionally, the selection of this hypothesis is supported by the restriction for hiring new staff, determined by a highly specialized selection profile. The available vacancies are mainly aimed at professionals with minimum experience and specific qualifications, which limits opportunities for recent graduates and other candidates. This factor emerges as one of the most relevant variables in the studied relationship, highlighting the impact of the selection profile on employment dynamics in the sector.

3.3 Hiring and retention policies (VI-3) and job stability (VD-2).

Formulation of the plithogenic hypothesis:

- Null hypothesis (Ho): Labor stability in the floriculture sector does not depend on hiring and retention policies (false hypothesis).
- Alternative hypothesis (H1): Hiring and retention policies increase labor stability (true hypothesis).
- Alternative hypothesis (H2): A clear relationship between hiring and retention policies and labor stability cannot be predicted (indeterminate hypothesis).
- Alternative hypothesis (H3): Hiring and retention policies focused on specific areas decrease labor stability (contradictory hypothesis).

Once the hypotheses are defined, the calculation of the structured observed and expected frequencies is carried out, as shown in Tables 8 and 9. Finally, the Chi-square test is applied to evaluate the discrepancy between the structured observed and expected values, in order to verify the proposed hypothesis.

Table 8: Structured observed frequencies. Source: Own elaboration.

Hiring and retention policies/job stability	C	(H + VH)	Medium (M)	(L + VL)	Total
Administrative + Post-Harvest (A+PH)	20	7	5	8	**40**
Cultivation (C)	25	15	30	10	**80**
Total	**45**	**22**	**35**	**18**	**120**

Table 9: Expected frequencies. Source: Own elaboration.

Hiring and retention policies/job stability	C	(H + VH)	Medium (M)	(L + VL)	Total
Administrative + Post Harvest (A+PH)	15.00	7.33	11.67	6.00	**40**
Cultivation (C)	30.00	14.67	23.33	12.00	**80**
Total	**45.00**	**22.00**	**35.00**	**18.00**	**120**

For df= 3 and α=0.05, a critical value of 7.815 is obtained, and the analysis of the samples results in X^2=9.237, which is greater than 7.815. Therefore, the null hypothesis is rejected, and the alternate hypothesis is accepted. In this case, the H3 or contradictory plithogenic hypothesis is selected, which has a weight of 42.50% (C) compared to the other plithogenic categories in the analyzed sample (see Figure 1).

This is due to the high concentration of employment in the cultivation area, which leads to an overload of work and increased labor turnover. This highlights that, although hiring and retention policies exist, their effectiveness could be compromised if employment is concentrated in a single sector, potentially leading to labor instability.

3.4 Recruitment capacity (VI-4) and economically active population (VD-5).

Formulation of the plithogenic hypothesis:

- Null hypothesis (Ho): The recruitment capacity in floriculture companies does not influence the number of economically active population employed in the sector (false hypothesis).
- Alternative hypothesis (H1): The higher the recruitment capacity in floriculture companies, the greater the proportion of the economically active population employed in the sector (true hypothesis).
- Alternative hypothesis (H2): The recruitment capacity and the employment of the economically active population in the sector do not show a constant relationship due to external factors (indeterminate hypothesis).
- Alternative hypothesis (H3): An increase in the recruitment capacity in the floriculture sector decreases the employment of the economically active population (contradictory hypothesis).

Once the hypotheses are established, the structured observed and expected frequencies are calculated, as detailed in Tables 10 and 11. Subsequently, the Chi-square test is performed to evaluate the discrepancy between the observed and expected values, in order to analyze the validity of the hypotheses proposed in the analyzed relationship.

Table 10: Structured observed frequencies. Source: Own elaboration.

Recruitment capacity / economically active population	C	(H + VH)	Medium (M)	(L + VL)	Total
Administrative + Post-harvest (A+PH)	9	12	10	9	**40**
Cultivation (C)	7	39	24	10	**80**
Total	**16**	**51**	**34**	**19**	**120**

Table 11: Expected frequencies. Source: Own elaboration.

Recruitment capacity / economically active population	C	(H + VH)	Medium (M)	(L + VL)	Total
Administrative + Post Harvest (A+PH)	5.33	17.00	11.33	6.33	**40**
Cultivation (C)	10.67	34.00	22.67	12.67	**80**
Total	**16.00**	**51.00**	**34.00**	**19.00**	**120**

For a critical value of 7.815, a value of $X^2 = 7.907 > 7.815$ was calculated with a significance level of $\alpha = 0.05$. Therefore, the null hypothesis is rejected and the alternative hypothesis is accepted. In this case, the H1 or true plithogenic hypothesis is selected, as it holds a weight of 42.50% (H + VH) over the other plithogenic categories in the analyzed sample (see Figure 1). Furthermore, it is established that the hiring capacity is effective in improving job stability for the economically active population in the floricultural sector. Thus, initiatives that promote this capacity are supported as a strategy for strengthening labor within the sector.

3.5 Analysis of the relationships between variables in the floricultural sector.

The relationship between labor demand and job stability was evaluated under the indeterminate plithogenic hypothesis (H2), which was accepted. The results indicate that fluctuations in labor demand, influenced by external factors such as economic changes and government policies, have a direct impact on job stability in the floricultural sector. This increases unemployment and underemployment rates, emphasizing the need to adjust hiring strategies specifically aimed at including new professionals and reducing barriers for recent graduates. It is also essential to implement measures to improve job security and working conditions within the sector.

On the other hand, the relationship between hiring and retention policies and job stability was evaluated under the contradictory plithogenic hypothesis (H3), which was also accepted. The results show that, although hiring and retention policies exist, the high concentration of employment in specific areas of the floricultural sector generates job instability. This is due to work overload and the limited diversification of job opportunities. Thus, it becomes necessary to reconsider current policies to distribute employment more equitably and promote greater job stability in the sector.

Finally, regarding the relationship between hiring capacity and the economically active population, the true plithogenic hypothesis (H1) was confirmed. It was observed that an increase in hiring capacity supports the integration of a greater proportion of the economically active population, thereby improving job stability. This highlights the importance of strengthening hiring capacities in the floricultural sector to reduce unemployment and underemployment rates by providing greater job opportunities and improving workers' conditions.

4. Conclusion

The plithogenic statistical analysis has revealed that fluctuations in labor demand within the floricultural sector negatively impact job stability by increasing unemployment and underemployment rates. Despite the existence of hiring and retention policies, their effectiveness is contradictory, suggesting the need to reconsider these policies to diversify job opportunities and improve employment distribution. Furthermore, it has been confirmed that greater hiring capacity is linked to higher integration of the economically active population. This underscores the need to strengthen hiring capacities as a strategy to improve working conditions and reduce unemployment in the sector.

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A Neutrosophic Approach to Analyzing Determining Factors in RIMPE Tax Collection in Ecuador

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Abstract. This study analyzed the factors influencing tax collection under the RIMPE regime in Ecuador, applying neutrosophic logic to address the uncertainty and ambiguity inherent in decision-making within fiscal analysis. The employed methodology included an adaptation of the ELECTRE I method with bipolar neutrosophic numbers, enabling the integration of diverse perspectives into the decision-making process. The results obtained highlighted the simplicity of the regime and the control of the SRI as the most influential factors in fiscal revenue collection. The approach applied in the study facilitates a more accurate and efficient evaluation of fiscal policies, promoting greater equity and efficiency in the tax system.

Keywords: fiscal revenue collection, RIMPE regime, neutrosophic logic, ELECTRE I, multi-criteria decision-making.

1 Introduction

Tax revenue plays a central role in the economic and social development of any nation, serving as the primary source of funding for essential public services such as healthcare, education, and security. In Ecuador, the Simplified Tax Regime for Entrepreneurs and Popular Businesses (RIMPE) emerges as a fiscal policy designed to simplify tax compliance and promote the economic formalization of traditionally marginalized sectors. Implemented in 2022, this regime replaced previous tax systems, such as the Simplified Tax Regime (RISE) and the Microenterprise Regime (RIM), intending to revitalize the economy and ensure a more equitable distribution of the tax burden. However, despite its significance, the factors influencing its effectiveness remain poorly understood, highlighting the need for a more in-depth analysis. [1]

Although RIMPE has gained attention for its potential to integrate small businesses into the tax system, the conditions determining its performance across the country's provinces may vary, offering opportunities for improvement in this indicator. In particular, the COVID-19 pandemic, with its devastating economic effects, has raised questions about how the exceptional circumstances of recent years have affected taxpayers' ability to adapt to this new scheme [2]. Moreover, Ecuador's socioeconomic and demographic heterogeneity suggests that fiscal revenue dynamics can vary significantly, requiring an analytical approach capable of capturing these differences and proposing effective solutions.[3]

In this context, informed decision-making becomes an indispensable tool for understanding and optimizing tax revenue collection. Multi-criteria decision-making (MCDM) methods play a critical role in this regard, as they integrate multiple factors and variables into an analytical framework that facilitates the identification of patterns, significant relationships, and priority areas for intervention [4], [5]. However, uncertainty is a constant in real-world scenarios, particularly in fiscal policy, where data are often incomplete, ambiguous, or contradictory [6]. In the case of RIMPE, such uncertainties can manifest in diverse ways, ranging from discrepancies in tax information to unforeseen impacts of external policies.

Neutrosophy, developed by philosopher and mathematician Florentin Smarandache in the late 20th century, has emerged as a significant theoretical framework for tackling uncertainty and ambiguity in decision-making across diverse domains, including science, business, and industry. This conceptual

framework posits that truth, untruth, and indeterminacy can coexist inside a single statement or argument, so encapsulating the inherent complexity present in many decision-making contexts [7,8].

Neutrosophic logic, by expanding the analytical capabilities of traditional methods, offers a robust framework for dealing with the limitations inherent in the real world. Its application in this study not only allows for a more accurate representation of reality but may also open up new opportunities for research and policy implementation.

The primary objective of this study is to identify the key factors influencing tax collection within the RIMPE regime in Ecuador using neutrosophic logic. In doing so, the study aims to provide a solid knowledge base to enhance the efficiency and equity of the tax system, contributing to the strengthening of public finances and the country's economic development.

To conduct this analysis, the study proposes an adaptation of the ELECTRE I method, expanded through a neutrosophic structure based on the incorporation of bipolar neutrosophic numbers [9]. This approach is well-suited to manage the uncertainties and ambiguities inherent in the field of study more precisely and effectively. The implementation of bipolar neutrosophic numbers enables the integration of diverse perspectives and levels of indeterminacy into the decision-making process [10], making it an essential tool for addressing the complexity of the dynamics associated with tax collection in Ecuador.

2.1 Preliminaries

Definition 1[11,12, 13]. Let X be a space of points or objects, with its generic elements represented as x. A single-valued neutrosophic set (SVNS) A in X is characterized by three membership functions: $T_A(x)$, indicating truth membership; $I_A(x)$ defining indeterminacy membership; and $F_A(x)$, indicating falsehood membership. Consequently, an SVNS A can be expressed as $A = \{x, T_{A(x)}, I_{A(x)}, F_{A(x)x} \in X\}$, where $T_{A(x)}, I_{A(x)}, F_{A(x)} \in [0,1]$ for each x in X. Furthermore, these membership functions satisfy the condition $0 \leq T_{A(x)} + I_{A(x)} + F_{A(x)} \leq 3$.

Definition 2 [14, 15, 16]. A bipolar neutrosophic set A in X is defined as an object of the form.

$$\tilde{A} = \{x, \langle T_A^+(x), I_A^+(x), F_{\tilde{A}}^+(x), T_{\tilde{A}}^-(x), I_{\tilde{A}}^-(x), F_{\tilde{A}}^-(x)\rangle \big| x \in X\}, \quad (1)$$

where the functions $T_A^+(x), I_A^+(x), F_{\tilde{A}}^+(x): X \to [0,1]$ represent the positive values of truth, indeterminacy, and falsehood membership, respectively, while $T_{\tilde{A}}^-(x), I_{\tilde{A}}^-(x), F_A^-(x): X \to [-1,0]$ correspond to their negative values.

The method used paper considers the existence of a set $S = \{S_1, S_2, \cdots, S_m\}$ representing m selection alternatives, and a set $T = \{T_1, T_2, \cdots, T_n\}$ denoting n attributes or evaluation criteria. Additionally, the weight vector associated with the evaluation criteria is denoted as $W = \left[w_{1w_2} \cdots w_n\right]^T$ where $0 \leq wj \leq 1$ and $\sum j = 1_{nwj} = 1$. Assuming the decision-maker assigns a rating to each alternative Si, (i=1,2, ⋯,m) concerning each attribute Tj, (j=1,2,⋯,n) in the form of bipolar neutrosophic sets (BNSs), which represent truth, falsity, and indeterminacy on positive and negative scales[17, 18].

The adapted ELECTRE I method incorporates these evaluations, leveraging concordance and discordance indices to rank alternatives [19]. This approach enhances the analysis by addressing ambiguity and complexity, providing a nuanced evaluation framework for fiscal policy.

3 Methods

3.1 Bipolar ELECRTE I method

The method used in this paper can be outlined shown in Figure 1.

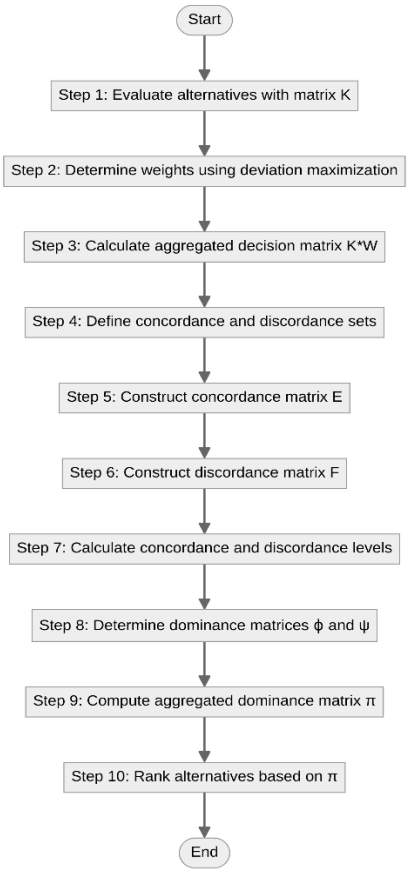

Figure 1. Decision-Making Process

Step 1. Each alternative is evaluated based on multiple criteria. The evaluation of each alternative concerning each criterion is represented using BNSs, which can be expressed in the decision matrix as:

$$K = [k_{ij}]_{m\times n} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix}$$

Each entry $k_{ij} =< T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} >$ is characterized as follows:

$T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}$, represent the positive truth, indeterminacy, and falsehood membership degrees, respectively, while $T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-}$ represent the negative truth, indeterminacy, and falsehood membership degrees. These values satisfy the constraints $T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+} \in [0,1]$, y $0 \leq T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} \leq 6$, where i=1,2,3,…,m and j=1,2,3,…,n.

Step 2. When the weights of the criteria are not evenly distributed and their values are unknown to the decision-maker, the deviation maximization method is employed to determine the unspecified criterion weights. Consequently, the weight of the attribute T_j is calculated as follows:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|\right)^2}}, \quad (2)$$

And the normalized weight of the attribute T_j is determined as described in Equation (3):

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|}{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|\right)}. \quad (3)$$

Step 3: The weighted bipolar neutrosophic aggregated decision matrix is computed by multiplying the attribute weights with the aggregated decision matrix as follows:

$$K * W = [k_{ij}^{w_j}]_{m\times n} = \begin{bmatrix} k_{11}^{w_1} & k_{12}^{w_2} & \cdots & k_{1n}^{w_n} \\ k_{21}^{w_1} & k_{22}^{w_2} & \cdots & k_{2n}^{w_n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ k_{m1}^{w_1} & k_{m2}^{w_2} & \cdots & k_{mn}^{w_n} \end{bmatrix} \quad (4)$$

Where

$$k_{ij}^{w_j} =< T_{ij}^{w_j+}, I_{ij}^{w_j+}, F_{ij}^{w_j+}, T_{ij}^{w_j-}, I_{ij}^{w_j-}, F_{ij}^{w_j-} >$$
$$=< 1-(1-T_{ij}^{+})^{w_j}, (I_{ij}^{+})^{w_j}, (F_{ij}^{+})^{w_j}, -(-T_{ij}^{-})^{w_j}, -(-I_{ij}^{-})^{w_j}, -(1-(1-(-F_{ij}^{-}))^{w_j}) >$$

Step 4: The bipolar neutrosophic concordance sets E_{xy} and discordance sets F_{xy} are defined as:

$$E_{xy} = \{1 \le j \le n | \rho_{xj} \ge \rho_{yj}\}, x \ne y, x, y = 1,2,\cdots,m,$$
$$F_{xy} = \{1 \le j \le n | \rho_{xj} \le \rho_{yj}\}, x \ne y, x, y = 1,2,\cdots,m, \quad (5)$$

where $\rho_{ij} = T_{ij}^{+} + I_{ij}^{+} + F_{ij}^{+} + T_{ij}^{-} + I_{ij}^{-} + F_{ij}^{-}, i = 1,2,\cdots,m$ and $j = 1,2,\cdots,n$

Step 5: The bipolar neutrosophic concordance matrix E is constructed as:

$$E = \begin{bmatrix} - & e_{12} & \cdot & \cdot & \cdot & e_{1m} \\ e_{21} & - & \cdot & \cdot & \cdot & e_{2m} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ e_{m1} & e_{m2} & \cdot & \cdot & \cdot & - \end{bmatrix}$$

where the bipolar concordance indices $e_{xy}{}'s$ are calculated using (6):

$$e_{xy} = \sum_{j\in E_{xy}} w_j \quad (6)$$

Step 6: The bipolar neutrosophic discordance matrix F is constructed as:

$$F = \begin{bmatrix} - & f_{12} & \cdot & \cdot & \cdot & f_{1m} \\ f_{21} & - & \cdot & \cdot & \cdot & f_{2m} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ f_{m1} & f_{m2} & \cdot & \cdot & \cdot & - \end{bmatrix},$$

where the bipolar discordance indices $f_{xy}{}'s$ are determined by:

$$f_{xy} = \frac{\max\limits_{j\in F_{xy}} \sqrt{\frac{1}{6n}\left\{\begin{matrix}(T_{xj}^{w_j+}-T_{yj}^{w_j+})^2+(I_{xj}^{w_j+}-I_{yj}^{w_j+})^2+(F_{xj}^{w_j+}-F_{yj}^{w_j+})^2+ \\ (T_{xj}^{w_j-}-T_{yj}^{w_j-})^2+(I_{xj}^{w_j-}-I_{yj}^{w_j-})^2+(F_{xj}^{w_j-}-F_{yj}^{w_j-})^2\end{matrix}\right\}}}{\max\limits_{j} \sqrt{\frac{1}{6n}\left\{\begin{matrix}(T_{xj}^{w_j+}-T_{yj}^{w_j+})^2+(I_{xj}^{w_j+}-I_{yj}^{w_j+})^2+(F_{xj}^{w_j+}-F_{yj}^{w_j+})^2+ \\ (T_{xj}^{w_j-}-T_{yj}^{w_j-})^2+(I_{xj}^{w_j-}-I_{yj}^{w_j-})^2+(F_{xj}^{w_j-}-F_{yj}^{w_j-})^2\end{matrix}\right\}}} \quad (7)$$

Step 7: The levels of concordance and discordance are calculated to rank the alternatives. The bipolar neutrosophic concordance level $\hat{e}$ is computed as:

$$\hat{e} = \frac{1}{m(m-1)} \sum_{\substack{x=1, \\ x \neq y}}^{m} \sum_{\substack{y=1, \\ y \neq x}}^{m} e_{xy} \tag{8}$$

Similarly, the bipolar neutrosophic discordance level $\hat{f}$ is given by:

$$\hat{f} = \frac{1}{m(m-1)} \sum_{\substack{x=1, \\ x \neq y}}^{m} \sum_{\substack{y=1, \\ y \neq x}}^{m} f_{xy} \tag{9}$$

Step 8: The bipolar neutrosophic concordance dominance matrix ϕ and discordance dominance matrix ψ, are determined as:

$$\phi = \begin{bmatrix} - & \phi_{12} & \cdot & \cdot & \cdot & \phi_{1m} \\ \phi_{21} & - & \cdot & \cdot & \cdot & \phi_{2m} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \phi_{m1} & \phi_{m2} & \cdot & \cdot & \cdot & - \end{bmatrix},$$

$$\psi = \begin{bmatrix} - & \psi_{12} & \cdot & \cdot & \cdot & \psi_{1m} \\ \psi_{21} & - & \cdot & \cdot & \cdot & \psi_{2m} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \psi_{m1} & \psi_{m2} & \cdot & \cdot & \cdot & - \end{bmatrix}$$

,

Where

$$\phi_{xy} = \begin{cases} 1, & \text{if} e_{xy} \geq \hat{e}, \\ 0, & \text{if} e_{xy} < \hat{e}. \end{cases} \tag{10}$$

$$\psi_{xy} = \begin{cases} 1, & \text{if} f_{xy} \leq \hat{f}, \\ 0, & \text{if} f_{xy} > \hat{f}. \end{cases} \tag{11}$$

Step 9: The aggregated bipolar neutrosophic dominance matrix π is computed by combining the matrices ϕ and ψ, as:

$$\pi = \begin{bmatrix} - & \pi_{12} & \cdot & \cdot & \cdot & \pi_{1m} \\ \pi_{21} & - & \cdot & \cdot & \cdot & \pi_{2m} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \pi_{m1} & \pi_{m2} & \cdot & \cdot & \cdot & - \end{bmatrix}$$

Where

$$\pi_{xy} = \phi_{xy}\psi_{xy} \tag{12}$$

Step 10: Finally, the alternatives are ranked based on the dominance values π_{xy}.
For each pair of alternatives S_x and S_y:
(a) A single arrow from S_x to S_y indicates S_x is preferred.
(b) Two arrows indicate indifference between S_x and S_y.

(c) No arrow signifies incomparability between S_x and S_y

4 Results

Based on the conducted literature review, four primary factors were identified as pivotal for determining the most influential ones regarding fiscal revenue collection under the RIMPE regime in Ecuador. The elements considered for evaluation include:

S1. Simplicity and clarity of the regime: The ease of understanding and applying tax regulations under the RIMPE framework emerges as a critical factor. A straightforward system that avoids overburdening microenterprises is likely to enhance compliance, thereby improving fiscal revenue collection.

S2. Macroeconomic conditions: Variables such as inflation, GDP growth, and employment levels significantly affect the purchasing power and income-generating capacity of microenterprises. A stable or growing economy generally correlates with increased fiscal revenues.

S3. Fiscal incentives and benefits: Tax exemptions, discounts, or other fiscal advantages provided under the RIMPE regime can influence microenterprises' willingness to register and fulfill their tax obligations, ultimately affecting revenue collection.

S4. Control and oversight by the Internal Revenue Service (SRI): The SRI's ability to monitor, detect, and penalize non-compliance is essential. Effective oversight and an appropriate sanctions policy contribute to higher compliance rates and improved fiscal revenue performance.

For the evaluation of these factors, a series of criteria will be considered.

C1. Impact on revenue collection efficiency: This criterion evaluates the extent to which each alternative contributes to maximizing fiscal revenue within the RIMPE regime.

C2. Ease of implementation: Measures the technical and operational feasibility of implementing each alternative, including the clarity of regulations, administrative costs, and the ability of microenterprises to adapt to the proposed changes.

C3. Tax equity: Analyzes how each alternative affects the fairness of the tax system, ensuring that fiscal burdens are proportionally distributed among microenterprises based on their economic capacity.

C4. Taxpayer acceptance: Assesses the perception and willingness of microenterprises to comply with the proposed measures, taking into account factors such as the simplicity of the regime and the fiscal benefits offered.

Based on the previously established criteria, the experts assessed the factors analyzed in the study. Table 1 presents the bipolar numerical decision matrix derived for this purpose.

Table 1: Decision matrix.

	C1	C2	C3	C4
S1	(0.3, 0.4, 0.2, -0.4, -0.3, -0.7)	(0.3, 0.5, 0.2, -0.3, -0.8, -0.5)	(0.8, 0.5, 0.7, -0.3, -0.4, -0.3)	(0.4, 0.4, 0.4, -0.7, -0.5, -0.4)
S2	(0.3, 0.4, 0.1, -0.5, -0.5, -0.5)	(0.3, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.4, 0.2, 0.5, -0.6, -0.3, -0.1)	(0.2, 0.5, 0.2, -0.5, -0.4, -0.2)
S3	(0.5, 0.1, 0.5, -0.3, -0.4, -0.4)	(0.1, 0.2, 0.3, -0.6, -0.2, -0.4)	(0.2, 0.3, 0.5, -0.3, -0.2, -0.3)	(0.4, 0.7, 0.3, -0.3, -0.5, -0.4)
S4	(0.5, 0.5, 0.3, -0.2, -0.1, -0.3)	(0.7, 0.4, 0.3, -0.1, -0.3, -0.4)	(0.6, 0.3, 0.6, -0.5, -0.4, -0.2)	(0.3, 0.3, 0.2, -0.1, -0.3, -0.1)

The weight vector for the evaluation criteria is expressed as $w = (0.3, 0.25, 0.2, 0.25)$. By integrating these weights with the initial decision matrix, the normalized matrix is derived. This process involves scaling each value in the decision matrix according to the corresponding criterion weight, as outlined in Equation (4). Based on this information, the bipolar neutrosophic concordance sets and discordance sets can be established, as presented in Tables 2 and 3.

Table 2: Bipolar neutrosophic concordance sets

$\mathbf{E_{xy}}$	1	2	3	4
$\mathbf{E_{1y}}$	{-}	{1, 2, 3}	{3}	{3}

$\mathbf{E_{xy}}$	1	2	3	4
$\mathbf{E_{2y}}$	{4}	{-}	{0}	{0}
$\mathbf{E_{3y}}$	{1, 2, 4}	{1, 2, 3, 4}	{-}	{3}
$\mathbf{E_{4y}}$	{1, 2, 4}	{1, 2, 3, 4}	{1, 2, 4}	{-}

Table 3. Bipolar neutrosophic discordance sets

$\mathbf{F_{xy}}$	1	2	3	4
$\mathbf{F_{1y}}$	-	{0}	{1, 2, 3, 4}	{1, 2, 3, 4}
$\mathbf{F_{2y}}$	{1, 2, 3, 4}	-	{1, 2, 3, 4}	{4}
$\boldsymbol{F_{3y}}$	{1,2}	{3}	-	{1, 2, 4}
$\boldsymbol{F_{4y}}$	{0}	{0}	{3}	-

Based on the collected data and applying equations (6) and (7), the calculation and construction of the bipolar neutrosophic concordance matrix E_{xy} and the bipolar neutrosophic discordance matrix F_{xy} are performed. These matrices provide a quantitative representation of the concordance and discordance relationships among the evaluated alternatives, thereby facilitating the decision-making process.

$$E_{xy} = \begin{bmatrix} - & 0.75 & 0.20 & 0.20 \\ 0.25 & - & 0.00 & 0.00 \\ 0.80 & 1.00 & - & 0.20 \\ 0.80 & 1.00 & 0.80 & - \end{bmatrix} \qquad F_{xy} = \begin{bmatrix} - & 0.67 & 1.00 & 1.00 \\ 1.00 & - & 1.00 & 0.00 \\ 1.00 & 0.00 & - & 1.00 \\ 0.41 & 0.00 & 0.444 & - \end{bmatrix}$$

Based on the calculations, the bipolar neutrosophic concordance and discordance levels were determined as 0.5 and 0.71, respectively. By applying equations (10) - (11), the dominance matrices for bipolar neutrosophic concordance and discordance were generated. These matrices support the ranking of alternatives and the selection of the most suitable strategy based on the established criteria.

$$\pi = \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 1 & - & 0 \\ 1 & 1 & 1 & - \end{bmatrix}$$

According to the results, an arrow from S1 to $S2$ $\pi_{\{12\}} = 1$ was observed, indicating a preference for *Simplicity and clarity of the regime* over *Macroeconomic conditions*. This result highlights the greater importance of regulatory simplicity compared to macroeconomic factors. Additionally, *Control and oversight by the SRI* demonstrated dominance over all other factors $\pi_{\{41\}} = \pi_{\{42\}} = \pi_{\{43\}} = 1$, highlighting its relevance as a key criterion.

The analysis also revealed that S2 and S3 lacked outgoing arrows toward other factors, indicating a lower influence compared to the remaining factors. These findings suggested that effective oversight was the most influential factor, emphasizing the critical role of efficient supervision in improving fiscal revenue collection.

5 Conclusion

The conducted study identified the most relevant factors influencing fiscal revenue collection under the RIMPE regime in Ecuador, highlighting the simplicity of the regulatory framework and the control exercised by the Internal Revenue Service (SRI) as key determinants. Regulatory simplicity was preferred over macroeconomic conditions, underscoring the importance of a clear and accessible framework to encourage tax compliance. The application of the ELECTRE I method, enhanced with bipolar neutrosophic numbers, proved effective in addressing the uncertainty and complexity inherent in fiscal analysis, enabling a more precise representation of relationships between factors. This approach demonstrated its usefulness in prioritizing strategies to improve fiscal revenue collection and promote equity within the tax system. Further integration of advanced multi-criteria methods is recommended to optimize tax policies, taking into account regional dynamics and the impact of changing socioeconomic contexts.

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Integration of Fuzzy Set Theory and Neutrosophic Sets for Qualitative and Quantitative Analysis in Inclusive Service to Native Communities of Ucayali

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Abstract. This research focuses on the examination of public records and their potential to deliver inclusive service to the native populations of Ucayali, a key difficulty in the administration of public services in varied and culturally rich areas. The study tackles a gap in the current literature, where standard techniques do not effectively examine how transparency and sufficiency of services influence inclusive service quality. Using methodologies based on neutrosophic set theory, the research presents a novel way to examine the efficacy and accessibility of public registry services in a particular cultural setting. The findings reveal that the combination of Management Transparency (TG) and Service Adequacy (ADS) has a considerable beneficial influence on Inclusive Service Quality (CSI). This research underlines the necessity of tackling both aspects concurrently to enhance service quality. The contribution of the study is twofold: first, it advances theoretical knowledge by integrating advanced methods for evaluating public services in multicultural contexts; Second, it provides concrete practical recommendations, such as administrative simplification, cultural training of staff, and infrastructure improvement, that can be implemented to make public registry services more accessible and effective for the native communities of Ucayali.

Keywords: Native Communities, Set Theoretical Methods, SQA, Fsqca, Neutrosophic Sets, Public Records.

1 Introduction

In the context of public administration and access to important services, the inclusion of Aboriginal communities in the public records system is of crucial relevance. This research focuses on the assessment of the efficacy and degree of inclusion of public records services for the native populations of Ucayali, utilizing methodologies based on neutrosophic set theory. This problem is significant given that effective inclusion in public registers not only provides access to rights and services but is also a key component for social development and administrative justice [1]. As public policies become more integrative, there is a need to understand how they are being implemented in specific contexts and whether these policies are truly reaching the populations they are targeted at [2]. Historically, public records have grown from basic handwritten annotations to more technologically advanced systems [3]. In the past, a lack of infrastructure and resources hampered access and participation, particularly in distant areas and for indigenous groups. With the growth of technology and rising awareness of the need for inclusive services, several nations have undertaken attempts to upgrade these systems [4]. However, despite these efforts, persistent impediments continue to impede the capacity of Native people to receive these services successfully [5]. Ucayali, in particular, represents an interesting case due to its cultural diversity and specific challenges in terms of accessibility and adequacy of public records services [6].

The central problem of this study can be formulated as follows: How can the effectiveness of public records be evaluated in terms of their inclusive service for the native communities of Ucayali, considering the complexity and diversity of their needs? Despite attempts to increase inclusion and access, existing evaluations frequently do not effectively reflect the lived experience of these populations and the unique hurdles they confront [7]. This work tackles this gap by employing approaches based on neutrosophic set theory to give a more thorough and nuanced evaluation. The aims of this research are, first of all, to assess the performance of the public records service in terms of inclusion and accessibility for the local populations of Ucayali using neutrosophic methodologies. Second, it aims to identify particular challenges that impact these populations and make practical solutions to promote inclusion and access. Finally, the purpose is to present an assessment methodology that may be utilized in other comparable situations to enhance the quality and equality of public services.

2. Preliminaries.

2.1. Public Records.

Public records are a crucial part of contemporary administration, representing rights, properties, and transactions that shape people's everyday lives. They serve as a formal framework for encapsulating and consolidating various aspects of society, such as property ownership and birth registration. Public records are essential for preventing confusion and promoting legal clarity, as they help in transferring rights and preventing legal confusion [8].

The history of public records dates back to ancient civilizations, with recording technologies evolving. However, these systems often face challenges such as lack of access, corruption, and inefficiency. To address these issues, it is essential to study how public records can be improved, including the use of emerging technologies like blockchain and digitization of documents. However, the adoption of these solutions must be carefully regulated to prevent exclusion or inequity [9].

Inclusion of all individuals, particularly those living in disadvantaged or rural regions, is a key feature of public records. Policies should focus on reducing obstacles that hinder individuals from obtaining registration services, ensuring no gaps perpetuate inequality. Data protection and secrecy are also crucial aspects of public records, and comprehensive procedures must be developed to secure information from illegal access and misuse. Collaboration between government bodies and the private sector is essential for the efficacy of public records. This comprehensive strategy should include the introduction of new technology, inclusion, and privacy.

2.2. Complexity and causality theory and neutrosophic sets.

Relationships between variables are typically more difficult than they appear and often express via patterns that are neither simple nor linear, as complexity theory argues [10]. This indicates that the same element might have distinct consequences depending on the context in which it happens. In this theory, three fundamental concepts stand out: conjunction, equifinality, and causal asymmetry:

- **Conjunction:** This concept focuses on the notion that numerous elements must work together to achieve an outcome, rather than working alone. In other words, the result is usually the result of the combination of different preconditions that interact with each other.
- **Equifinality:** According to this principle, a system can reach the same final state through different paths and initial conditions. That is, there is no single way to reach a certain result; Different trajectories can lead to the same endpoint.
- **Causal asymmetry:** This principle states that while certain conditions can contribute to the occurrence of a result, the absence of those conditions does not guarantee that the result will not occur

This shows that the relationship between factors is complex and non-linear. These principles emphasize that the connection between conditions and outcomes is complex and not always predictable. Furthermore, neutrosophy offers a valuable perspective for understanding these complicated relationships by incorporating the indeterminacy and uncertainty that are inherent to social phenomena [11]. Neutrosophic set theory, with its ability to handle ambiguity, provides a more nuanced and deeper view of causality in these complex contexts.

2.3. Neutrosophic Liker Scales

Surveys employing neutrosophy Likert scales [12, 13, 14] proficiently assess the variability of opinions and their impact on public policy and social discourse, delineating areas of consensus, dissent, and ambivalence.

We give the essential definitions and notions pertaining to neutrosophic sets and single-valued neutrosophic sets.

Definition 1 ([15]). Let U be a discursive universe. N ={(x , T (x), I (x), F (x)): $x \in U$ } is a neutrosophic set, denoted by a truth membership function, $TN : U \rightarrow]0-, 1+[$; an indeterminacy membership function, $IN : U \rightarrow]0-, 1+[$; and a falsity membership function, $FN : U \rightarrow]0-, 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the universe of discourse U

Definition 2 ([16, 17]). Let U be a discursive universe. A single-valued neutrosophic set is defined as N ={(x , T (x), I (x), F (x)): $x \in U$ }, which is identified by a truth membership function, $TN : U \rightarrow [0, 1]$; indeterminacy membership function, $IN : U \rightarrow [0, 1]$; and falsity membership function, $FN : U \rightarrow [0, 1]$, with $0 \leq TN$ (x)+ IN (x)+ FN (x)≤ 3

Responses are categorized based on the aggregate of the True, Indeterminate, and False components utilizing neutrosophic scales with single value neutrosophic sets.:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are derived due to the frequent incompleteness or contradictions in the opinions. This classification is a benefit of employing neutrosophic approaches, as it facilitates a more nuanced comprehension of varying degrees of truth, indeterminacy, and falsity in responses.

3. Proposed framework

Each stage of the diagram is accompanied by a brief explanation that facilitates the general understanding of the process. The steps involved in the proposed methodology are broken down below:

Definition of the result: The first stage involves identifying and precisely describing the specific phenomenon, event, or condition that you want to investigate. This phase is essential because it establishes the focus and framework for all subsequent analyses. Clearly defining the outcome not only focuses the study on what you want to examine but also provides a solid foundation on which to build the subsequent stages of the analysis.

Development of neutrosophic Likert scales: In this phase, neutrosophic Likert scales are designed to measure both the result and the associated variables. Unlike traditional Likert scales, which use a fixed range of values (e.g., 1 to 5) to evaluate responses, neutrosophic scales introduce an additional dimension by including elements of truth, indeterminacy, and falsehood. Each option on the scale is expressed by a triplet (T, I, F), where T represents the degree of truth, I the degree of indeterminacy, and F is the degree of falsehood. This approach provides a more nuanced and detailed interpretation of participants' responses and attitudes, allowing the complexity of their opinions to be captured more accurately.

Data collection: The final stage consists of collecting relevant data on the cases in question, using a variety of indicators or measures that are related to the defined outcome. It is essential that the data be complete and accurately reflect the variables being studied. For this collection, neutrosophic Likert scales will be used in questionnaires and surveys. This method not only provides a richer data set, but also allows the complexity of respondents' opinions and attitudes to be captured in a more sophisticated way. The use of neutrosophic scales facilitates a deeper and more detailed understanding of the perceptions and responses of the participants, helping to unravel the complexity inherent to the phenomenon investigated.

Fuzzification: Finally, the obtained neutrosophic sets are transformed into equivalent fuzzy sets, following the procedure described in [18]. This step is essential for subsequent analysis, allowing you to handle the uncertainty and ambiguity inherent in the collected data. Be $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$a NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$,where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the similarity equation proposed in,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

Since the range of the similarity measure function is the unit interval [0,1], μA(x) ∈ [0,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [0,1] and therefore satisfies the property of a membership function of a fuzzy set (FS).

Analysis: Conduct fsQCA to determine the combinations of factors or conditions linked to the presence or extent of the outcome. The fsQCA software for Windows is utilized for data processing. [19].

The configuration's validity is assessed by measuring consistency and coverage values. Consistency quantifies the reliability of the pathways in generating the intended outcome. Coverage denotes the extent to which the outcome is elucidated by this configuration of paths. [20]:

$$Consistency\ (Y_i \leq\ X_i) = \frac{\sum \min\ (X_i, Y_i)}{\sum Y_i} \quad (2)$$

$$Coverage\ (Y_i \leq\ X_i) = \frac{\sum \min\ (X_i, Y_i)}{\sum X_i} \quad (3)$$

where:

$X_i$is the membership value of case i in the set of causal conditions.

$Y_i$is the membership value of case iii in the result set.

Both are employed in comparative analysis to assess the links formed between individual conditions, combinations of conditions, path configurations, and the outcome. Values exceeding 0.8 are typically regarded as signs of a robust association. [21].

4. Results.

For the study of public records and their inclusive service to the native communities of Ucayali, applying research methods based on the theory of neutrosophic sets, here is a simulated case with all the necessary calculations and data:

Definition of the Result

Defined result: Inclusive Service Quality (CSI).

Variables considered:

- **Access to Public Records (ARP):** Measurement of the ease with which native communities can access public records.
- **Adequacy of Services (ADS):** Evaluates the adequacy of the services offered in terms of inclusion and cultural adaptation.
- **Management Transparency (TG):** The degree to which the management of public records is transparent and accessible to native communities.

Community Survey

A survey was conducted with a group of 10 community leaders in Ucayali. The results are presented in Table 1.

Table 1. Survey Data

Leader	Access to Public Records (ARP)	Adequacy of Services (ADS)	Transparency in Management (TG)	Inclusive Service Quality (CSI)
1	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)
2	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
3	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.1)
4	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)
5	(0.8, 0.1, 0.1)	(0.6, 0.3, 0.1)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)
6	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
7	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.1)
8	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)
9	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)
10	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)

Fuzzification is performed using Equation 1 to convert the neutrosophic sets to equivalent fuzzy sets.

Table 2. Fuzzy Values

Leader	Access to Public Records (ARP)	Adequacy of Services (ADS)	Transparency in Management (TG)	Inclusive Service Quality (CSI)
1	0.60	0.50	0.85	0.60
2	0.50	0.60	0.65	0.60
3	0.55	0.80	0.75	0.55
4	0.50	0.50	0.65	0.50
5	0.80	0.60	0.90	0.80

Leader	Access to Public Records (ARP)	Adequacy of Services (ADS)	Transparency in Management (TG)	Inclusive Service Quality (CSI)
6	0.50	0.60	0.65	0.60
7	0.60	0.50	0.70	0.60
8	0.55	0.60	0.85	0.55
9	0.50	0.60	0.55	0.60
10	0.60	0.50	0.65	0.60

The consistency and coverage of the conditions are analyzed using fsQCA.

Table 3. Analysis of Necessary Conditions

Tested conditions	Consistency	Coverage
Access to Public Records (ARP)	0.5357	0.5857
Adequacy of Services (ADS)	0.5786	0.6000
Transparency in Management (TG)	0.6500	0.6571
Inclusive Service Quality (CSI)	0.6057	0.6286

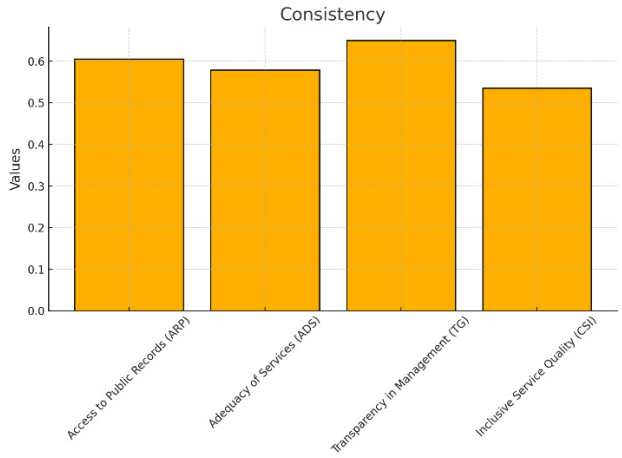

Figure 1. Analysis of Necessary Conditions (Consistency)

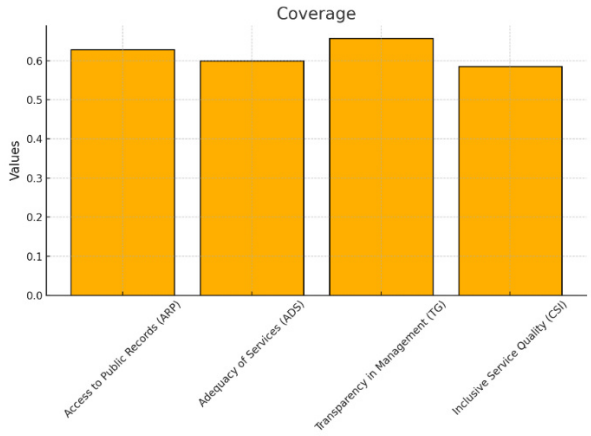

Figure 2. Analysis of Necessary Conditions (Coverage)

Set Coincidence Analysis

Table 4. Set Coincidence Analysis

Conditions	Coincidence
ARP, ADS, TG	0.5500
ARP, ADS	0.5250
ARP, T.G.	0.5700
A.D.S., T.G.	0.5850

Superset Analysis

Table 5. Results of Superset/Superset Analysis

Terms	Consistency	Coverage	Set
ARP, ADS, TG	0.5357	0.5857	0.5607
ARP, ADS	0.5250	0.5500	0.5375
ARP, T.G.	0.5700	0.5750	0.5725
A.D.S., T.G.	0.5850	0.5900	0.5875
ARP	0.5357	0.5700	0.5529
ADS	0.5786	0.6000	0.5893
T.G.	0.6500	0.6571	0.6536

The investigation demonstrates that Management Transparency (TG) and Service Adequacy (ADS) have the highest influence on Inclusive Service Quality (CSI). The greatest matches are discovered in the combination of TG and ADS , showing that these parameters have a significant and

positive association with the intended result. Based on the findings collected, the following steps are suggested:

- **Optimization of the Public Registry Process**

Administrative Simplification: It is proposed to simplify the administrative processes of public documents to promote access for native populations. This may involve digitizing records and building online platforms that enable communities to access services without

These findings suggest that Transparency in Management and Adequacy of Services are critical factors to improve Inclusive Service Quality. The high coincidence between TG and ADS implies that when both elements are treated well, a considerable gain in service quality is obtained. This may be because more openness and appropriateness in service management promotes more fair and effective access. Our findings are in accord with prior research that stresses the relevance of transparency and adequacy in service delivery. Previous studies has demonstrated that openness in public administration may promote user confidence and improve service efficiency. However, they vary from several research that has shown that openness alone is not enough without proper supporting infrastructure. Our research expands this approach by demonstrating that the combination of openness and sufficiency of services leads in a stronger influence on the quality of the included service. A key weakness of this research is the likely difference in the impression of service quality across various native populations. The absence of longitudinal data also limits determining long-term consequences. The findings imply that future studies should further examine how the combination of openness and appropriateness may be enhanced.

5. Conclusions

This research demonstrates that Management Transparency (TG) and Service Adequacy (ADS) have a considerable influence on Inclusive Service Quality (CSI). The research reveals that the combination of these elements causes a considerable increase in the quality of the service supplied to native populations. This conclusion shows the need of addressing these two characteristics simultaneously to optimize the efficacy of public services. The practical relevance of these results is considerable. Recommendations derived from our analysis, such as administrative simplification, staff training, and cultural adaptation of services, offer clear guidelines to improve the accessibility and efficiency of public records. Implementing these suggestions can facilitate more equitable access to services and better respond to the specific needs of Native communities, ultimately contributing to a more inclusive and effective service system. The study makes a significant contribution to the field by demonstrating how the combination of transparency and appropriateness in service management can substantially improve inclusive service quality. Although previous studies have highlighted the importance of each of these factors individually, our work expands this knowledge by showing how their joint interaction offers more robust results. This innovation in evaluation methodology and the identification of the most impactful factors provide a solid foundation for future research and practice in improving public services. However, the research has several limitations that must be noted. Variability in perceptions of service quality between different communities and lack of longitudinal data prevent a full assessment of long-term impact. These limitations should be taken into account when applying the results and designing additional research. For future research, it is recommended to explore complementary methods that can offer a broader perspective on how to optimize the combination of transparency and appropriateness in different contexts. Expanding the study to other regions and populations could also help validate and generalize these results, providing a more comprehensive view of improving inclusive public services. Continued evaluation and implementation of reforms based on these findings are crucial to ensuring that public record services are more accessible and effective in the future.

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Study of Sound Pressure Levels through the Creation of Noise Maps in the Urban Area of Latacunga City using Plithogenic n-Superhypergraphs

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Abstract. Acoustic pollution has become an increasing problem in urban areas, and its monitoring is crucial for protecting public health. In Latacunga, the presence of noise sources has affected the quality of life of its inhabitants. Therefore, the study focused on monitoring sound pressure levels and identifying the areas most affected by acoustic pollution. The Plithogenic n-SuperHyperGraphs were used to analyze the interactions between noise sources, impacted areas, and regulatory frameworks. Among the results, it was observed that measurements were taken at 12 critical points, especially in high-traffic and commercial activity areas. It was also indicated that there is a priority for implementing public policies to manage noise, including the regulation of sources and promoting solutions such as acoustic barriers. Additionally, managing acoustic pollution using technological tools and advanced methodologies was recommended.

Keywords: Acoustic pollution, critical noise zones, stratified population groups, plithogenic n-SuperHyperGraphs.

1 Introduction

Acoustic pollution is an unpleasant and inarticulate sensation that generates negative and harmful effects on human health [1]. This phenomenon is generally considered unacceptable and represents one of the major environmental issues globally [2], as it directly affects public health. Prolonged exposure to elevated noise levels can trigger adverse effects such as increased stress levels, hypertension, hearing loss, concentration difficulties, and sleep disorders [3]. Additionally, acoustic pollution affects the nervous system, and the myocardium, and may even cause cerebrovascular alterations.

Therefore, it is essential to establish equivalent sound pressure levels at critical impact points to ensure the protection of public health and environmental well-being [4]. Acoustic impact assessment is fundamental for preventing and controlling the effects of noise on the population and the environment [5][6]. Urban planning and traffic management are key elements in mitigating the density and morphology of noise effects both on the population and the ecosystem [7][8]. As such, regulatory agencies at the global and national levels have established technical standards for maximum permissible noise levels to preserve environmental balance and public health.

In Ecuador, for instance, the Ministry of Environment, Water, and Ecological Transition, together with other competent entities, has defined acceptable noise ranges, which vary between 55 dB and 70 dB, depending on the type of area. These include residential areas, public service zones, industrial zones, and agricultural or commercial areas [9]. These regulations are based on the country's environmental classification and are governed by the Unified Text of Secondary Environmental Legislation (TULSMA) and its reforms.

In the city of Latacunga, various studies have highlighted the issue of noise and its impact on residents [10]. Previous research has revealed that noise generated by vehicular traffic on the city's main arteries exceeds the limits established by local and national legislation [11]. Recent studies have also shown that acoustic pollution negatively affects residents' quality of life, emphasizing the need for measures to mitigate its impact [12]. Thus, the objective of this study is to monitor the equivalent sound

pressure levels in the urban area of Latacunga. To achieve this, it is proposed to use a Delta OHM brand sound level meter, which enables the creation of environmental noise maps to identify the areas most affected by acoustic pollution. Additionally, the study aims to identify and manage the structures of elements (vertices) and groups of elements (supervertex) that influence the reduction of decibel (dB) levels in the city using Plithogenic n-SuperHyperGraphs.

2 Materials and Methods

The study, conducted in Latacunga, Ecuador, analyzed acoustic pollution in critical areas such as commercial, educational, and high-traffic zones. Tools such as sound level meters, GPS, and specialized software were used to monitor and calculate specific noise levels, following normative methods and adjusting for impulsive noise and low frequencies. The results included acoustic maps that identified critical areas, providing key information for urban management.

The Plithogenic n-SuperHyperGraphs approach was used to model the relationships between noise sources, locations, and impacts, integrating elements of uncertainty into the study [13]. This method allowed for the identification of complex patterns, as well as optimization in decision-making. Despite limitations in selecting specific areas, the results provide a solid foundation for reducing noise and improving the quality of life in the city.

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in [14] [15]

First, an n-SuperHyperGraph is defined as follows:

Given $V = \{V_1, V_2, \cdots, V_{\mathrm{m}}\}$, where $1 \leq \mathrm{m} \leq \infty$ is a set of vertices, containing *Single Vertices* which are classical, *Indeterminate Vertices* that are unclear, vague, or partially known, and the *Null Vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, …, $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq \mathrm{n} \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq \mathrm{n} \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1)- SuperVertex,* and the others can be *r- SuperVertex* for $r \leq \mathrm{n} - 1$.

E_n contains the following types of edges:

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq \mathrm{n}$).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),

- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

  Within the framework of the theory of Plithogenic n-SuperHyperGraphs, there are the following concepts:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervene,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights [16].

3 Results

3.1 Structure of the Plithogenic n-SuperHyperGraph.

For the development, the Plithogenic n-SuperHyperGraph structure of the study was modeled, determining four priority vertices (see figures 1 to 4). Among the vertices, three are simple vertices (V_1, V_2, and V_3), and one is indeterminate (V_4), because the knowledge of regulations and environmental legislation is partially known by the study population.

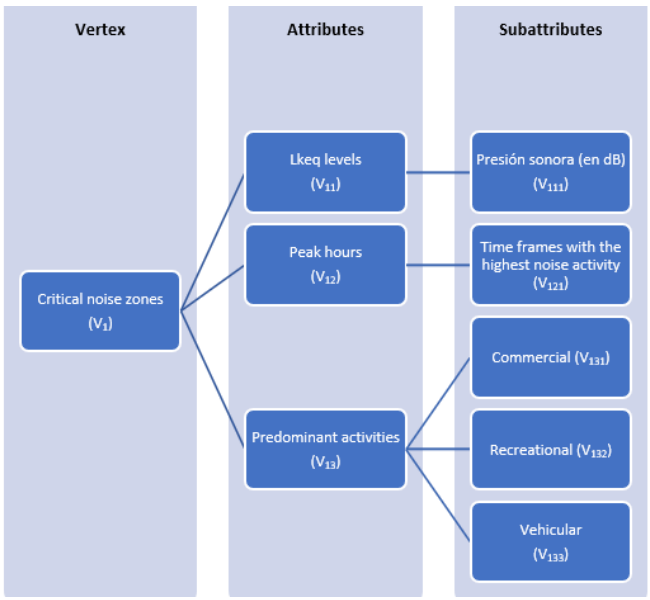

Figure 1: Critical noise zones (V_1).

In Figure 1, the areas affected by environmental pollution are represented as a vertex V_1. All its elements constitute intermediate-encompassing vertices. For this study, the predominant activities were limited to the scope of the study in the analyzed city of Latacunga.

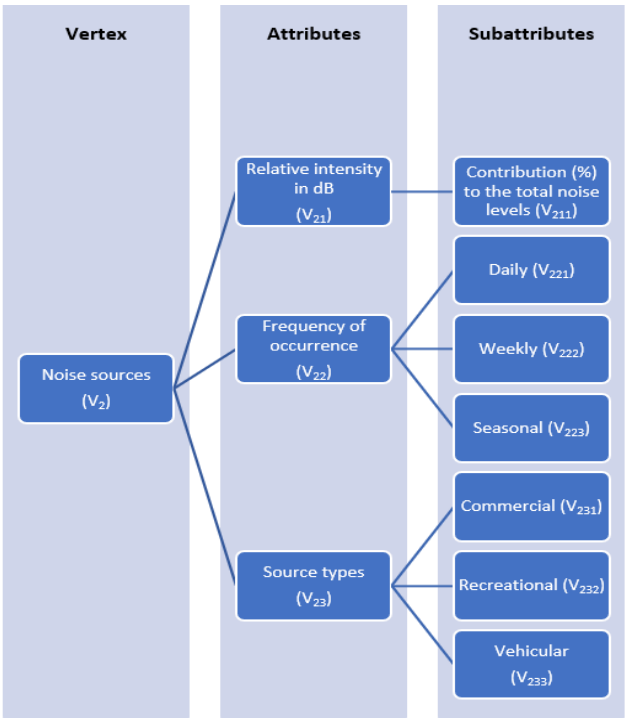

Figure 2: Noise sources (V_2).

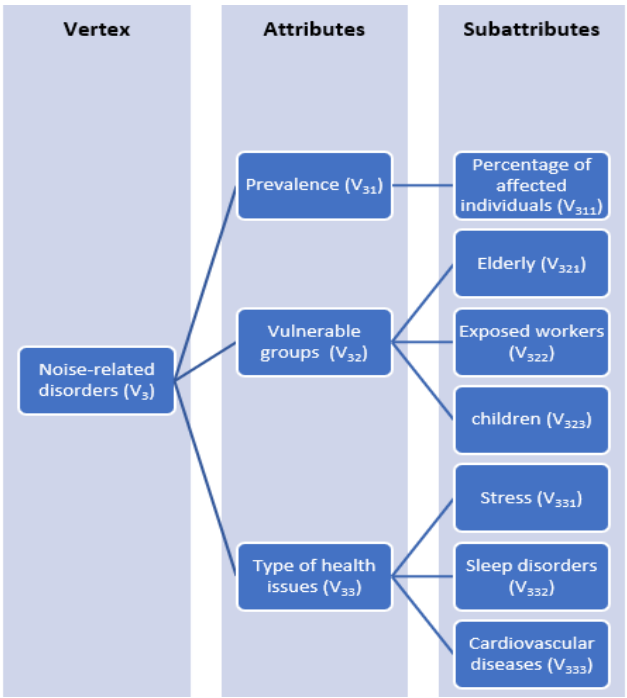

Figure 3: Noise-related disorders (V_3).

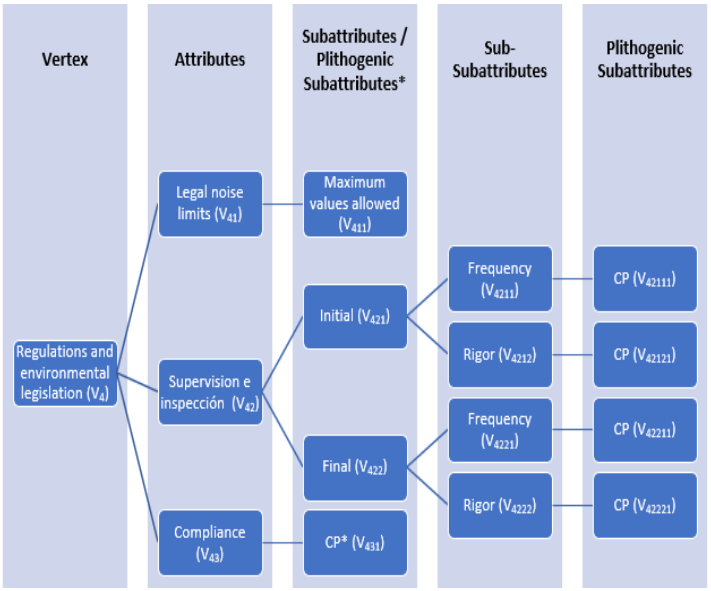

Figure 4: Regulations and environmental legislation (V_4).

In Figure 4, the vertices V_{431}, as well as V_{42111}, V_{42121}, V_{42211}, and V_{42221}, represent plithogenic sub-attributes used to determine or evaluate the vertex. For example, supervision encompasses elements such as inspections conducted (which may include ministerial visits, audits, controls, etc.). These inspections have a frequency and rigor (depending on the scope and depth of the inspection carried out). Therefore, both are characterized by plithogenic criteria, which in this study are defined by the labels CP (very high, high, medium, low, and very low). Similarly, compliance with these regulations does not occur immediately after a supervision visit. Hence, it is also defined by CP* (not fulfilled, partially fulfilled, and fulfilled, where the scale can be extended based on the stages of compliance or non-conformities to be addressed). Therefore, V_4 is considered an indeterminate or partially defined vertex. On the other hand, V_{421} and V_{422} are considered entry and exit encompassing vertices, respectively, while the others are considered intermediate encompassing vertices.

3.2 Geographic location of critical noise impact points (Vertex V_1).

Twelve critical impact points related to noise were identified, which were recognized as significant sources of acoustic pollution in productive activities (see Table 1). Among these sources, the constant flow of vehicles, traffic congestion, the use of sound amplification equipment for advertising, and the gathering of people near educational institutions stand out. Most of these points are concentrated in the historic center of the city.

For the monitoring process, four measurement points were selected and evaluated over three days, between November 2022 and January 2023, during a period of high commercial mobility. The noise measurements were organized according to the similar characteristics of the points, which facilitated the comparison of data between them (see Table 1).

Table 1: Measurement of timeline data. Source: Own elaboration.

Points	Reference location	Measurement day	Measurement schedule
P1	Traffic light at the terminal	Friday	19:00-20:00
P2	Vicente León Park	Friday	16:00-17:00
P3	General Hospital of Latacunga	Monday	16:00-17:00
P4	La Cocha	Friday	12:00-13:00

Points	Reference location	Measurement day	Measurement schedule
P5	Malteria	Monday	07:00-08:00
P6	San Felipe Park	Monday	19:00-20:00
P7	Canada Plaza	Thursday	07:00-08:00
P8	Simón Rodríguez Avenue	Thursday	19:00-20:00
P9	El Salto	Thursday	16:00-17:00
P10	U.E. Ramón Barba Naranjo	Thursday	12:00-13:00
P11	ESPE	Friday	07:00-08:00
P12	Traffic light at the intersection of Eugenio Espejo Avenue and Eloy Alfaro Avenue	Monday	12:00-13:00

3.3 Interactions of SuperVertex (V_1, V_2, V_3 and V_4) on research results.

3.3.1 Environmental diagnosis of the study area.

The identification of critical points was carried out through an evaluation of the measurement procedures, the equipment to be used, and the environment in which the measurements took place. A significant impact on health was evidenced, particularly in the more vulnerable population groups, such as the elderly, who experience high stress levels due to noise. It also revealed 43 areas of the city where elevated noise levels were perceived during daytime hours (7:00 to 21:00). These areas are primarily attributed to intense vehicular traffic during peak hours, with the "El Salto" Commercial Plaza being identified as having the highest noise levels. Of these 43 areas, 12 were selected for further study. Additionally, the results showed that the population was unfamiliar with noise isolation measures, although many suggested that green barriers could be an effective option to mitigate the noise (see Figure 5).

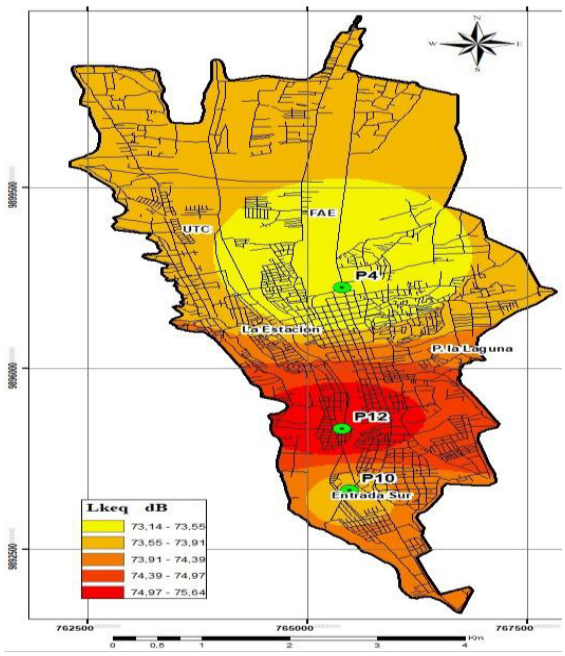

Figure 5: Areas with the highest noise levels in Latacunga.

3.3.2 Corrected equivalent continuous sound pressure level (Lkeq).

During the seven-week monitoring period, 84 measurements were taken, of which only 5% met the maximum noise levels set by regulations. In specific weeks, measurements within the limit were recorded at some points, such as point 2 during weeks 2 and 7, and points 4 and 7 during week 5. The

remaining 95% of the measurements exceeded the permitted limits. This excess in noise levels is attributed to the high concentration of people and vehicles in commercial areas, shopping centers, and parks during the Christmas season.

3.3.3 Statistical analysis: Chi-Square test and graph determination.

A statistical analysis was performed using the chi-square test to compare the average values of the corrected equivalent continuous sound pressure level (Lkeq) with the expected values, defined according to the maximum permissible noise levels for commercial land use. The results of the analysis showed that the calculated chi-square value was 10.53, with 12 degrees of freedom. According to the chi-square distribution table, the critical value was 19.68. Since the calculated value falls within the rejection region, the null hypothesis, which stated that the noise levels were influenced by land use, is rejected. This suggests that noise is an independent variable in urban planning, as it does not directly affect compliance with sound-level regulations.

According to the results, Lkeq levels above the permitted limits are mainly due to daily activities in the city, such as mobility, commercial activities, and recreation. Tolerance zones for noise are also defined based on the land use of each sector and the schedule established in regulations, such as daytime and nighttime hours.

Furthermore, noise maps are presented, indicating the areas with the highest Lkeq levels, obtained during peak hours in the morning, afternoon, and evening at different points in the central canton. Qualitative analyses are presented through the comparison of noise measurements between three different points at the same time and week.

In the time frame from 7:00 to 8:00, it was observed that points located in areas such as ESPE and Canada Plaza exhibited high noise levels due to vehicular circulation and the traffic of students near educational institutions. During the 16:00 to 17:00 interval, the critical point identified was "El Salto," with a level of 82.5 dB, influenced by vehicular traffic and the influx of people due to commercial activities and the proximity of government establishments.

In the 19:00 to 20:00 period, greater impact was observed in sectors such as San Felipe, where levels close to 75 dB were recorded due to the presence of bars, restaurants, and shops that increase noise during nighttime hours. Finally, between 12:00 and 13:00, the point at the intersection of Eugenio Espejo and Eloy Alfaro avenues showed levels of 77.76 dB, primarily due to vehicular flow generated by the proximity of bus stops and congestion at traffic lights.

Graph analysis: In Figure 6, a representation of the neutrosophic-directed graph from the research conducted is shown. It identifies the supervertex (V_{13}, V_{23}) aimed at determining the most affected areas, zones, or critical points by pollution. On the other hand, the n-SuperHyperEdge contains the indeterminate vertex (V_4), which represents that a significant portion of the population in the study has partial knowledge of the regulations governing and controlling noise pollution. Additionally, the supervertex (V_{11}, V_{12}, V_{21}, and V_{22}) focuses on determining the exposure levels of the city to noise pollution, as well as the sound pressure levels encountered. In fact, this grouping, together with the vertex V_3, defines the consequences of exposure to pollution for the city's residents. Another visual characteristic observed in the graph is the dominance among its elements, where vertices V_{11} and V_{23} represent the prevalence over the other elements of the supervertex.

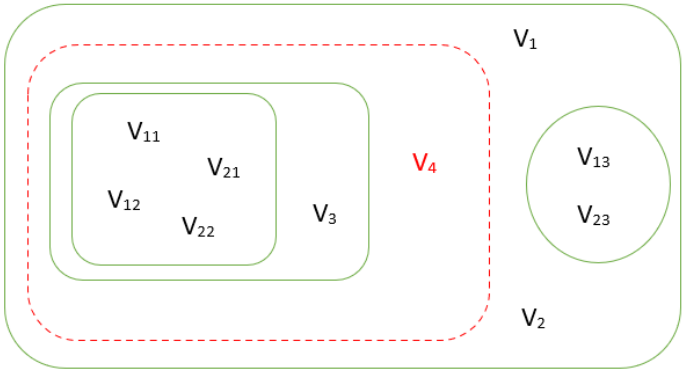

Figure 6: Representation of the neutrosophic directed graph in the study.

The graph analysis allows for the inclusion of other points to consider, such as promoting healthy lifestyle habits, educating the population on the effects of noise and existing regulations, and encouraging the use of quieter technologies. These are some of the measures that contribute to reducing decibel levels, as well as ensuring compliance with acceptable sound pressure limits. Additionally, it is crucial to study, validate, and properly manage noise to ensure the protection of the health and environmental well-being of the population.

It is therefore imperative to adopt concrete actions to reduce the impact of noise in Latacunga and improve the well-being of its residents. Among the recommended measures are the regulation of vehicular traffic, the promotion of more sustainable transportation options, and the installation of acoustic barriers in the most affected areas.

On the other hand, public policies must focus on the sustainable management of noise by implementing measures that not only regulate noise sources but also promote citizen awareness of the impact of noise on health and well-being. The creation of noise maps and continuous monitoring of sound pressure levels are key tools to identify the most affected areas and guide the necessary interventions.

4 Conclusion

The research has demonstrated the utility of the Plithogenic n-SuperHyperGraph model in addressing acoustic pollution in Latacunga by identifying the interactions between noise sources, exposure levels, and regulatory knowledge. The results highlighted the need to strengthen public policies with tools such as noise maps and continuous monitoring, as well as awareness strategies to mitigate the impact on the health and well-being of the population. It not only has local applications but also provides a foundation for future projections, with the inclusion of intelligent monitoring technologies and the evaluation of the effectiveness of interventions in noise reduction.

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Uncertain Automata and Uncertain Graph Grammar

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Abstract. Graph theory has been widely studied, resulting in numerous applications across various felds. Among its many topics, Automata and Graph Grammar have emerged as signifcant areas of research. This paper delves into these concepts, emphasizing their adaptation o uncertain frameworks like Fuzzy, Neutro- sophic, Vague, Turiyam Neutrosophic, and Plithogenic systems. By integrating uncertainty into traditional graph theoretical models, the paper aims to address ongoing research challenges and expand the scope of these models.

Keywords: Neutrosophic set, Fuzzy set, Graph Grammar, Automata

1. Introduction

1.1. *Graph Grammar and Automata*

Graph theory has been a subject of extensive research, leading to numerous applications across various domains [19,39–41,78,122,179]. Among its diverse topics, Graph Automata and Graph Grammar stand out as key areas of study within graph theory and its related concepts. This paper addresses Automata and Graph Grammar.

Automata are mathematical models representing abstract machines that process input sequences, transition between states, and generate outputs according to predefined rules [18,33,82,110,131,132]. Extensive research has been conducted on automata, resulting in various related concepts, including Finite Automata [35,124], Pushdown Automata [5,54,175,180], Infinite Automata [123, 177, 178], Linear-Bounded Automata [116], and Weighted Automata [42, 43, 113].

Graph Grammar is a formal system that defines transformation rules for modifying graph structures, modeling dynamic changes by adding, deleting, or replacing vertices and edges. It

has been extensively studied, with many related concepts explored [47,48,48,49,128]. Examples include Triple Graph Grammars [101,103,134,135], Context-Free Graph Grammars [6,53,121, 181], NLC Graph Grammars [46,57,84,85], Apex Graph Grammars [51,52,100], and Attributed Graph Grammars [75,129].

1.2. *Uncertain concepts*

Various concepts for handling uncertainty are continuously being researched to address the unpredictability of the world [7–11, 13, 149, 151, 152, 160, 165, 168, 170–172]. Among them, this paper focuses on fuzzy sets and neutrosophic sets. A fuzzy set assigns each element a membership degree between 0 and 1, representing partial membership [37, 44, 45, 87, 176, 188, 191, 192]. A neutrosophic set assigns three values—truth, indeterminacy, and falsity—to each element, with values ranging from 0 to 1 [14, 28, 50, 117, 149, 150, 172, 182]. A Turiyam Neutrosophic set assigns four values: truth, indeterminacy, falsity, and liberal state, all within the [0,1] range [69, 145–147]. These set concepts play significant roles in modeling uncertainty across various fields [61, 63, 64].

Concepts for handling uncertainty have been proposed not only in set theory but also in other fields. For example, in graph theory, various concepts have been explored, such as fuzzy graphs [115,127], neutrosophic graphs [12,15,29,60,80,83,90,130,159,169], vague graphs [27,126,133], Turiyam Neutrosophic graphs [69–71], and Plithogenic graphs [58, 102, 109]. Based on these points, research on Uncertain concepts is essential.

1.3. *Our contribution*

Based on the above, research on Graph Grammar and Automata is both significant and valuable. There remains considerable scope for further exploration, especially regarding uncertain concepts within Graph Grammar and Automata. Therefore, this paper will examine the concepts of Graph Grammar and Automata in the contexts of Fuzzy, Neutrosophic, Vague, Turiyam Neutrosophic, and Plithogenic frameworks.

This paper's structure is briefly outlined as follows. Section 2 examines Uncertain Automata, Section 3 explores Graph Grammar, and Section 4 discusses future perspectives.

2. **Automata**

Automata are mathematical models that represent computational machines or systems, processing input sequences to change states and produce outputs [18, 33, 82, 110, 131, 132]. In this section, we examine the concepts of Automata in the contexts of Fuzzy, Neutrosophic, Vague, Turiyam Neutrosophic, and Plithogenic frameworks.

2.1. *Fuzzy general finite Automata*

Fuzzy General Finite Automata are a fuzzy variant of General Finite Automata, widely explored in various studies [2,21,34]. In the context of automata theory and its application to fuzzy concepts, several other models have been developed, including Universal Fuzzy Automata [107], Fuzzy Rough Automata [119,186], LB-valued General Fuzzy Automata [4], BL-General Fuzzy Automata [1,3,139], Intuitionistic General Fuzzy Automata [142], Vector General Fuzzy Automata [140,141], Fuzzy Cellular Automata [16,23,73], Circular Fuzzy Cellular Automata [24], and Fuzzy Tree Automata [106]. The definitions and theorems, including related concepts, are provided below.

Definition 2.1. (cf. [82]) A *Finite Automaton (FA)* is defined as a 5-tuple:

$$A = (Q, \Sigma, \delta, q_0, F),$$

where:

- Q is a finite set of states.
- Σ is a finite alphabet of input symbols.
- $\delta : Q \times \Sigma \to Q$ is the state transition function, mapping a state and input symbol to a new state.
- $q_0 \in Q$ is the start state.
- $F \subseteq Q$ is the set of accepting (or final) states.

An FA processes an input string $w = a_1 a_2 \cdots a_n$ from Σ^* as follows:

(1) Begin in the start state q_0.
(2) For each symbol a_i in w, compute the new state $q_{i+1} = \delta(q_i, a_i)$.
(3) If the computation ends in a state $q \in F$, the automaton *accepts* w; otherwise, it *rejects* w.

Definition 2.2. [188,189] A *fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \to [0,1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is called a *fuzzy relation on* τ if

$$\delta(y,z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Example 2.3. Let $Y = \{1, 2, 3, 4, 5\}$ be a finite universe, and consider the following fuzzy set τ defined on Y:

$$\tau(y) = \begin{cases} 0.8 & \text{if } y = 1, \\ 0.6 & \text{if } y = 2, \\ 0.4 & \text{if } y = 3, \\ 0.2 & \text{if } y = 4, \\ 0.0 & \text{if } y = 5. \end{cases}$$

Now, consider a fuzzy relation δ defined on $Y \times Y$:

$$\delta(y, z) = \begin{cases} 0.5 & \text{if } (y, z) = (1, 2) \text{ or } (2, 1), \\ 0.4 & \text{if } (y, z) = (2, 3) \text{ or } (3, 2), \\ 0.2 & \text{if } (y, z) = (3, 4) \text{ or } (4, 3), \\ 0.0 & \text{otherwise.} \end{cases}$$

We can verify that δ is a fuzzy relation on τ by checking the condition:

$$\delta(y, z) \leq \min\{\tau(y), \tau(z)\}.$$

For instance:

- For $(y, z) = (1, 2)$, $\delta(1, 2) = 0.5$ and $\min\{\tau(1), \tau(2)\} = \min\{0.8, 0.6\} = 0.6$, so $0.5 \leq 0.6$.
- For $(y, z) = (2, 3)$, $\delta(2, 3) = 0.4$ and $\min\{\tau(2), \tau(3)\} = \min\{0.6, 0.4\} = 0.4$, so $0.4 \leq 0.4$.
- For $(y, z) = (3, 4)$, $\delta(3, 4) = 0.2$ and $\min\{\tau(3), \tau(4)\} = \min\{0.4, 0.2\} = 0.2$, so $0.2 \leq 0.2$.

Thus, δ is a valid fuzzy relation on τ.

Definition 2.4. [2, 21, 34] A *General Fuzzy Automaton (GFA)* is defined as an eight-tuple machine:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where:

- Q is a finite set of states, $Q = \{q_1, q_2, \ldots, q_n\}$.
- Σ is a finite set of input symbols, $\Sigma = \{a_1, a_2, \ldots, a_m\}$.
- $\tilde{R}$ is the set of fuzzy start states, $\tilde{R} \subseteq \tilde{P}(Q)$, where $\tilde{P}(Q)$ is the fuzzy power set of Q.
- Z is a finite set of output symbols, $Z = \{b_1, b_2, \ldots, b_k\}$.
- $\tilde{\delta} : (Q \times [0, 1]) \times \Sigma \times Q \xrightarrow{F_1(\mu,\delta)} [0, 1]$ is the *augmented fuzzy transition function*, where μ is the membership value of a predecessor state, and δ is the transition weight.

- $\omega : Q \to Z$ is the non-fuzzy output function.
- $F_1 : [0,1] \times [0,1] \to [0,1]$ is the *membership assignment function*, with common forms such as $F_1(\mu, \delta) = \max(\mu, \delta)$, $\min(\mu, \delta)$, or $\frac{\mu+\delta}{2}$.
- $F_2 : [0,1]^* \to [0,1]$ is the *multi-membership resolution function*, used to resolve simultaneous transitions to the same state.

Definition 2.5. The membership value of the state q_j at time $t+1$ is given by:

$$\mu_{t+1}(q_j) = \tilde{\delta}((q_i, \mu_t(q_i)), a_k, q_j) = F_1(\mu_t(q_i), \delta(q_i, a_k, q_j)),$$

where:

- $\mu_t(q_i)$ is the membership value of state q_i at time t.
- $\delta(q_i, a_k, q_j)$ is the transition weight from q_i to q_j on input a_k.

Definition 2.6. If there are multiple transitions to the active state q_j, the membership values v_i from different transitions are combined using F_2:

$$\mu_{t+1}(q_j) = F_2\left(\{v_i\}_{i=1}^n\right) = F_2\left(\{F_1(\mu_t(q_i), \delta(q_i, a_k, q_j))\}_{i=1}^n\right),$$

where:

- n is the number of simultaneous transitions to state q_j at time $t+1$.

Proposition 2.7. *A General Fuzzy Automaton (GFA) is a generalization of a Finite Automaton (FA).*

Proof. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton. Define a GFA $\tilde{F}$ as follows:

- Use the same set of states: $Q_{\text{GFA}} = Q_{\text{FA}}$.
- Use the same alphabet: $\Sigma_{\text{GFA}} = \Sigma_{\text{FA}}$.
- Set the fuzzy start state: $\tilde{R} = \{q_0 \mapsto 1, q \mapsto 0 \text{ for all } q \neq q_0\}$.
- Define the fuzzy transition function:

$$\tilde{\delta}((q,1), a, q') = \begin{cases} 1 & \text{if } \delta(q,a) = q', \\ 0 & \text{otherwise.} \end{cases}$$

- Use a trivial output function: $\omega(q) = b_0$ for some fixed $b_0 \in Z$.
- Define $F_1(\mu, \delta) = \delta$, treating δ as a deterministic transition.
- Use F_2 as the identity function to avoid multi-membership conflicts.

With these definitions:

(1) Any deterministic transition in A is encoded as a transition with membership degree 1 in $\tilde{F}$.

(2) The computation of $\tilde{F}$ simulates the state changes of A exactly, as F_1 and F_2 are designed to preserve deterministic behavior.

Thus, any FA is a special case of a GFA where the fuzzy parameters are restricted to deterministic values. Therefore, $\tilde{F}$ generalizes A. □

2.2. *Neutrosophic general finite automata*

Neutrosophic General Finite Automata represent a Neutrosophic adaptation of General Finite Automata, extensively studied in various works [99]. Within the scope of automata theory and its application to Neutrosophic concepts, several models have emerged, including Composite Neutrosophic Finite Automata [98], Reverse Neutrosophic Automata [97], Single-Valued Neutrosophic Automata [94, 95, 112], and Interval Neutrosophic Automata [91–93, 96, 97]. The definitions and theorems, including related concepts, are provided below.

Definition 2.8. [149] Let X be a given set. A Neutrosophic Set A on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degree of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.9. (cf. [94, 95, 99, 112]) A *General Neutrosophic Automaton (GNA)* is defined as an eight-tuple machine:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where:

- Q, Σ, Z, and ω are defined as in the GFA.
- $\tilde{R} = \{(q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q)) \mid q \in R\}$ is the set of *neutrosophic start states*, where:
  - $\mu_1^0(q)$ is the initial truth-membership value of state q,
  - $\mu_2^0(q)$ is the initial indeterminacy-membership value of state q,
  - $\mu_3^0(q)$ is the initial falsity-membership value of state q.
- $\tilde{\delta} : (Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma \times Q \xrightarrow{F_1(\mu,\delta)} [0,1] \times [0,1] \times [0,1]$ is the *neutrosophic augmented transition function*, where $\mu = (\mu_1, \mu_2, \mu_3)$ and $\delta = (\delta_1, \delta_2, \delta_3)$ represent the truth, indeterminacy, and falsity components.
- $F_1 = (F_1^{\wedge}, F_1^{\wedge\vee}, F_1^{\vee})$, where:
  - $F_1^{\wedge}(\mu_1, \delta_1)$ represents the truth-membership assignment,

  - $F_1^{\wedge\vee}(\mu_2, \delta_2)$ represents the indeterminacy-membership assignment,
  - $F_1^{\vee}(\mu_3, \delta_3)$ represents the falsity-membership assignment.
- $F_2 = (F_2^{\wedge}, F_2^{\wedge\vee}, F_2^{\vee})$ is the *multi-membership resolution function*, resolving truth, indeterminacy, and falsity components.

Definition 2.10. The neutrosophic membership values of state q_j at time $t+1$ are calculated as:

$$\mu_{1,t+1}(q_j) = \tilde{\delta}_1((q_i, \mu_{1,t}(q_i)), a_k, q_j) = F_1^{\wedge}(\mu_{1,t}(q_i), \delta_1(q_i, a_k, q_j)),$$
$$\mu_{2,t+1}(q_j) = \tilde{\delta}_2((q_i, \mu_{2,t}(q_i)), a_k, q_j) = F_1^{\wedge\vee}(\mu_{2,t}(q_i), \delta_2(q_i, a_k, q_j)),$$
$$\mu_{3,t+1}(q_j) = \tilde{\delta}_3((q_i, \mu_{3,t}(q_i)), a_k, q_j) = F_1^{\vee}(\mu_{3,t}(q_i), \delta_3(q_i, a_k, q_j)),$$

where:

- $\mu_{1,t}(q_i)$, $\mu_{2,t}(q_i)$, and $\mu_{3,t}(q_i)$ represent the truth, indeterminacy, and falsity membership values of state q_i at time t.

Definition 2.11. The simultaneous transitions to the same state q_j are resolved by F_2:

$$\mu_{1,t+1}(q_j) = F_2^{\wedge}\left(\{F_1^{\wedge}(\mu_{1,t}(q_i), \delta_1(q_i, a_k, q_j))\}_{i=1}^{n}\right),$$
$$\mu_{2,t+1}(q_j) = F_2^{\wedge\vee}\left(\{F_1^{\wedge\vee}(\mu_{2,t}(q_i), \delta_2(q_i, a_k, q_j))\}_{i=1}^{n}\right),$$
$$\mu_{3,t+1}(q_j) = F_2^{\vee}\left(\{F_1^{\vee}(\mu_{3,t}(q_i), \delta_3(q_i, a_k, q_j))\}_{i=1}^{n}\right).$$

Proposition 2.12. *A General Neutrosophic Automaton (GNA) is a generalization of a General Fuzzy Automaton (GFA).*

Proof. Let $\tilde{F}_{\text{GFA}} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be a General Fuzzy Automaton. Define a GNA $\tilde{F}_{\text{GNA}} = (Q, \Sigma, \tilde{R}_{\text{N}}, Z, \tilde{\delta}_{\text{N}}, \omega, F_1^{\text{N}}, F_2^{\text{N}})$ as follows:

- Use the same set of states, input symbols, and output symbols: $Q_{\text{GNA}} = Q_{\text{GFA}}$, $\Sigma_{\text{GNA}} = \Sigma_{\text{GFA}}$, $Z_{\text{GNA}} = Z_{\text{GFA}}$.
- Define the neutrosophic start states:

$$\tilde{R}_{\text{N}} = \{(q, \mu_1, \mu_2, \mu_3) \mid q \in Q, \mu_1 = \mu, \mu_2 = 0, \mu_3 = 1 - \mu \text{ for } \mu \in \tilde{R}\}.$$

- Extend the fuzzy transition function $\tilde{\delta}$ to the neutrosophic transition function $\tilde{\delta}_{\text{N}}$:

$$\tilde{\delta}_{\text{N}}((q, \mu_1, \mu_2, \mu_3), a, q') = \begin{cases} (\tilde{\delta}(q, a, q'), 0, 1 - \tilde{\delta}(q, a, q')) & \text{if } \tilde{\delta}(q, a, q') \neq 0, \\ (0, 0, 1) & \text{otherwise.} \end{cases}$$

- Define F_1^{N} to operate component-wise:

$$F_1^{\text{N}}((\mu_1, \mu_2, \mu_3), (\delta_1, \delta_2, \delta_3)) = (F_1(\mu_1, \delta_1), 0, 1 - F_1(\mu_1, \delta_1)).$$

- Use F_2^{N} as a component-wise extension of F_2:

$$F_2^{\mathrm{N}}(\{(\mu_{1,i},\mu_{2,i},\mu_{3,i})\}) = (F_2(\{\mu_{1,i}\}), 0, 1 - F_2(\{\mu_{1,i}\})).$$

By construction, $\tilde{F}_{\mathrm{GNA}}$ reproduces the behavior of $\tilde{F}_{\mathrm{GFA}}$ when restricted to the fuzzy case, where indeterminacy is always 0 and falsity is the complement of truth. Thus, GNA generalizes GFA. □

Proposition 2.13. *A General Neutrosophic Automaton (GNA) is a generalization of a Finite Automaton (FA).*

Proof. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton. Define a GNA $\tilde{F}_{\mathrm{GNA}}$ as follows:

- Use the same set of states and input symbols: $Q_{\mathrm{GNA}} = Q_{\mathrm{FA}}$, $\Sigma_{\mathrm{GNA}} = \Sigma_{\mathrm{FA}}$.
- Define the neutrosophic start state:

$$\tilde{R}_{\mathrm{N}} = \{(q, 1, 0, 0) \text{ if } q = q_0, (q, 0, 0, 1) \text{ otherwise.}\}.$$

- Define the neutrosophic transition function:

$$\tilde{\delta}_{\mathrm{N}}((q,\mu_1,\mu_2,\mu_3), a, q') = \begin{cases} (1,0,0) & \text{if } \delta(q,a) = q', \\ (0,0,1) & \text{otherwise.} \end{cases}$$

- Use trivial membership assignment and resolution functions:

$$F_1^{\mathrm{N}}((\mu_1,\mu_2,\mu_3),(\delta_1,\delta_2,\delta_3)) = (\delta_1, 0, 1-\delta_1),$$

$$F_2^{\mathrm{N}}(\{(\mu_{1,i},\mu_{2,i},\mu_{3,i})\}) = (1,0,0).$$

Under this construction:

(1) Each state in A maps to a neutrosophic state in $\tilde{F}_{\mathrm{GNA}}$, with truth membership 1 and falsity membership 0 when active.
(2) The deterministic transitions of A are captured by the neutrosophic transition function with truth membership 1.

Thus, $\tilde{F}_{\mathrm{GNA}}$ reproduces the behavior of A as a special case where indeterminacy is always 0, truth is binary, and falsity is the complement of truth. Therefore, GNA generalizes FA. □

2.3. *Turiyam Neutrosophic general finite automata*

Turiyam Neutrosophic general finite automata represent a Turiyam Neutrosophic adaptation of General Finite Automata. The definitions and theorems, including related concepts, are provided below.

Definition 2.14. [69, 145] For Turiyam Neutrosophic sets μ and ν on E:

- *Order Relation:* $\mu \leq \nu$ if for all $e \in E$:

$$T_\mu(e) \leq T_\nu(e), \quad I_\mu(e) \geq I_\nu(e), \quad F_\mu(e) \geq F_\nu(e), \quad L_\mu(e) \geq L_\nu(e).$$

- *Maximum (Join):*

$$(\mu \vee \nu)(e) = (\max\{T_\mu(e), T_\nu(e)\},\ \min\{I_\mu(e), I_\nu(e)\},\ \min\{F_\mu(e), F_\nu(e)\},\ \min\{L_\mu(e), L_\nu(e)\}).$$

- *Minimum (Meet):*

$$(\mu \wedge \nu)(e) = (\min\{T_\mu(e), T_\nu(e)\},\ \max\{I_\mu(e), I_\nu(e)\},\ \max\{F_\mu(e), F_\nu(e)\},\ \max\{L_\mu(e), L_\nu(e)\}).$$

- *Support of* μ*:*

$$\text{supp}(\mu) = \{e \in E : T_\mu(e) > 0\}.$$

- *Minimal Truth-Membership Value:*

$$m_T(\mu) = \min\{T_\mu(e) : e \in \text{supp}(\mu)\}.$$

Definition 2.15. A *Turiyam Neutrosophic General Finite Automaton (TGFA)* is an eight-tuple machine defined as:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where:

(1) *States*:

- Q is a finite set of states:

$$Q = \{q_1, q_2, \ldots, q_n\}.$$

(2) *Input Symbols*:

- Σ is a finite set of input symbols:

$$\Sigma = \{a_1, a_2, \ldots, a_m\}.$$

(3) *Turiyam Neutrosophic Start States*:

- $\tilde{R}$ is the set of Turiyam Neutrosophic start states:

$$\tilde{R} = \left\{ \left(q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q), \mu_4^0(q)\right) \,\middle|\, q \in R \right\},$$

where:

- $R \subseteq Q$ is the set of initial states.

- $\mu_1^0(q)$ is the initial truth-membership value of state q.
- $\mu_2^0(q)$ is the initial indeterminacy-membership value of state q.
- $\mu_3^0(q)$ is the initial falsity-membership value of state q.
- $\mu_4^0(q)$ is the initial liberal state-membership value of state q.

(4) *Output Symbols*:

- Z is a finite set of output symbols:

$$Z = \{b_1, b_2, \ldots, b_k\}.$$

(5) *Turiyam Neutrosophic Transition Function*:

- $\tilde{\delta} : (Q \times [0,1]^4) \times \Sigma \times Q \to [0,1]^4$ is the augmented Turiyam Neutrosophic transition function, defined by:

$$\tilde{\delta}\left((q_i, \mu_t(q_i)), a_k, q_j\right) = \left(\tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3, \tilde{\delta}_4\right),$$

where:

- $\mu_t(q_i) = (\mu_{1,t}(q_i), \mu_{2,t}(q_i), \mu_{3,t}(q_i), \mu_{4,t}(q_i))$ represents the Turiyam Neutrosophic membership values of state q_i at time t.
- $\delta(q_i, a_k, q_j) = (\delta_1(q_i, a_k, q_j), \delta_2(q_i, a_k, q_j), \delta_3(q_i, a_k, q_j), \delta_4(q_i, a_k, q_j))$ are the Turiyam Neutrosophic transition weights.
- The components are computed using the membership assignment functions:

$$\begin{aligned}
\tilde{\delta}_1 &= F_1^{\wedge}\left(\mu_{1,t}(q_i), \delta_1(q_i, a_k, q_j)\right), \\
\tilde{\delta}_2 &= F_1^{\wedge\vee}\left(\mu_{2,t}(q_i), \delta_2(q_i, a_k, q_j)\right), \\
\tilde{\delta}_3 &= F_1^{\vee}\left(\mu_{3,t}(q_i), \delta_3(q_i, a_k, q_j)\right), \\
\tilde{\delta}_4 &= F_1^{\mathrm{L}}\left(\mu_{4,t}(q_i), \delta_4(q_i, a_k, q_j)\right).
\end{aligned}$$

(6) *Output Function*:

- $\omega : (Q \times [0,1]^4) \to Z$ is the non-fuzzy output function.

(7) *Membership Assignment Functions*:

- $F_1 = \left(F_1^{\wedge}, F_1^{\wedge\vee}, F_1^{\vee}, F_1^{\mathrm{L}}\right)$, where:
  - $F_1^{\wedge} : [0,1] \times [0,1] \to [0,1]$ assigns the truth-membership value.
  - $F_1^{\wedge\vee} : [0,1] \times [0,1] \to [0,1]$ assigns the indeterminacy-membership value.
  - $F_1^{\vee} : [0,1] \times [0,1] \to [0,1]$ assigns the falsity-membership value.
  - $F_1^{\mathrm{L}} : [0,1] \times [0,1] \to [0,1]$ assigns the liberal state-membership value.

(8) *Multi-Membership Resolution Functions*:

- $F_2 = \left(F_2^{\wedge}, F_2^{\wedge\vee}, F_2^{\vee}, F_2^{\mathrm{L}}\right)$, where:
  - Each F_2 component resolves multiple membership values when there are simultaneous transitions to the same state.

Definition 2.16. The Turiyam Neutrosophic membership values of state q_j at time $t+1$ are calculated as:

$$\begin{aligned}
\mu_{1,t+1}(q_j) &= \tilde{\delta}_1\left((q_i, \mu_{1,t}(q_i)), a_k, q_j\right) = F_1^{\wedge}\left(\mu_{1,t}(q_i), \delta_1(q_i, a_k, q_j)\right), \\
\mu_{2,t+1}(q_j) &= \tilde{\delta}_2\left((q_i, \mu_{2,t}(q_i)), a_k, q_j\right) = F_1^{\wedge\vee}\left(\mu_{2,t}(q_i), \delta_2(q_i, a_k, q_j)\right), \\
\mu_{3,t+1}(q_j) &= \tilde{\delta}_3\left((q_i, \mu_{3,t}(q_i)), a_k, q_j\right) = F_1^{\vee}\left(\mu_{3,t}(q_i), \delta_3(q_i, a_k, q_j)\right), \\
\mu_{4,t+1}(q_j) &= \tilde{\delta}_4\left((q_i, \mu_{4,t}(q_i)), a_k, q_j\right) = F_1^{\mathrm{L}}\left(\mu_{4,t}(q_i), \delta_4(q_i, a_k, q_j)\right).
\end{aligned}$$

Definition 2.17. If there are multiple transitions to the same state q_j, the membership values are resolved using F_2:

$$\begin{aligned}
\mu_{1,t+1}(q_j) &= F_2^{\wedge}\left(\left\{F_1^{\wedge}\left(\mu_{1,t}(q_i), \delta_1(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right), \\
\mu_{2,t+1}(q_j) &= F_2^{\wedge\vee}\left(\left\{F_1^{\wedge\vee}\left(\mu_{2,t}(q_i), \delta_2(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right), \\
\mu_{3,t+1}(q_j) &= F_2^{\vee}\left(\left\{F_1^{\vee}\left(\mu_{3,t}(q_i), \delta_3(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right), \\
\mu_{4,t+1}(q_j) &= F_2^{\mathrm{L}}\left(\left\{F_1^{\mathrm{L}}\left(\mu_{4,t}(q_i), \delta_4(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right),
\end{aligned}$$

where:

- n is the number of simultaneous transitions to state q_j at time $t+1$.
- $q_i \in Q$ are the predecessor states.

Theorem 2.18. *A Turiyam Neutrosophic General Finite Automaton (TGFA) can be transformed into a Fuzzy General Finite Automaton (GFA) or a Neutrosophic General Finite Automaton (GNA) by appropriately redefining its transition and membership functions.*

Proof. Let $\tilde{F}^T$ be a Turiyam Neutrosophic General Finite Automaton defined as:

$$\tilde{F}^T = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}^T, \omega, F_1^T, F_2^T),$$

where:

- $\tilde{\delta}^T : (Q \times [0,1]^4) \times \Sigma \times Q \to [0,1]^4$ is the augmented Turiyam Neutrosophic transition function.
- F_1^T and F_2^T are the membership assignment and resolution functions, respectively, with four components: truth, indeterminacy, falsity, and liberal state.

We consider Transformation to a Fuzzy General Finite Automaton. A Fuzzy General Finite Automaton $\tilde{F}^F$ is defined as:

$$\tilde{F}^F = (Q, \Sigma, \tilde{R}^F, Z, \tilde{\delta}^F, \omega, F_1^F, F_2^F),$$

where:

- The fuzzy start states $\tilde{R}^F$ are obtained from $\tilde{R}$ in the TGFA by retaining only the truth-membership component:

$$\tilde{R}^F = \{(q, \mu_1^0(q)) \mid (q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q), \mu_4^0(q)) \in \tilde{R}\}.$$

- The fuzzy transition function $\tilde{\delta}^F$ is derived from the truth component of $\tilde{\delta}^T$:

$$\tilde{\delta}^F \left((q_i, \mu_{1,t}(q_i)), a_k, q_j\right) = \tilde{\delta}_1 \left((q_i, \mu_{1,t}(q_i)), a_k, q_j\right).$$

- The membership assignment function F_1^F corresponds to the truth-assignment component $F_1^\wedge$ of the TGFA:

$$F_1^F(\mu, \delta) = F_1^\wedge(\mu, \delta).$$

- The multi-membership resolution function F_2^F is defined as:

$$F_2^F \left(\{v_i\}\right) = F_2^\wedge \left(\{v_i\}\right),$$

  where v_i are the truth-membership values from simultaneous transitions.

Thus, the TGFA $\tilde{F}^T$ reduces to a GFA $\tilde{F}^F$, retaining only the truth-membership component and the corresponding fuzzy transition and resolution functions.

Next, we consider Transformation to a Neutrosophic General Finite Automaton. A Neutrosophic General Finite Automaton $\tilde{F}^N$ is defined as:

$$\tilde{F}^N = (Q, \Sigma, \tilde{R}^N, Z, \tilde{\delta}^N, \omega, F_1^N, F_2^N),$$

where:

- The neutrosophic start states $\tilde{R}^N$ are obtained from $\tilde{R}$ in the TGFA by retaining the truth, indeterminacy, and falsity components:

$$\tilde{R}^N = \{(q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q)) \mid (q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q), \mu_4^0(q)) \in \tilde{R}\}.$$

- The neutrosophic transition function $\tilde{\delta}^N$ is derived from the first three components of $\tilde{\delta}^T$:

$$\tilde{\delta}^N \left((q_i, \mu_t(q_i)), a_k, q_j\right) = \left(\tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3\right).$$

- The membership assignment function F_1^N is defined by the truth, indeterminacy, and falsity components of the TGFA:

$$F_1^N = \left(F_1^\wedge, F_1^{\wedge\vee}, F_1^\vee\right).$$

- The multi-membership resolution function F_2^N is defined as:

$$F_2^N = \left(F_2^\wedge, F_2^{\wedge\vee}, F_2^\vee\right).$$

Thus, the TGFA $\tilde{F}^T$ reduces to a GNA $\tilde{F}^N$, retaining the truth, indeterminacy, and falsity components, along with their corresponding neutrosophic transition and resolution functions.

By appropriately redefining the start states, transition functions, membership assignment, and resolution functions, a Turiyam Neutrosophic General Finite Automaton (TGFA) can be transformed into either a Fuzzy General Finite Automaton (GFA) or a Neutrosophic General Finite Automaton (GNA). □

2.4. *Vague General Finite Automata*

Vague general finite automata represent a Vague adaptation of General Finite Automata. The definitions and theorems, including related concepts, are provided below.

Definition 2.19. (cf. [30, 32, 56, 72, 111, 125, 184]) A *Vague Set (VS) A* on a set X is defined as:

$$A = (t_A, f_A),$$

where:

- $t_A : X \to [0, 1]$ is the *truth-membership function*.
- $f_A : X \to [0, 1]$ is the *falsity-membership function*.

For any element $x \in X$, the following condition holds:

$$0 \leq t_A(x) + f_A(x) \leq 1.$$

Definition 2.20. A *Vague General Finite Automaton (VGFA)* is defined as an eight-tuple:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where:

(1) *States*:

- Q is a finite set of states:

$$Q = \{q_1, q_2, \ldots, q_n\}.$$

(2) *Input Symbols*:

- Σ is a finite set of input symbols:

$$\Sigma = \{a_1, a_2, \ldots, a_m\}.$$

(3) *Vague Start States*:

- $\tilde{R}$ is the set of vague start states:

$$\tilde{R} = \left\{ \left(q, \mu_t^0(q), \mu_f^0(q)\right) \,\middle|\, q \in R \right\},$$

where:

- $R \subseteq Q$ is the set of initial states.
- $\mu_t^0(q)$ is the initial truth-membership value of state q.
- $\mu_f^0(q)$ is the initial falsity-membership value of state q.
- For all $q \in R$:

$$0 \leq \mu_t^0(q) + \mu_f^0(q) \leq 1.$$

(4) *Output Symbols*:

- Z is a finite set of output symbols:

$$Z = \{b_1, b_2, \ldots, b_k\}.$$

(5) *Vague Transition Function*:

- $\tilde{\delta} : (Q \times [0,1]^2) \times \Sigma \times Q \to [0,1]^2$ is the augmented vague transition function, defined by:

$$\tilde{\delta}\left((q_i, \mu_t(q_i), \mu_f(q_i)), a_k, q_j\right) = \left(\tilde{\delta}_t, \tilde{\delta}_f\right),$$

where:

- $\mu_t(q_i)$ and $\mu_f(q_i)$ are the truth and falsity membership values of state q_i.
- $\delta(q_i, a_k, q_j) = (\delta_t(q_i, a_k, q_j), \delta_f(q_i, a_k, q_j))$ are the transition weights.
- The components are computed using the membership assignment function F_1:

$$\tilde{\delta}_t = F_1^{\wedge}\left(\mu_t(q_i), \delta_t(q_i, a_k, q_j)\right),$$
$$\tilde{\delta}_f = F_1^{\vee}\left(\mu_f(q_i), \delta_f(q_i, a_k, q_j)\right).$$

(6) *Output Function*:

- $\omega : Q \to Z$ is the output function.

(7) *Membership Assignment Functions*:

- $F_1 = (F_1^{\wedge}, F_1^{\vee})$, where:
  - $F_1^{\wedge} : [0,1] \times [0,1] \to [0,1]$ assigns the truth-membership value.
  - $F_1^{\vee} : [0,1] \times [0,1] \to [0,1]$ assigns the falsity-membership value.
  - Common forms for $F_1^{\wedge}$ and $F_1^{\vee}$ include:

$$F_1^{\wedge}(\mu_t, \delta_t) = \min(\mu_t, \delta_t), \quad F_1^{\vee}(\mu_f, \delta_f) = \max(\mu_f, \delta_f).$$

(8) *Multi-Membership Resolution Function*:

- $F_2 = (F_2^{\wedge}, F_2^{\vee})$, where:
  - $F_2^{\wedge} : [0,1]^n \to [0,1]$ resolves multiple truth-membership values.
  - $F_2^{\vee} : [0,1]^n \to [0,1]$ resolves multiple falsity-membership values.
  - Common forms include:

$$F_2^{\wedge}\left(\{\mu_t^i\}_{i=1}^n\right) = \max_i \mu_t^i, \quad F_2^{\vee}\left(\{\mu_f^i\}_{i=1}^n\right) = \min_i \mu_f^i.$$

The membership values of the state q_j at time $t+1$ are calculated as:

$$\mu_{t+1}^{t}(q_j) = F_2^{\wedge}\left(\left\{F_1^{\wedge}\left(\mu_t^t(q_i), \delta_t(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right),$$
$$\mu_{t+1}^{f}(q_j) = F_2^{\vee}\left(\left\{F_1^{\vee}\left(\mu_t^f(q_i), \delta_f(q_i, a_k, q_j)\right)\right\}_{i=1}^{n}\right),$$

where:

- n is the number of transitions leading to q_j at time $t+1$.
- $\mu_t^t(q_i)$ and $\mu_t^f(q_i)$ are the truth and falsity membership values of state q_i at time t.
- $\delta_t(q_i, a_k, q_j)$ and $\delta_f(q_i, a_k, q_j)$ are the transition truth and falsity weights.

Theorem 2.21. *A Neutrosophic General Finite Automaton (GNA) can be transformed into a Vague General Finite Automaton (VGFA) by appropriately redefining its transition and membership functions.*

Proof. Let $\tilde{F}^N$ be a Neutrosophic General Finite Automaton defined as:

$$\tilde{F}^N = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}^N, \omega, F_1^N, F_2^N),$$

where:

- $\tilde{\delta}^N : (Q \times [0,1]^3) \times \Sigma \times Q \to [0,1]^3$ is the neutrosophic transition function, which includes truth, indeterminacy, and falsity components.
- F_1^N and F_2^N are the membership assignment and resolution functions, respectively, with three components: truth, indeterminacy, and falsity.

A Vague General Finite Automaton $\tilde{F}^V$ is defined as:

$$\tilde{F}^V = (Q, \Sigma, \tilde{R}^V, Z, \tilde{\delta}^V, \omega, F_1^V, F_2^V),$$

where:

- *Vague Start States:* The vague start states $\tilde{R}^V$ are obtained from the neutrosophic start states $\tilde{R}^N$ by combining the truth and falsity components, while ignoring the indeterminacy component:

  $$\tilde{R}^V = \{(q, \mu_t^0(q), \mu_f^0(q)) \mid (q, \mu_1^0(q), \mu_2^0(q), \mu_3^0(q)) \in \tilde{R}^N\},$$

  where:

  $$\mu_t^0(q) = \mu_1^0(q),$$
  $$\mu_f^0(q) = \mu_3^0(q).$$

- *Vague Transition Function:* The vague transition function $\tilde{\delta}^V$ is derived from the neutrosophic transition function $\tilde{\delta}^N$ by retaining only the truth and falsity components:

  $$\tilde{\delta}^V\left((q_i, \mu_t(q_i), \mu_f(q_i)), a_k, q_j\right) = \left(\tilde{\delta}_t, \tilde{\delta}_f\right),$$

where:

$$\tilde{\delta}_t = \tilde{\delta}_1 \left((q_i, \mu_t(q_i)), a_k, q_j \right),$$
$$\tilde{\delta}_f = \tilde{\delta}_3 \left((q_i, \mu_f(q_i)), a_k, q_j \right).$$

- *Membership Assignment Function:* The vague membership assignment function F_1^V is defined by the truth and falsity components of the neutrosophic membership assignment function F_1^N:

$$F_1^V = \left(F_1^{\wedge}, F_1^{\vee} \right),$$

where:

$$F_1^{\wedge}(\mu_t, \delta_t) = F_1^{\wedge}(\mu_1, \delta_1),$$
$$F_1^{\vee}(\mu_f, \delta_f) = F_1^{\vee}(\mu_3, \delta_3).$$

- *Multi-Membership Resolution Function:* The vague multi-membership resolution function F_2^V is derived from the neutrosophic resolution function F_2^N, retaining only the truth and falsity components:

$$F_2^V = \left(F_2^{\wedge}, F_2^{\vee} \right),$$

where:

$$F_2^{\wedge}\left(\{v_t^i\} \right) = F_2^{\wedge}\left(\{v_1^i\} \right),$$
$$F_2^{\vee}\left(\{v_f^i\} \right) = F_2^{\vee}\left(\{v_3^i\} \right).$$

By appropriately redefining the start states, transition functions, membership assignment, and resolution functions, a Neutrosophic General Finite Automaton (GNA) can be transformed into a Vague General Finite Automaton (VGFA). □

Theorem 2.22. *A Vague General Finite Automaton (VGFA) is a generalization of a General Fuzzy Automaton (GFA).*

Proof. Let $\tilde{F}_{\text{GFA}} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be a General Fuzzy Automaton. Define a VGFA $\tilde{F}_{\text{VGFA}} = (Q, \Sigma, \tilde{R}_{\text{V}}, Z, \tilde{\delta}_{\text{V}}, \omega, F_1^{\text{V}}, F_2^{\text{V}})$ as follows:

- Use the same set of states, input symbols, and output symbols:

$$Q_{\text{VGFA}} = Q_{\text{GFA}}, \quad \Sigma_{\text{VGFA}} = \Sigma_{\text{GFA}}, \quad Z_{\text{VGFA}} = Z_{\text{GFA}}.$$

- Define the vague start states:

$$\tilde{R}_{\text{V}} = \left\{ (q, \mu, 1-\mu) \mid \mu \in \tilde{R} \right\}.$$

- Extend the fuzzy transition function $\tilde{\delta}$ to the vague transition function $\tilde{\delta}_{\mathrm{V}}$:

$$\tilde{\delta}_{\mathrm{V}}((q,\mu_t,\mu_f),a,q') = \begin{cases} (\tilde{\delta}(q,a,q'), 1-\tilde{\delta}(q,a,q')) & \text{if } \tilde{\delta}(q,a,q') \neq 0, \\ (0,1) & \text{otherwise.} \end{cases}$$

- Define F_1^{V} to operate on truth and falsity components:

$$F_1^{\mathrm{V}}((\mu_t,\mu_f),(\delta_t,\delta_f)) = (\min(\mu_t,\delta_t), \max(\mu_f,\delta_f)).$$

- Use F_2^{V} to resolve multi-membership values:

$$F_2^{\mathrm{V}}(\{(\mu_{t,i},\mu_{f,i})\}) = \left(\max_i \mu_{t,i}, \min_i \mu_{f,i}\right).$$

By construction, $\tilde{F}_{\mathrm{VGFA}}$ reproduces the behavior of $\tilde{F}_{\mathrm{GFA}}$ when the falsity membership is the complement of truth membership. Thus, VGFA generalizes GFA. □

Theorem 2.23. *A Vague General Finite Automaton (VGFA) is a generalization of a Finite Automaton (FA).*

Proof. Let $A = (Q,\Sigma,\delta,q_0,F)$ be a finite automaton. Define a VGFA $\tilde{F}_{\mathrm{VGFA}} = (Q,\Sigma,\tilde{R}_{\mathrm{V}},Z,\tilde{\delta}_{\mathrm{V}},\omega,F_1^{\mathrm{V}},F_2^{\mathrm{V}})$ as follows:

- Use the same set of states and input symbols:

$$Q_{\mathrm{VGFA}} = Q_{\mathrm{FA}}, \quad \Sigma_{\mathrm{VGFA}} = \Sigma_{\mathrm{FA}}.$$

- Define the vague start states:

$$\tilde{R}_{\mathrm{V}} = \{(q,1,0) \text{ if } q = q_0,\ (q,0,1) \text{ otherwise.}\}.$$

- Define the vague transition function:

$$\tilde{\delta}_{\mathrm{V}}((q,\mu_t,\mu_f),a,q') = \begin{cases} (1,0) & \text{if } \delta(q,a) = q', \\ (0,1) & \text{otherwise.} \end{cases}$$

- Use trivial membership assignment functions:

$$F_1^{\mathrm{V}}((\mu_t,\mu_f),(\delta_t,\delta_f)) = (\delta_t,\delta_f).$$

- Define F_2^{V} as follows:

$$F_2^{\mathrm{V}}(\{(\mu_{t,i},\mu_{f,i})\}) = \left(\max_i \mu_{t,i}, \min_i \mu_{f,i}\right).$$

Under this construction:

(1) Each state in A maps to a vague state in $\tilde{F}_{\mathrm{VGFA}}$, with truth membership 1 and falsity membership 0 when active.

(2) The deterministic transitions of A are captured by the vague transition function with truth membership 1.

Thus, $\tilde{F}_{\mathrm{VGFA}}$ reproduces the behavior of A as a special case where truth and falsity membership values are binary. Therefore, VGFA generalizes FA. □

2.5. *Plithogenic General Finite Automaton*

Plithogenic General Finite Automaton represent a Plithogenic adaptation of General Finite Automata. The definitions and theorems, including related concepts, are provided below.

Definition 2.24. [156, 157] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v.
- $pdf : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)*.
- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

(1) *Reflexivity of Contradiction Function*:

$$pCF(a, a) = 0$$

(2) *Symmetry of Contradiction Function*:

$$pCF(a, b) = pCF(b, a)$$

Example 2.25. (cf. [61]) Here, $s, t \in \{1, 2, 3, 4\}$.

- When $s = t = 1$, PS is called a *Plithogenic Fuzzy Set* and is denoted by PFS.
- When $s = 2, t = 1$, PS is called a *Plithogenic Intuitionistic Fuzzy Set* and is denoted by $PIFS$.
- When $s = 3, t = 1$, PS is called a *Plithogenic Neutrosophic Set* and is denoted by PNS.
- When $s = 4, t = 1$, PS is called a *Plithogenic Turiyam Neutrosophic Set* and is denoted by $PTuS$.

Definition 2.26. A *Plithogenic General Finite Automaton (PGFA)* is defined as an eight-tuple:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where:

(1) *States*:

- Q is a finite set of states:

$$Q = \{q_1, q_2, \ldots, q_n\}.$$

(2) *Input Symbols*:

- Σ is a finite set of input symbols:

$$\Sigma = \{a_1, a_2, \ldots, a_m\}.$$

(3) *Plithogenic Start States*:

- $\tilde{R}$ is the set of plithogenic start states:

$$\tilde{R} = \{(q, v_q, \mathrm{DAF}_R(q, v_q)) \,|\, q \in R\},$$

where:

- $R \subseteq Q$ is the set of initial states.
- $v_q \in V$ is the attribute value of state q.
- $\mathrm{DAF}_R(q, v_q) \in [0,1]^s$ is the Degree of Appurtenance Function (DAF) for state q.

(4) *Output Symbols*:

- Z is a finite set of output symbols:

$$Z = \{b_1, b_2, \ldots, b_k\}.$$

(5) *Plithogenic Transition Function*:

- $\tilde{\delta} : (Q \times V \times [0,1]^s) \times \Sigma \times Q \to [0,1]^s$ is the augmented plithogenic transition function, defined by:

$$\tilde{\delta}\left((q_i, v_{q_i}, \mu(q_i)), a_k, q_j\right) = \mathrm{DAF}_\delta\left((q_i, q_j), (v_{q_i}, v_{q_j}), a_k\right),$$

where:

- $\mu(q_i)$ is the Degree of Appurtenance of state q_i.
- $v_{q_i}, v_{q_j} \in V$ are the attribute values of states q_i and q_j.
- DAF_δ is determined using the Degree of Contradiction Function (DCF) and the DAF of the states.

(6) *Output Function*:

- $\omega : Q \to Z$ is the output function.

(7) *Membership Assignment Function*:

- $F_1 : [0,1]^s \times [0,1]^s \to [0,1]^s$ assigns the degree of appurtenance for transitions, considering the DAF and DCF.

(8) *Multi-Membership Resolution Function*:

- $F_2 : ([0,1]^s)^n \to [0,1]^s$ resolves multiple degrees of appurtenance when there are simultaneous transitions to the same state.

The degree of appurtenance of state q_j at time $t+1$ is calculated as:

$$\mu_{t+1}(q_j) = F_2\left(\left\{F_1\left(\mu_t(q_i), \mathrm{DAF}_\delta\left((q_i,q_j),(v_{q_i},v_{q_j}),a_k\right)\right)\right\}_{i=1}^{n}\right),$$

where:

- n is the number of transitions leading to q_j at time $t+1$.
- $\mu_t(q_i)$ is the degree of appurtenance of state q_i at time t.
- $\mathrm{DAF}_\delta\left((q_i,q_j),(v_{q_i},v_{q_j}),a_k\right)$ incorporates the attribute values and degrees of contradiction.

Theorem 2.27. *A Plithogenic General Finite Automaton (PGFA) can be transformed into a Neutrosophic General Finite Automaton (GNA), a Vague General Finite Automaton (VGFA), a Fuzzy General Finite Automaton (GFA), and a Turiyam Neutrosophic General Finite Automaton (TGFA) by appropriately selecting the parameters s and t in the Plithogenic set and redefining its transition and membership functions accordingly.*

Proof. A Plithogenic General Finite Automaton is defined using Plithogenic sets, which generalize various types of fuzzy sets by introducing an attribute with possible values and corresponding degrees of appurtenance and contradiction. By specializing the parameters s and t in the Plithogenic set, we can obtain different types of automata.

A *Plithogenic Set PS* is defined as:

$$PS = (P, v, Pv, \mathrm{DAF}, \mathrm{DCF}),$$

.

By choosing specific values for s and t, the Plithogenic set reduces to various fuzzy sets:

- $s = t = 1$: Plithogenic Fuzzy Set (PFS).
- $s = 2, t = 1$: Plithogenic Intuitionistic Fuzzy Set (PIFS).
- $s = 3, t = 1$: Plithogenic Neutrosophic Set (PNS).
- $s = 4, t = 1$: Plithogenic Turiyam Neutrosophic Set (PTuS).
- $s = 2, t = 1$ with additional constraints: Vague Set.

A *Plithogenic General Finite Automaton (PGFA)* is defined as an eight-tuple:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

with states, input symbols, plithogenic start states, output symbols, plithogenic transition function, output function, membership assignment function, and multi-membership resolution function.

We will show that by selecting appropriate values of s and t and redefining functions, a PGFA can be transformed into each of the other automata.

We consider Transformation to a Fuzzy General Finite Automaton (GFA). Set $s = t = 1$. The Plithogenic set becomes a Plithogenic Fuzzy Set (PFS). The Degree of Appurtenance Function (DAF) and Degree of Contradiction Function (DCF) reduce to functions with a single value.

- *States*: Remain the same.
- *Start States*: The DAF now assigns a single membership degree $\mu(q) \in [0, 1]$ to each state q.
- *Transition Function*: The DAF for transitions assigns a single membership degree to each transition, similar to the transition weights in a GFA.
- *Membership Assignment Function* F_1: Becomes $F_1 : [0, 1] \times [0, 1] \to [0, 1]$.
- *Multi-Membership Resolution Function* F_2: Becomes $F_2 : [0, 1]^n \to [0, 1]$.

Thus, the PGFA reduces to a GFA, with the fuzzy membership degrees being the degrees of appurtenance from the Plithogenic set.

We consider Transformation to a Neutrosophic General Finite Automaton (GNA). Set $s = 3$, $t = 1$. The Plithogenic set becomes a Plithogenic Neutrosophic Set (PNS).

- *States*: Remain the same.
- *Start States*: The DAF assigns a vector $(\mu_T(q), \mu_I(q), \mu_F(q)) \in [0, 1]^3$ to each state q, representing truth, indeterminacy, and falsity membership degrees.
- *Transition Function*: The DAF for transitions assigns a vector of degrees to each transition.
- *Membership Assignment Function* F_1: Becomes $F_1 : [0, 1]^3 \times [0, 1]^3 \to [0, 1]^3$, handling the neutrosophic components.
- *Multi-Membership Resolution Function* F_2: Becomes $F_2 : ([0, 1]^3)^n \to [0, 1]^3$.

Thus, the PGFA reduces to a GNA, with the neutrosophic membership degrees derived from the degrees of appurtenance in the Plithogenic set.

We consider Transformation to a Vague General Finite Automaton (VGFA). Set $s = 2$, $t = 1$, and ensure that for each element x:

$$0 \leq \mu_T(x) + \mu_F(x) \leq 1.$$

This corresponds to a Vague Set.

- *States*: Remain the same.
- *Start States*: The DAF assigns a pair $(\mu_T(q), \mu_F(q)) \in [0, 1]^2$ to each state q, representing truth and falsity membership degrees.

- *Transition Function*: The DAF for transitions assigns pairs of degrees to each transition.
- *Membership Assignment Function* F_1: Becomes $F_1 : [0,1]^2 \times [0,1]^2 \to [0,1]^2$.
- *Multi-Membership Resolution Function* F_2: Becomes $F_2 : ([0,1]^2)^n \to [0,1]^2$.

Thus, the PGFA reduces to a VGFA, with the vague membership degrees obtained from the degrees of appurtenance in the Plithogenic set.

We consider Transformation to a Turiyam Neutrosophic General Finite Automaton (TGFA). Set $s = 4$, $t = 1$. The Plithogenic set becomes a Plithogenic Turiyam Neutrosophic Set (PTuS).

- *States*: Remain the same.
- *Start States*: The DAF assigns a vector $(\mu_T(q), \mu_I(q), \mu_F(q), \mu_L(q)) \in [0,1]^4$ to each state q, representing truth, indeterminacy, falsity, and liberal membership degrees.
- *Transition Function*: The DAF for transitions assigns a vector of degrees to each transition.
- *Membership Assignment Function* F_1: Becomes $F_1 : [0,1]^4 \times [0,1]^4 \to [0,1]^4$.
- *Multi-Membership Resolution Function* F_2: Becomes $F_2 : ([0,1]^4)^n \to [0,1]^4$.

Thus, the PGFA reduces to a TGFA, with the Turiyam Neutrosophic membership degrees derived from the degrees of appurtenance in the Plithogenic set.

By selecting appropriate values for s and t in the Plithogenic set and redefining the functions accordingly, a Plithogenic General Finite Automaton can be transformed into a Neutrosophic, Vague, Fuzzy, or Turiyam Neutrosophic General Finite Automaton. □

3. Graph Grammar

Graph Grammar is a formal framework that defines rules for graph transformations, enabling the representation of dynamic changes in graph structures.We explore the concepts of Graph Grammar within the contexts of Fuzzy, Neutrosophic, Vague, Turiyam, and Plithogenic frameworks. First, we present the fundamental definitions of Graph Grammar. The definitions are provided below.

Definition 3.1 (Graph)**.** [41] A *graph* G is a mathematical structure that represents relationships between objects. It consists of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge connects a pair of vertices. Formally, a graph is represented as $G = (V, E)$, where V is the set of vertices and E is the set of edges.

Definition 3.2 (Degree)**.** [41] Let $G = (V, E)$ be a graph. The *degree* of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges connected to v. For undirected graphs, the degree is given by:

$$\deg(v) = |\{e \in E \mid v \in e\}|.$$

For directed graphs, the *in-degree* $\deg^-(v)$ refers to the number of edges directed towards v, while the *out-degree* $\deg^+(v)$ represents the number of edges directed away from v.

Definition 3.3. (cf. [137]) An *empty graph* $G = (V, E)$ is a graph with a non-empty set of vertices V and an empty set of edges E. Formally:

$$G = (V, \emptyset),$$

where $V \neq \emptyset$ and $E = \emptyset$.

Definition 3.4. A *null graph* $G = (V, E)$ is a graph with an empty vertex set V and consequently an empty edge set E. Formally:

$$G = (\emptyset, \emptyset).$$

Definition 3.5 (Graph Grammar)**.** (cf. [48, 48, 128]) Graph Grammar is a formal framework for representing dynamic transformations of graph structures. It defines a set of rules for graph rewriting and consists of the following components:

(1) *Vertex and Edge Labels:*
- Vertex label set: $\Sigma_V = N_V \cup T_V$, where:
  - N_V: Non-terminal labels.
  - T_V: Terminal labels.
- Edge label set: $\Sigma_E = N_E \cup T_E$, where:
  - N_E: Non-terminal labels.
  - T_E: Terminal labels.

(2) *Start Graph:* The start graph G_S serves as the initial graph for transformations.

(3) *Production Rules:* A finite set of production rules P defines the transformations, where each rule $p \in P$ is represented as:

$$p : G_L \xleftarrow{l} H \xrightarrow{r} G_R,$$

where:
- G_L: Left-hand side graph, representing the pattern to match.
- G_R: Right-hand side graph, representing the replacement graph.
- H: Interface graph, representing the common subgraph used for gluing G_L and G_R.

(4) *Isomorphic Mappings:* The interface graph H is associated with the following label-preserving isomorphisms:

$$\phi_L : H' \to \phi_L(H) \subseteq G_L, \quad \phi_R : H' \to \phi_R(H) \subseteq G_R,$$

where:
- ϕ_L: Maps H to G_L.

- ϕ_R: Maps H to G_R.

Remark 3.6. A production rule p searches for a subgraph in G isomorphic to G_L. Upon matching, it replaces G_L with G_R, preserving the common structure H by gluing G_R at the interface defined by H.

Example 3.7 (Simple Graph Grammar)**.** Consider a simple Graph Grammar GG $= (T, G_0, \text{Rule})$:

- *Type Graph* T:
  - Vertex labels: $\Sigma_V = \{A, B\}$.
  - Edge labels: $\Sigma_E = \{a\}$.
- *Start Graph* G_0:
  - Vertices: $V = \{v_1\}$ with label A.
  - Edges: None.
- *Production Rule* p:

$$p : G_L \xleftarrow{l} H \xrightarrow{r} G_R,$$

  where:
  - G_L:
    - Vertices: $\{v\}$ with label A.
    - Edges: None.
  - G_R:
    - Vertices: $\{v_1, v_2\}$ with labels A, B, respectively.
    - Edges: $\{e\}$ connecting v_1 to v_2 with label a.
  - H:
    - Vertices: $\{v\}$ with label A.
    - Edges: None.
  - Mappings:
    - $l : H \to G_L$, identity on v.
    - $r : H \to G_R$, maps v to v_1.

Applying the production rule p to G_0 results in a graph with one A-labeled vertex connected by an a-labeled edge to a new B-labeled vertex.

Theorem 3.8. *Graph Grammar can represent both the empty graph and the null graph.*

Proof. Graph Grammar provides a framework for defining graph structures and their transformations through a set of rules. We show that both the empty graph and the null graph can be represented using Graph Grammar.

The empty graph $G = (V, E)$ is defined as having a non-empty vertex set V and an empty edge set $E = \emptyset$. Using Graph Grammar, we can construct G as follows:

(1) **Type Graph T:** Define a vertex label set $\Sigma_V = \{A\}$ and an edge label set $\Sigma_E = \emptyset$.
(2) **Start Graph G_S:** Initialize G_S with vertices $V = \{v_1, v_2, \ldots, v_n\}$, where $n > 0$, and $E = \emptyset$.
(3) **Production Rules:** No rules are needed, as the empty graph does not include any edges or transformations.

Thus, G_S itself represents the empty graph.

The null graph $G = (\emptyset, \emptyset)$ is defined as having no vertices and no edges. Using Graph Grammar, we can construct G as follows:

(1) **Type Graph T:** Define an empty vertex label set $\Sigma_V = \emptyset$ and an edge label set $\Sigma_E = \emptyset$.
(2) **Start Graph G_S:** Initialize $G_S = (\emptyset, \emptyset)$.
(3) **Production Rules:** No rules are needed, as the null graph is static and requires no transformations.

Thus, $G_S = (\emptyset, \emptyset)$ directly represents the null graph.

Graph Grammar can represent both the empty graph and the null graph by appropriately defining the vertex and edge label sets, the start graph, and the (optional) production rules. □

3.1. *Fuzzy Graph Grammar*

Fuzzy Graph Grammar is an extension of Graph Grammar with added fuzzy conditions, and it has been explored in several studies [22, 79, 118, 120, 183]. The definitions, including related concepts, are provided below.

Definition 3.9. [127] A *fuzzy graph* G is defined as:

$$G = ((V_G, \mathrm{pert}_V), (E_G, \mathrm{pert}_E), \mathrm{source}_G, \mathrm{target}_G),$$

where:

- V_G is a fuzzy set of vertices with a membership function $\mathrm{pert}_V : V_G \to [0, 1]$, which assigns a membership degree to each vertex.
- E_G is a fuzzy set of edges with a membership function $\mathrm{pert}_E : E_G \to [0, 1]$, which assigns a membership degree to each edge.
- $\mathrm{source}_G : E_G \to V_G$ is the *source mapping* of edges.
- $\mathrm{target}_G : E_G \to V_G$ is the *target mapping* of edges.

The membership degrees $\mathrm{pert}_V(v)$ and $\mathrm{pert}_E(e)$ indicate the presence or influence of vertices $v \in V_G$ and edges $e \in E_G$, respectively.

Definition 3.10. [22] Let

$$G = ((V_G, \text{pert}_V), (E_G, \text{pert}_E), \text{source}_G, \text{target}_G)$$

and

$$H = \big((V_H, \text{pert}'_V), (E_H, \text{pert}'_E), \text{source}_H, \text{target}_H\big)$$

be two fuzzy graphs. A *fuzzy graph morphism* $f : G \to H$ is a pair of fuzzy set morphisms (f_V, f_E) such that:

$$f_V \circ \text{source}_G = \text{source}_H \circ f_E \quad \text{and} \quad f_V \circ \text{target}_G = \text{target}_H \circ f_E,$$

where:

- $f_V : V_G \to V_H$ maps vertices from G to H.
- $f_E : E_G \to E_H$ maps edges from G to H.
- The mappings f_V and f_E must preserve the source and target of edges, as well as the membership degrees.

Definition 3.11. [22] A *typed fuzzy graph* G_T is defined as a tuple:

$$G_T = (G, t_G, T),$$

where:

- G is a fuzzy graph.
- T is a *fuzzy type graph* that characterizes the types of vertices and edges allowed in the grammar.
- $t_G : G \to T$ is a total fuzzy graph morphism, ensuring that each fuzzy graph in the grammar conforms to the fuzzy type graph T.

Definition 3.12. [22] A *fuzzy rule* p in FGG is defined as a tuple:

$$p : L \xleftarrow{l} K \xrightarrow{r} R,$$

where:

- L, K, R are fuzzy graphs typed over the fuzzy type graph T.
- l and r are total fuzzy graph morphisms, representing the left-hand side, interface, and right-hand side of the rule, respectively.

The following sets are associated with the application of a fuzzy rule:

- *Deleted vertices:* $\text{RuleDel}_V^p = V_L \setminus \text{rng}(l_V)$.
- *Deleted edges:* $\text{RuleDel}_E^p = E_L \setminus \text{rng}(l_E)$.
- *Preserved vertices:* $\text{RulePres}_V^p = V_L \setminus \text{RuleDel}_V^p$.
- *Preserved edges:* $\text{RulePres}_E^p = E_L \setminus \text{RuleDel}_E^p$.
- *Created vertices:* $\text{RuleCreated}_V^p = V_R \setminus \text{rng}(r_V)$.

- *Created edges:* $\text{RuleCreated}_E^p = E_R \setminus \text{rng}(r_E)$.

Definition 3.13. [22] A *Typed Fuzzy Graph Grammar (FGG)* is defined as a tuple:

$$\text{FGG} = (T, G_0, \text{Rule}),$$

where:

- T is the fuzzy type graph.
- G_0 is the initial fuzzy graph typed over T, representing the initial state of the system.
- Rule is a set of fuzzy rules that describe the state transitions allowed within the system.

The behavior of a fuzzy graph grammar is determined by the application of these fuzzy rules, enabling the transition from one fuzzy graph to another by modifying vertices and edges according to the specified rules.

Example 3.14 (Fuzzy Graph Grammar)**.** Consider a Fuzzy Graph Grammar $\text{FGG} = (T', G_0', \text{Rule}')$:

- *Type Graph* T':
  - Vertices with membership functions pert_V:
    - Vertex A with $\text{pert}_V(A) = 1$.
    - Vertex B with $\text{pert}_V(B) = 1$.
  - Edges with membership functions pert_E:
    - Edge a with $\text{pert}_E(a) = 1$.
- *Start Graph* G_0':
  - Vertices: $V = \{v_1\}$ with label A and $\text{pert}_V(v_1) = 0.8$.
  - Edges: None.
- *Fuzzy Production Rule* p':

$$p' : L \xleftarrow{l} K \xrightarrow{r} R,$$

  where:
  - L:
    - Vertices: $\{v\}$ with label A and $\text{pert}_V(v) = \alpha$.
    - Edges: None.
  - R:
    - Vertices: $\{v_1, v_2\}$ with labels A, B and $\text{pert}_V(v_1) = \alpha$, $\text{pert}_V(v_2) = \beta$.
    - Edges: $\{e\}$ connecting v_1 to v_2 with label a and $\text{pert}_E(e) = \gamma$.
  - K:
    - Vertices: $\{v\}$ with label A and $\text{pert}_V(v) = \alpha$.
    - Edges: None.
  - Mappings:

$*$ $l : K \to L$, identity on v.

$*$ $r : K \to R$, maps v to v_1.

Applying the fuzzy production rule p' to G'_0 results in:

- The original vertex v_1 retains its membership degree $\text{pert}_V(v_1) = 0.8$.
- A new vertex v_2 with label B and membership degree $\text{pert}_V(v_2) = 0.6$ (assuming $\beta = 0.6$).
- A new edge e connecting v_1 and v_2 with membership degree $\text{pert}_E(e) = 0.7$ (assuming $\gamma = 0.7$).

The membership degrees represent the uncertainty or partial presence of vertices and edges in the fuzzy graph. This example illustrates how Fuzzy Graph Grammar extends Graph Grammar by incorporating fuzzy membership values.

Theorem 3.15. *A Fuzzy Graph Grammar generalizes a Graph Grammar by allowing vertices and edges to have membership degrees between 0 and 1. A standard Graph Grammar is a special case of a Fuzzy Graph Grammar where all membership degrees are either 0 or 1.*

Proof. Let $\text{GG} = (T, G_0, \text{Rule})$ be a Graph Grammar, where:

- Vertices and edges are unweighted (membership degrees of 1 for existing elements, and implicitly 0 for non-existing elements).
- T is the type graph, G_0 is the initial graph, and Rule is the set of production rules.

We can construct a corresponding Fuzzy Graph Grammar $\text{FGG} = (T', G'_0, \text{Rule}')$ as follows:

- For each vertex v in T, set $\text{pert}_V(v) = 1$ in T'.
- For each edge e in T, set $\text{pert}_E(e) = 1$ in T'.
- Similarly, define G'_0 and Rule' by assigning membership degrees of 1 to all vertices and edges present in G_0 and Rule.

In this construction, the fuzzy membership functions pert_V and pert_E take only values in $\{0, 1\}$. Therefore, the behavior of FGG replicates that of GG, demonstrating that Graph Grammar is a special case of Fuzzy Graph Grammar. Hence, Fuzzy Graph Grammar generalizes Graph Grammar by allowing membership degrees in the interval $[0, 1]$. □

Theorem 3.16. *A Fuzzy Graph Grammar can represent both the empty graph and the null graph.*

Proof. Fuzzy Graph Grammar extends Graph Grammar by allowing vertices and edges to have membership degrees in the interval $[0, 1]$. We demonstrate how a Fuzzy Graph Grammar can represent both the empty graph and the null graph.

The empty graph $G = (V, E)$ has a non-empty vertex set V and an empty edge set $E = \emptyset$.

Using Fuzzy Graph Grammar, we define:

(1) **Type Graph T:** Define a vertex label set $\Sigma_V = \{A\}$ and an edge label set $\Sigma_E = \emptyset$.
(2) **Initial Fuzzy Graph G_0:** Initialize G_0 with:
   - A fuzzy vertex set $V_G = \{v_1, v_2, \ldots, v_n\}$ with membership function $\text{pert}_V(v_i) = 1$ for all v_i.
   - A fuzzy edge set $E_G = \emptyset$.
(3) **Production Rules:** No rules are needed, as the empty graph contains no edges or transformations.

Since the membership function assigns $\text{pert}_E(e) = 0$ for all edges e, the resulting fuzzy graph G_0 represents the empty graph.

The null graph $G = (\emptyset, \emptyset)$ has both an empty vertex set and an empty edge set.

Using Fuzzy Graph Grammar, we define:

(1) **Type Graph T:** Define an empty vertex label set $\Sigma_V = \emptyset$ and an edge label set $\Sigma_E = \emptyset$.
(2) **Initial Fuzzy Graph G_0:** Initialize G_0 with:
   - $V_G = \emptyset$.
   - $E_G = \emptyset$.
(3) **Production Rules:** No rules are needed, as the null graph is static.

Since $V_G = \emptyset$ and $E_G = \emptyset$, the resulting fuzzy graph G_0 directly represents the null graph.

Fuzzy Graph Grammar can represent both the empty graph and the null graph by appropriately defining the type graph, the initial fuzzy graph, and the membership functions. This demonstrates the expressiveness of Fuzzy Graph Grammar in capturing these fundamental graph structures. □

3.2. *Neutrosophic Graph Grammar*

Neutrosophic Graph Grammar is an extension of Graph Grammar with added Neutrosophic conditions. The definitions and theorems, including related concepts, are provided below.

Definition 3.17. [169] A *Neutrosophic Graph* G is defined as a quadruple:

$$G = (V, E, \sigma, \mu),$$

where:

- V is a finite set of vertices.
- E is a set of edges, where each edge connects two vertices in V.

- $\sigma : V \to [0,1]^3$ assigns to each vertex $v \in V$ a triple $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, representing the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* degrees, respectively.
- $\mu : E \to [0,1]^3$ assigns to each edge $e \in E$ a triple $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.

These membership degrees satisfy the condition for all $v \in V$ and $e \in E$:

$$0 \leq \sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3, \quad 0 \leq \mu_T(e) + \mu_I(e) + \mu_F(e) \leq 3.$$

Additionally, for every edge $e = (v_i, v_j) \in E$:

$$\mu_T(e) \leq \min\{\sigma_T(v_i), \sigma_T(v_j)\}.$$

Definition 3.18. Let $G = (V_G, E_G, \sigma_G, \mu_G)$ and $H = (V_H, E_H, \sigma_H, \mu_H)$ be two neutrosophic graphs. A *Neutrosophic Graph Morphism* $f : G \to H$ is a pair of functions (f_V, f_E) where:

- $f_V : V_G \to V_H$ maps vertices of G to vertices of H.
- $f_E : E_G \to E_H$ maps edges of G to edges of H.
- The mappings preserve the incidence relations:

$$f_V(\text{source}_G(e)) = \text{source}_H(f_E(e)), \quad f_V(\text{target}_G(e)) = \text{target}_H(f_E(e)), \quad \forall e \in E_G.$$

- The membership degrees satisfy:

$$\sigma_G(v) \leq \sigma_H(f_V(v)), \quad \mu_G(e) \leq \mu_H(f_E(e)), \quad \forall v \in V_G, e \in E_G.$$

Definition 3.19. A *Typed Neutrosophic Graph* G_T is a triple:

$$G_T = (G, t_G, T),$$

where:

- G is a neutrosophic graph.
- T is a *Neutrosophic Type Graph* specifying allowed types and membership degrees for vertices and edges.
- $t_G : G \to T$ is a total neutrosophic graph morphism, ensuring that G conforms to the type graph T.

Definition 3.20. A *Neutrosophic Rule* p is defined as:

$$p : L \xleftarrow{l} K \xrightarrow{r} R,$$

where:

- L, K, R are neutrosophic graphs typed over the neutrosophic type graph T.
- l and r are total neutrosophic graph morphisms.

The following sets are associated with the rule p:

- *Deleted Vertices*: $\text{Del}_V^p = V_L \setminus \text{Im}(l_V)$.

- *Deleted Edges*: $\mathrm{Del}_E^p = E_L \setminus \mathrm{Im}(l_E)$.
- *Preserved Vertices*: $\mathrm{Pres}_V^p = \mathrm{Im}(l_V)$.
- *Preserved Edges*: $\mathrm{Pres}_E^p = \mathrm{Im}(l_E)$.
- *Created Vertices*: $\mathrm{Cre}_V^p = V_R \setminus \mathrm{Im}(r_V)$.
- *Created Edges*: $\mathrm{Cre}_E^p = E_R \setminus \mathrm{Im}(r_E)$.

Definition 3.21. A *Typed Neutrosophic Graph Grammar (NGG)* is a triple:

$$\mathrm{NGG} = (T, G_0, \mathrm{Rule}),$$

where:

- T is the neutrosophic type graph.
- G_0 is the initial neutrosophic graph typed over T.
- Rule is a set of neutrosophic rules.

The grammar defines the possible transformations of neutrosophic graphs through the application of rules, modeling the dynamic behavior of systems with neutrosophic uncertainty.

Theorem 3.22. *A Neutrosophic Graph Grammar generalizes a Fuzzy Graph Grammar by allowing vertices and edges to have truth, indeterminacy, and falsity membership values, whereas a Fuzzy Graph Grammar is limited to truth-membership only.*

Proof. Let $\mathrm{FGG} = (T, G_0, \mathrm{Rule})$ be a Fuzzy Graph Grammar where:

- Vertices and edges have fuzzy membership values $\mathrm{pert}_V : V \to [0, 1]$ and $\mathrm{pert}_E : E \to [0, 1]$.

Define a corresponding Neutrosophic Graph Grammar $\mathrm{NGG} = (T, G_0, \mathrm{Rule})$ where:

- Each vertex $v \in V$ and edge $e \in E$ is assigned neutrosophic membership values $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$ and $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$, satisfying:

$$0 \leq \sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3, \quad 0 \leq \mu_T(e) + \mu_I(e) + \mu_F(e) \leq 3.$$

When $\sigma_I(v) = 0$ and $\sigma_F(v) = 0$ for all $v \in V$, and similarly $\mu_I(e) = 0$ and $\mu_F(e) = 0$ for all $e \in E$, the NGG reduces to the FGG. Therefore, NGG generalizes FGG by introducing indeterminacy and falsity memberships, enabling richer representation of uncertainty. □

Corollary 3.23. *A Neutrosophic Graph Grammar can represent both the empty graph and the null graph.*

Proof. The proof follows almost the same method as for the Fuzzy Graph Grammar. □

3.3. *Turiyam Neutrosophic Graph Grammar*

Turiyam Neutrosophic Graph Grammar is an extension of Graph Grammar with added Turiyam Neutrosophic conditions. The definitions and theorems, including related concepts, are provided below.

Definition 3.24 (Turiyam Neutrosophic Graph)**.** [69–71,144] A *Turiyam Neutrosophic Graph* G^T is defined as:

$$G^T = (V^T, E^T),$$

where:

- V^T is the set of vertices with Turiyam Neutrosophic membership functions.
- E^T is the set of edges with Turiyam Neutrosophic membership functions.

For each vertex $v \in V^T$, there are mappings:

$$T(v), I(v), F(v), L(v) : V^T \to [0, 1],$$

where:

- $T(v)$ is the truth-membership degree.
- $I(v)$ is the indeterminacy-membership degree.
- $F(v)$ is the falsity-membership degree.
- $L(v)$ is the liberal state-membership degree.

These degrees satisfy:

$$0 \leq T(v) + I(v) + F(v) + L(v) \leq 4, \quad \forall v \in V^T.$$

Similarly, for each edge $e = (v_i, v_j) \in E^T$, there are mappings:

$$T(e), I(e), F(e), L(e) : E^T \to [0, 1],$$

satisfying:

$$0 \leq T(e) + I(e) + F(e) + L(e) \leq 4, \quad \forall e \in E^T.$$

Definition 3.25. Let $G = (V_G^T, E_G^T)$ and $H = (V_H^T, E_H^T)$ be two Turiyam Neutrosophic graphs. A *Turiyam Neutrosophic Graph Morphism* $f : G \to H$ is a pair of functions (f_V, f_E) where:

- $f_V : V_G^T \to V_H^T$.
- $f_E : E_G^T \to E_H^T$.
- The mappings preserve the incidence relations:

$$f_V(\text{source}_G(e)) = \text{source}_H(f_E(e)), \quad f_V(\text{target}_G(e)) = \text{target}_H(f_E(e)), \quad \forall e \in E_G^T.$$

- The Turiyam Neutrosophic membership degrees satisfy:

$$T_G(v) \leq T_H(f_V(v)), \quad I_G(v) \geq I_H(f_V(v)),$$
$$F_G(v) \geq F_H(f_V(v)), \quad L_G(v) \geq L_H(f_V(v)), \quad \forall v \in V_G^T,$$
$$T_G(e) \leq T_H(f_E(e)), \quad \text{and similarly for } I, F, L.$$

Definition 3.26. A *Typed Turiyam Neutrosophic Graph* G_T^T is a triple:

$$G_T^T = (G^T, t_G, T^T),$$

where:

- G^T is a Turiyam Neutrosophic graph.
- T^T is a *Turiyam Neutrosophic Type Graph* specifying allowed types and Turiyam Neutrosophic membership degrees.
- $t_G : G^T \to T^T$ is a total Turiyam Neutrosophic graph morphism.

Definition 3.27. A *Turiyam Neutrosophic Rule* p is defined as:

$$p : L^T \xleftarrow{l} K^T \xrightarrow{r} R^T,$$

where:

- L^T, K^T, R^T are Turiyam Neutrosophic graphs typed over the Turiyam Neutrosophic type graph T^T.
- l and r are total Turiyam Neutrosophic graph morphisms.

Definition 3.28. A *Typed Turiyam Neutrosophic Graph Grammar (TGG)* is a triple:

$$\text{TGG} = (T^T, G_0^T, \text{Rule}),$$

where:

- T^T is the Turiyam Neutrosophic type graph.
- G_0^T is the initial Turiyam Neutrosophic graph typed over T^T.
- Rule is a set of Turiyam Neutrosophic rules.

The grammar defines transformations of Turiyam Neutrosophic graphs, capturing systems with Turiyam Neutrosophic uncertainty involving truth, indeterminacy, falsity, and liberal states.

Theorem 3.29. *A Turiyam Neutrosophic Graph Grammar (TGG) can be transformed into a Neutrosophic Graph Grammar (NGG) and a Fuzzy Graph Grammar (FGG) by appropriately redefining the membership functions to match the specific requirements of each grammar type.*

Proof. Turiyam Neutrosophic Graph Grammar extends the conventional graph grammar by incorporating four membership components: truth, indeterminacy, falsity, and liberal state.

By selectively adjusting or omitting these components, we can transform a TGG into either an NGG or an FGG.

A *Turiyam Neutrosophic Graph* G^T is defined as:

$$G^T = (V^T, E^T),$$

where:

- V^T is the set of vertices with Turiyam Neutrosophic membership functions.
- E^T is the set of edges with Turiyam Neutrosophic membership functions.

For each vertex $v \in V^T$, there are mappings:

$$T(v), I(v), F(v), L(v) : V^T \to [0, 1].$$

Similarly, for each edge $e = (v_i, v_j) \in E^T$, there are mappings:

$$T(e), I(e), F(e), L(e) : E^T \to [0, 1].$$

These membership degrees satisfy:

$$0 \leq T(v) + I(v) + F(v) + L(v) \leq 4, \quad 0 \leq T(e) + I(e) + F(e) + L(e) \leq 4.$$

We will show how a TGG can be transformed into an NGG or an FGG by redefining the membership functions and adjusting the grammar rules accordingly.

We consider Transformation to a Neutrosophic Graph Grammar (NGG). To transform a TGG into an NGG, we set the liberal state membership degree to zero:

$$L(v) = 0, \quad L(e) = 0, \quad \forall v \in V^T, \, e \in E^T.$$

The Turiyam Neutrosophic graph G^T becomes a Neutrosophic Graph G, where each vertex v and edge e is represented by a triple of membership degrees:

$$\sigma(v) = (T(v), I(v), F(v)), \quad \mu(e) = (T(e), I(e), F(e)).$$

- The truth-membership, indeterminacy-membership, and falsity-membership components are preserved.
- The conditions for a Neutrosophic Graph are satisfied:

$$0 \leq \sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3, \quad 0 \leq \mu_T(e) + \mu_I(e) + \mu_F(e) \leq 3.$$

- The rules and morphisms of the Turiyam Neutrosophic graph grammar are adjusted by ignoring the liberal state component, resulting in the structure of an NGG.

We consider Transformation to a Fuzzy Graph Grammar (FGG). To transform a TGG into an FGG, we retain only the truth-membership component:

$$I(v) = 0, \quad F(v) = 0, \quad L(v) = 0, \quad \forall v \in V^T,$$
$$I(e) = 0, \quad F(e) = 0, \quad L(e) = 0, \quad \forall e \in E^T.$$

The Turiyam Neutrosophic graph G^T becomes a Fuzzy Graph G, where each vertex v and edge e is represented by a single membership degree:

$$\text{pert}_V(v) = T(v), \quad \text{pert}_E(e) = T(e).$$

- The truth-membership degree serves as the fuzzy membership degree for vertices and edges.
- The conditions for a Fuzzy Graph are satisfied:

$$0 \leq \text{pert}_V(v) \leq 1, \quad 0 \leq \text{pert}_E(e) \leq 1.$$

- The rules and morphisms of the Turiyam Neutrosophic graph grammar are adjusted to include only the truth-membership component, resulting in the structure of an FGG.

By redefining the membership components and adjusting the grammar rules, a Turiyam Neutrosophic Graph Grammar can be transformed into either a Neutrosophic Graph Grammar or a Fuzzy Graph Grammar, depending on which components are retained or omitted. □

Corollary 3.30. *A Turiyam Neutrosophic Graph Grammar can represent both the empty graph and the null graph.*

Proof. The proof follows almost the same method as for the Fuzzy Graph Grammar. □

3.4. *Vague Graph Grammar*

Vague Graph Grammar is an extension of Graph Grammar with added Vague conditions.

Definition 3.31. [81, 190] A *Vague Set (VS) A* on a set X is defined as:

$$A = (t_A, f_A),$$

where:

- $t_A : X \to [0, 1]$ is the *truth-membership function.*
- $f_A : X \to [0, 1]$ is the *falsity-membership function.*

For any element $x \in X$, the following condition holds:

$$0 \leq t_A(x) + f_A(x) \leq 1.$$

Definition 3.32. (cf. [104, 143, 185]) A *Vague Graph (VG)* is defined as a pair:

$$G = (A, B),$$

where:

- $A = (t_A, f_A)$ is a vague set on the vertices V, representing the vertices' truth and falsity degrees.
- $B = (t_B, f_B)$ is a vague set on the edges $E \subseteq V \times V$, representing the edges' truth and falsity degrees.

For each edge $ab \in E$, the following conditions hold:

(1) *Truth Membership Condition:*

$$t_B(ab) \leq \min(t_A(a), t_A(b)).$$

(2) *Falsity Membership Condition:*

$$f_B(ab) \geq \max(f_A(a), f_A(b)).$$

Definition 3.33. Let $G = (A, B)$ and $H = (A', B')$ be two vague graphs. A *Vague Graph Morphism* $f : G \to H$ consists of functions $f_V : V \to V'$ and $f_E : E \to E'$ such that:

- For all $v \in V$:

$$t_A(v) \leq t_{A'}(f_V(v)), \quad f_A(v) \geq f_{A'}(f_V(v)).$$

- For all $e = (u, v) \in E$:

$$f_E(e) = (f_V(u), f_V(v)),$$
$$t_B(e) \leq t_{B'}(f_E(e)),$$
$$f_B(e) \geq f_{B'}(f_E(e)).$$

Definition 3.34. A *Typed Vague Graph* G_T is a triple:

$$G_T = (G, t_G, T),$$

where:

- G is a vague graph.
- T is a vague type graph.
- $t_G : G \to T$ is a total vague graph morphism.

Definition 3.35. A *Vague Rule* p is defined as:

$$p : L \xleftarrow{l} K \xrightarrow{r} R,$$

where:

- L, K, R are typed vague graphs over the vague type graph T.

- l and r are total vague graph morphisms.

Definition 3.36. A *Vague Graph Grammar (VGG)* is a triple:

$$\text{VGG} = (T, G_0, \text{Rule}),$$

where:

- T is the vague type graph.
- G_0 is the initial typed vague graph over T.
- Rule is a set of vague rules.

Theorem 3.37. *A Neutrosophic Graph Grammar (NGG) can be transformed into a Vague Graph Grammar (VGG) by appropriately redefining the neutrosophic membership functions to match the vague membership conditions.*

Proof. Neutrosophic Graph Grammar (NGG) extends conventional graph grammar by incorporating three membership components: truth, indeterminacy, and falsity. To transform an NGG into a VGG, we need to adjust the membership components, reducing the indeterminacy component to zero while retaining the truth and falsity components.

A *Neutrosophic Graph* G is defined as:

$$G = (V, E, \sigma, \mu).$$

The membership degrees satisfy the following conditions for all vertices $v \in V$ and edges $e \in E$:

$$0 \leq \sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3, \quad 0 \leq \mu_T(e) + \mu_I(e) + \mu_F(e) \leq 3.$$

To transform a Neutrosophic Graph Grammar into a Vague Graph Grammar, we redefine the membership functions by eliminating the indeterminacy component and retaining only the truth and falsity components.

We consider Vague Graph Transformation.

(1) *Vertex Transformation:*

For each vertex $v \in V$, we define the vague membership functions as:

$$t_A(v) = \sigma_T(v), \quad f_A(v) = \sigma_F(v).$$

The indeterminacy component is set to zero:

$$\sigma_I(v) = 0.$$

(2) *Edge Transformation:*

For each edge $e \in E$, we define the vague membership functions as:

$$t_B(e) = \mu_T(e), \quad f_B(e) = \mu_F(e).$$

Similarly, the indeterminacy component is set to zero:

$$\mu_I(e) = 0.$$

With these transformations, the vague graph $G = (A, B)$ is formed, where:

- $A = (t_A, f_A)$ represents the vague membership degrees for vertices.
- $B = (t_B, f_B)$ represents the vague membership degrees for edges.

We consider Vague Membership Conditions. The conditions for a Vague Graph are satisfied as follows:

(1) For each vertex $v \in V$:

$$0 \leq t_A(v) + f_A(v) \leq 1,$$

since $\sigma_T(v) + \sigma_F(v) \leq 1$ holds for all vertices in the Neutrosophic Graph.

(2) For each edge $ab \in E$, the vague truth and falsity conditions hold:

$$t_B(ab) \leq \min(t_A(a), t_A(b)),$$
$$f_B(ab) \geq \max(f_A(a), f_A(b)).$$

We consider Vague Graph Grammar Structure. The transformed vague graph G conforms to the structure of a Vague Graph Grammar (VGG):

- The type graph, initial graph, and rules in the NGG are modified by ignoring the indeterminacy component.
- The VGG is defined as:

$$\text{VGG} = (T, G_0, \text{Rule}),$$

where:

  - T is the vague type graph.
  - G_0 is the initial typed vague graph over T.
  - Rule is a set of vague rules.

By setting the indeterminacy components to zero and preserving the truth and falsity components, a Neutrosophic Graph Grammar can be effectively transformed into a Vague Graph Grammar. □

Corollary 3.38. *A Vague Graph Grammar can represent both the empty graph and the null graph.*

Proof. The proof follows almost the same method as for the Fuzzy Graph Grammar. □

3.5. *Plithogenic Graph Grammar*

Plithogenic Graph Grammar is an extension of Graph Grammar with added Plithogenic conditions.

Definition 3.39. [156, 157, 174] A *Plithogenic Graph* PG is defined as:

$$PG = (PM, PN),$$

where:

(1) *Plithogenic Vertex Set* $PM = (M, l, Ml, adf, aCf)$:
- $M \subseteq V$ is the set of vertices.
- l is an attribute associated with the vertices.
- Ml is the set of possible attribute values.
- $adf : M \times Ml \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* for vertices.
- $aCf : Ml \times Ml \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* for vertices.

(2) *Plithogenic Edge Set* $PN = (N, m, Nm, bdf, bCf)$:
- $N \subseteq E$ is the set of edges.
- m is an attribute associated with the edges.
- Nm is the set of possible attribute values.
- $bdf : N \times Nm \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* for edges.
- $bCf : Nm \times Nm \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* for edges.

The Plithogenic Graph PG must satisfy:

(1) *Edge Appurtenance Constraint*:
For all $(x,a),(y,b) \in M \times Ml$:

$$bdf\left((xy),(a,b)\right) \leq \min\{adf(x,a), adf(y,b)\},$$

where $xy \in N$.

(2) *Contradiction Function Constraint*:
For all $(a,b),(c,d) \in Nm \times Nm$:

$$bCf\left((a,b),(c,d)\right) \leq \min\{aCf(a,c), aCf(b,d)\}.$$

(3) *Reflexivity and Symmetry of Contradiction Functions*:

$$\begin{aligned} aCf(a,a) &= 0, && \forall a \in Ml, \\ aCf(a,b) &= aCf(b,a), && \forall a,b \in Ml, \\ bCf(a,a) &= 0, && \forall a \in Nm, \\ bCf(a,b) &= bCf(b,a), && \forall a,b \in Nm. \end{aligned}$$

Definition 3.40. Let $PG = (PM, PN)$ and $PG' = (PM', PN')$ be two plithogenic graphs. A *Plithogenic Graph Morphism* $f : PG \to PG'$ consists of functions:

- $f_V : M \to M'$ mapping vertices.
- $f_E : N \to N'$ mapping edges.

The mappings satisfy:

- Preservation of incidence relations: if $e = xy \in N$, then $f_E(e) = f_V(x)f_V(y) \in N'$.
- Attributes and degrees of appurtenance and contradiction are appropriately mapped and preserved.

Definition 3.41. A *Typed Plithogenic Graph* PG_T is a triple:

$$PG_T = (PG, t_{PG}, TPG),$$

where:

- PG is a plithogenic graph.
- TPG is a plithogenic type graph specifying allowed types and attribute values.
- $t_{PG} : PG \to TPG$ is a total plithogenic graph morphism.

Definition 3.42. A *Plithogenic Rule* p is defined as:

$$p : L \xleftarrow{l} K \xrightarrow{r} R,$$

where:

- L, K, R are typed plithogenic graphs over the plithogenic type graph TPG.
- l and r are total plithogenic graph morphisms.

Definition 3.43. A *Plithogenic Graph Grammar (PGG)* is a triple:

$$\text{PGG} = (TPG, PG_0, \text{Rule}),$$

where:

- TPG is the plithogenic type graph.
- PG_0 is the initial typed plithogenic graph over TPG.
- Rule is a set of plithogenic rules.

Theorem 3.44. *A Plithogenic Graph Grammar (PGG) can be transformed into a Neutrosophic Graph Grammar (NGG), a Vague Graph Grammar (VGG), a Fuzzy Graph Grammar (FGG), and a Turiyam Neutrosophic Graph Grammar (TGG) by appropriately selecting the parameters s and t in the Plithogenic set and redefining its functions accordingly.*

Proof. A Plithogenic Graph Grammar is defined using Plithogenic sets, which generalize various types of fuzzy sets by introducing attributes with possible values and corresponding degrees of appurtenance and contradiction. By specializing the parameters s and t in the Plithogenic set, we can obtain different types of graph grammars.

A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, \mathrm{DAF}, \mathrm{DCF}),$$

.

A *Plithogenic Graph Grammar (PGG)* is defined as:

$$\mathrm{PGG} = (TPG, PG_0, \mathrm{Rule}),$$

where:

- TPG is the plithogenic type graph.
- PG_0 is the initial typed plithogenic graph over TPG.
- Rule is a set of plithogenic rules.

We will show that by selecting appropriate values of s and t and redefining functions, a PGG can be transformed into each of the other graph grammars.

We consider Transformation to a Fuzzy Graph Grammar (FGG). Set $s = t = 1$. The Plithogenic set becomes a Plithogenic Fuzzy Set (PFS). The Degree of Appurtenance Function (DAF) and Degree of Contradiction Function (DCF) reduce to functions with a single value.

- *Vertices and Edges*: The plithogenic vertex set $PM = (M, l, Ml, \mathrm{adf}, \mathrm{aCf})$ reduces to a fuzzy vertex set, where $\mathrm{adf} : M \times Ml \to [0, 1]$ assigns a single membership degree $\mu(v) \in [0, 1]$ to each vertex v.
- The plithogenic edge set $PN = (N, m, Nm, \mathrm{bdf}, \mathrm{bCf})$ reduces similarly for edges.
- *Membership Assignment*: The attributes l and m can be considered constants or omitted since they do not affect the fuzzy membership degrees.
- *Rules and Morphisms*: The rules and morphisms simplify accordingly, with membership degrees represented by single values.

Thus, the PGG reduces to a Fuzzy Graph Grammar (FGG), with the fuzzy membership degrees derived from the degrees of appurtenance in the Plithogenic set.

We consider Transformation to a Neutrosophic Graph Grammar (NGG). Set $s = 3$, $t = 1$. The Plithogenic set becomes a Plithogenic Neutrosophic Set (PNS).

- *Vertices and Edges*: The DAF assigns a vector $(\mu_T(v), \mu_I(v), \mu_F(v)) \in [0,1]^3$ to each vertex v, representing truth, indeterminacy, and falsity membership degrees.
- *Edges*: Similar assignments are made for edges.
- *Membership Constraints*: The degrees satisfy $0 \leq \mu_T(v) + \mu_I(v) + \mu_F(v) \leq 3$, which aligns with the neutrosophic membership conditions.
- *Rules and Morphisms*: The rules and morphisms are adjusted to handle the neutrosophic components, mapping vectors of degrees instead of single values.

Thus, the PGG reduces to a Neutrosophic Graph Grammar (NGG), with the neutrosophic membership degrees derived from the degrees of appurtenance in the Plithogenic set.

We consider Transformation to a Vague Graph Grammar (VGG). Set $s = 2$, $t = 1$, and impose the constraint that for each element x:

$$0 \leq \mu_T(x) + \mu_F(x) \leq 1.$$

This corresponds to a Vague Set.

- *Vertices and Edges*: The DAF assigns a pair $(\mu_T(v), \mu_F(v)) \in [0,1]^2$ to each vertex v, representing truth and falsity membership degrees.
- *Membership Constraints*: The sum of the degrees does not exceed 1, matching the conditions of a Vague Set.
- *Edges*: Similar assignments and constraints apply to edges.
- *Rules and Morphisms*: Adjusted to handle the pair of membership degrees.

Thus, the PGG reduces to a Vague Graph Grammar (VGG), with the vague membership degrees obtained from the degrees of appurtenance in the Plithogenic set.

We consider Transformation to a Turiyam Neutrosophic Graph Grammar (TGG). Set $s = 4$, $t = 1$. The Plithogenic set becomes a Plithogenic Turiyam Neutrosophic Set (PTuS).

- *Vertices and Edges*: The DAF assigns a vector $(\mu_T(v), \mu_I(v), \mu_F(v), \mu_L(v)) \in [0,1]^4$ to each vertex v, representing truth, indeterminacy, falsity, and liberal state membership degrees.
- *Membership Constraints*: The degrees satisfy $0 \leq \mu_T(v) + \mu_I(v) + \mu_F(v) + \mu_L(v) \leq 4$.
- *Edges*: Similar assignments and constraints apply to edges.
- *Rules and Morphisms*: Adjusted to handle the four-component membership degrees.

Thus, the PGG reduces to a Turiyam Neutrosophic Graph Grammar (TGG), with the Turiyam Neutrosophic membership degrees derived from the degrees of appurtenance in the Plithogenic set.

By selecting appropriate values for s and t in the Plithogenic set and redefining the functions accordingly, a Plithogenic Graph Grammar can be transformed into a Neutrosophic Graph

Grammar, a Vague Graph Grammar, a Fuzzy Graph Grammar, or a Turiyam Neutrosophic Graph Grammar. □

Corollary 3.45. *A Plithogenic Graph Grammar can represent both the empty graph and the null graph.*

Proof. The proof follows almost the same method as for the Fuzzy Graph Grammar. □

4. Future prospects

Future prospects are described below.

4.1. *Soft Graph Grammar and Uncertain Hypergraph grammar*

In the future, we plan to explore Soft Graph Grammar. A Soft Graph is a graph that integrates the conditions of a Soft set, while Soft Graph Grammar extends this concept to graph grammar. Numerous studies, including various extensions, have been conducted on Soft sets [38, 55, 59, 66, 68, 89, 155, 166, 187]. Although still in the conceptual stage, the definition is provided below.

Definition 4.1. [17, 108, 114] A *soft set* (F, C) over a universe U is a parameterized family of subsets of U. It is defined as a mapping:

$$F : C \to P(U),$$

where C is a non-empty subset of parameters E, and $P(U)$ denotes the power set of U. For each parameter $c \in C$, $F(c) \subseteq U$ is called the set of c-approximate elements of the soft set (F, C).

Definition 4.2. (cf. [86, 138]) A *soft graph* is defined as a 4-tuple $G = (G', S, T, A)$, where:

(1) $G' = (V, E)$ is a simple graph with vertex set V and edge set E.
(2) A is a non-empty set of parameters.
(3) (S, A) is a soft set over V, where $S : A \to P(V)$.
(4) (T, A) is a soft set over E, where $T : A \to P(E)$.
(5) For each parameter $a \in A$, the pair $F(a) = (S(a), T(a))$ forms a subgraph of G'.

A soft graph can also be represented as:

$$G = (G', S, T, A) = \{F(a) : a \in A\}.$$

We now define soft graph morphisms, which are mappings between soft graphs that preserve the soft structure.

Definition 4.3. Let $G = (G', S, T, A)$ and $H = (H', S', T', B)$ be two soft graphs. A *soft graph morphism* $f : G \to H$ consists of:

(1) A graph homomorphism $f_G : G' \to H'$, which consists of mappings:
- $f_V : V \to V'$ between the vertex sets.
- $f_E : E \to E'$ between the edge sets, satisfying $f_E(e) = (f_V(u), f_V(v))$ for all $e = (u, v) \in E$.

(2) A function $f_A : A \to B$ between the parameter sets.

Such that for all $a \in A$:

(1) $f_V(S(a)) \subseteq S'(f_A(a))$.
(2) $f_E(T(a)) \subseteq T'(f_A(a))$.

Definition 4.4. A *typed soft graph* is a triple $G_T = (G, t_G, T)$, where:

(1) $G = (G', S, T, A)$ is a soft graph.
(2) $T = (G'_T, S_T, T_T, A_T)$ is a soft graph called the *soft type graph.*
(3) $t_G : G \to T$ is a total soft graph morphism, meaning that t_G maps G onto T while preserving the soft graph structure.

Definition 4.5 (Soft Graph Grammar)**.** A *Soft Graph Grammar (SGG)* is a triple:

$$\text{SGG} = (T, G_0, \text{Rule}),$$

where:

- T: Soft type graph specifying permissible labels and parameters.
- G_0: Initial soft graph typed over T.
- Rule: A set of soft rules, where each soft rule p is:

$$p : L \xleftarrow{l} K \xrightarrow{r} R,$$

with L, K, R typed soft graphs over the same soft type graph T, and l, r being total soft graph morphisms.

Theorem 4.6. *Soft Graph Grammar generalizes Graph Grammar.*

Proof. Let $\text{GG} = (T, G_0, \text{Rule})$ be a Graph Grammar with:

- T: Type graph.
- G_0: Initial graph.
- Rule: Set of production rules, $p : G_L \xleftarrow{l} H \xrightarrow{r} G_R$.

To show that GG can be represented as an SGG, define:

- $G' = (V, E)$: Underlying graph structure of G_0.
- $A = \{a\}$: Single parameter.

- $S(a) = V$: Vertex set of G_0 associated with the parameter a.
- $T(a) = E$: Edge set of G_0 associated with the parameter a.
- $F(a) = (S(a), T(a)) = (V, E)$: Subgraph corresponding to the parameter a.

Thus, G_0 is represented as a soft graph $G = (G', S, T, A)$. Similarly, each production rule $p : G_L \overset{l}{\leftarrow} H \overset{r}{\rightarrow} G_R$ is represented as:

$$p : L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R,$$

where L, K, R are soft graphs with a single parameter a, and l, r are mappings preserving the soft structure.

Since every component of GG can be translated into a corresponding component of an SGG, it follows that GG is a special case of SGG with a single parameter. Therefore, SGG generalizes GG. □

Corollary 4.7. *A Soft Graph Grammar can represent both the empty graph and the null graph.*

Proof. The proof follows almost the same method as for the Fuzzy Graph Grammar. □

Furthermore, in the future, we aim to explore Uncertain Hypergraph Grammar, which extends the Uncertain Graph Grammar presented in this paper to hypergraphs (cf. [36, 76, 77, 88, 105, 136, 173]). In the future, we also intend to study Graph Grammar and Automata in the context of Superhypergraphs (cf. [62, 67, 74, 159, 164, 167]).

4.2. *Fuzzy off/over/under automata*

In relation to Fuzzy Sets, the concepts of Fuzzy offset, Fuzzy overset, and Fuzzy underset have been recently introduced [154]. Inspired by these notions, we aim to extend these ideas to the domain of Fuzzy Automata and explore their implications. Although still in the conceptual phase, the definitions are presented as follows.

Definition 4.8 (Fuzzy Overset)**.** (cf. [154]) Let X be a universe of discourse. A *Fuzzy Overset* $\tilde{A}$ in X is defined as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X,\ \mu_{\tilde{A}}(x) \in [0, \Omega]\},$$

where $\Omega > 1$ represents the *Overlimit*, allowing membership degrees greater than 1. There exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) > 1$.

Definition 4.9 (Fuzzy Underset)**.** (cf. [154]) Let X be a universe of discourse. A *Fuzzy Underset* $\tilde{A}$ in X is defined as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X,\ \mu_{\tilde{A}}(x) \in [\Psi, 1]\},$$

where $\Psi < 0$ is the *Underlimit*, allowing membership degrees below 0. There exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) < 0$.

Definition 4.10 (Fuzzy Offset)**.** (cf. [154]) Let X be a universe of discourse. A *Fuzzy Offset* $\tilde{A}$ in X is defined as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X,\ \mu_{\tilde{A}}(x) \in [\Psi, \Omega]\},$$

where $\Omega > 1$ and $\Psi < 0$. There exist elements $x, y \in X$ such that $\mu_{\tilde{A}}(x) > 1$ and $\mu_{\tilde{A}}(y) < 0$.

Definition 4.11 (Fuzzy Over General Finite Automata)**.** A *Fuzzy Over General Finite Automaton (FO-GFA)* is an eight-tuple machine:

$$\tilde{F}_{\text{over}} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where all components are defined as in a General Fuzzy Automaton (GFA), except:

- The membership functions of states and transitions, $\mu(q)$ and $\tilde{\delta}$, map to $[0, \Omega]$, where $\Omega > 1$.
- There exists at least one state $q \in Q$ or transition (q_i, a, q_j) such that $\mu(q) > 1$ or $\tilde{\delta} > 1$.

Definition 4.12 (Fuzzy Under General Finite Automata)**.** A *Fuzzy Under General Finite Automaton (FU-GFA)* is an eight-tuple machine:

$$\tilde{F}_{\text{under}} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where all components are defined as in a General Fuzzy Automaton (GFA), except:

- The membership functions of states and transitions, $\mu(q)$ and $\tilde{\delta}$, map to $[\Psi, 1]$, where $\Psi < 0$.
- There exists at least one state $q \in Q$ or transition (q_i, a, q_j) such that $\mu(q) < 0$ or $\tilde{\delta} < 0$.

Definition 4.13 (Fuzzy Off General Finite Automata)**.** A *Fuzzy Off General Finite Automaton (FOFF-GFA)* is an eight-tuple machine:

$$\tilde{F}_{\text{off}} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where all components are defined as in a General Fuzzy Automaton (GFA), except:

- The membership functions of states and transitions, $\mu(q)$ and $\tilde{\delta}$, map to $[\Psi, \Omega]$, where $\Psi < 0$ and $\Omega > 1$.
- There exists at least one state $q \in Q$ or transition (q_i, a, q_j) such that $\mu(q) > 1$, $\mu(q) < 0$, $\tilde{\delta} > 1$, or $\tilde{\delta} < 0$.

Theorem 4.14. *A Fuzzy Off General Finite Automaton (FOFF-GFA) can be transformed into a standard Fuzzy General Finite Automaton (GFA) by restricting its membership degrees to the interval* $[0, 1]$.

Proof. Let $\tilde{F}_{\text{off}} = (Q, \Sigma, \tilde{R}_{\text{off}}, Z, \tilde{\delta}_{\text{off}}, \omega, F_1, F_2)$ be a FOFF-GFA. We construct a corresponding GFA $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ through the following steps:

Each state $q \in Q$ in $\tilde{R}_{\text{off}}$ has a membership degree $\mu_{\text{off}}(q) \in [\Psi, \Omega]$, where $\Psi < 0$ and $\Omega > 1$. Transform $\mu_{\text{off}}(q)$ to $\mu(q) \in [0, 1]$ as follows:

$$\mu(q) = \min(\max(\mu_{\text{off}}(q), 0), 1).$$

The transformed values define the new set of fuzzy start states $\tilde{R}$.

For each transition in $\tilde{\delta}_{\text{off}}$, the membership value $\tilde{\delta}_{\text{off}}((q_i, \mu(q_i)), a, q_j) \in [\Psi, \Omega]$ is restricted to $[0, 1]$ as:

$$\tilde{\delta}((q_i, \mu(q_i)), a, q_j) = \min(\max(\tilde{\delta}_{\text{off}}((q_i, \mu(q_i)), a, q_j), 0), 1).$$

This restriction ensures all transition membership values fall within the interval $[0, 1]$.

The input alphabet Σ, output symbols Z, output function ω, and the membership assignment F_1 and resolution F_2 functions remain unchanged, as they are already defined independently of the interval boundaries.

The resulting automaton $\tilde{F}$ satisfies the definition of a standard GFA because all membership values—both for states and transitions—are now confined to $[0, 1]$. This ensures the transformed automaton adheres to the constraints of a GFA.

Therefore, any FOFF-GFA $\tilde{F}_{\text{off}}$ can be equivalently represented as a GFA $\tilde{F}$ by applying the above restrictions. □

Theorem 4.15. *A Fuzzy Off General Finite Automaton generalizes both the Fuzzy Over General Finite Automaton and the Fuzzy Under General Finite Automaton.*

Proof. This is evident. □

Theorem 4.16. *Every Finite Automaton can be represented as a Fuzzy Off Automaton. Therefore, Fuzzy Off Automata generalize Finite Automata.*

Proof. Let $A = (Q, \Sigma, \delta, q_0, F)$ be a Finite Automaton, where $\delta : Q \times \Sigma \to Q$.

We define a Fuzzy Off Automaton $\tilde{A}_{\text{off}} = (Q, \Sigma, \tilde{\delta}_{\text{off}}, q_0, F)$ with the transition function:

$$\tilde{\delta}_{\text{off}}(q, a, q') = \begin{cases} 1, & \text{if } \delta(q, a) = q', \\ 0, & \text{otherwise.} \end{cases}$$

Here, the membership degrees are within $\{0, 1\}$, which is a subset of $[\Psi, \Omega]$ with $\Psi < 0$ and $\Omega > 1$. This FOA behaves identically to the original FA, with crisp transitions represented by membership degrees of 1 (existing transitions) or 0 (non-existing transitions).

Therefore, every FA can be viewed as a special case of an FOA, proving that FOA generalize FA. □

4.3. *Single-Valued Neutrosophic off automata*

The definition of the Single-Valued Neutrosophic OffSet is also included below [20,31,65,148, 153, 161–163]. We intend to further explore the concept by extending it to automata, specifically by developing and studying the Single-Valued Neutrosophic Off General Neutrosophic Automaton. Additionally, we aim to apply the principles of the Single-Valued Neutrosophic OffSet to Graph Grammar, investigating its potential applications and implications.

Definition 4.17 (Single-Valued Neutrosophic OffSet)**.** [163] A *Single-Valued Neutrosophic OffSet*, denoted $A_{\text{off}} \subseteq U_{\text{off}}$, is a set within a universe of discourse U_{off} in which certain elements may possess neutrosophic degrees—truth, indeterminacy, or falsity—that extend beyond the standard limits, either above 1 or below 0. It is formally defined as:

$$A_{\text{off}} = \{(x, \langle T(x), I(x), F(x)\rangle) \mid x \in U_{\text{off}}, \exists\ (T(x) > 1 \text{ or } F(x) < 0)\},$$

where:

- $T(x)$, $I(x)$, and $F(x)$ denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of each $x \in U_{\text{off}}$.
- $T(x), I(x), F(x) \in [\Psi, \Omega]$, where $\Omega > 1$ (termed the *OverLimit*) and $\Psi < 0$ (termed the *UnderLimit*), allow the possibility for $T(x)$, $I(x)$, or $F(x)$ to take values beyond the conventional bounds of $[0, 1]$.

Definition 4.18 (Single-Valued Neutrosophic Off General Neutrosophic Automaton)**.** A *Single-Valued Neutrosophic Off General Neutrosophic Automaton (SVNO-GNA)* is defined as an eight-tuple machine:

$$\tilde{F}_{\text{off}} = (Q, \Sigma, \tilde{R}_{\text{off}}, Z, \tilde{\delta}_{\text{off}}, \omega, F_1, F_2),$$

where:

- Q, Σ, Z, and ω are defined as in the General Neutrosophic Automaton (GNA).
- $\tilde{R}_{\text{off}} = \{(q, T_0(q), I_0(q), F_0(q)) \mid q \in R\}$ is the set of *off neutrosophic start states*, where:
  - $T_0(q)$ is the truth-membership degree of state q,
  - $I_0(q)$ is the indeterminacy-membership degree of state q,
  - $F_0(q)$ is the falsity-membership degree of state q,

and at least one of $T_0(q) > 1$ or $F_0(q) < 0$.

- $\tilde{\delta}_{\text{off}} : (Q \times [\Psi, \Omega]^3) \times \Sigma \times Q \to [\Psi, \Omega]^3$ is the *neutrosophic off augmented transition function*, where:
  - $[\Psi, \Omega]$ allows membership degrees beyond conventional bounds, with $\Psi < 0$ and $\Omega > 1$.
  - $T(x) > 1$, $F(x) < 0$, or both are possible for states or transitions.
- F_1 and F_2 are the membership assignment and resolution functions, adapted to handle $[\Psi, \Omega]$ intervals.

Theorem 4.19. *A Single-Valued Neutrosophic Off General Neutrosophic Automaton (SVNO-GNA) can be transformed into a standard Neutrosophic General Neutrosophic Automaton (GNA) by restricting its membership degrees within the interval* $[0, 1]$.

Proof. Let $\tilde{F}_{\text{off}} = (Q, \Sigma, \tilde{R}_{\text{off}}, Z, \tilde{\delta}_{\text{off}}, \omega, F_1, F_2)$ be an SVNO-GNA. We construct a corresponding GNA $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ by the following steps:

For each state $q \in Q$, let the truth-membership, indeterminacy-membership, and falsity-membership degrees in $\tilde{R}_{\text{off}}$ be:

$$T_0(q), I_0(q), F_0(q) \in [\Psi, \Omega],$$

where $\Psi < 0$ and $\Omega > 1$. Restrict these values to the interval $[0, 1]$ as follows:

$$T(q) = \min(\max(T_0(q), 0), 1),$$

$$I(q) = \min(\max(I_0(q), 0), 1),$$

$$F(q) = \min(\max(F_0(q), 0), 1).$$

The modified membership degrees $T(q), I(q), F(q)$ define the set of neutrosophic start states $\tilde{R}$.

For each transition $\tilde{\delta}_{\text{off}}((q_i, \mu(q_i)), a, q_j) = (\delta_T, \delta_I, \delta_F)$, where:

$$\delta_T, \delta_I, \delta_F \in [\Psi, \Omega],$$

restrict the values to $[0, 1]$ as follows:

$$\delta'_T = \min(\max(\delta_T, 0), 1),$$

$$\delta'_I = \min(\max(\delta_I, 0), 1),$$

$$\delta'_F = \min(\max(\delta_F, 0), 1).$$

Define the restricted transition function $\tilde{\delta}$ using these values.

The output function ω and the membership assignment and resolution functions F_1, F_2 remain unchanged, as they operate within the specified intervals.

By restricting all membership degrees and transition values to $[0,1]$, $\tilde{F}_{\text{off}}$ is transformed into a standard GNA $\tilde{F}$ without altering its structural or functional properties. Therefore, the transformation is valid. □

Theorem 4.20. *Every Fuzzy Off Automaton can be represented as a Neutrosophic Off Automaton. Therefore, Neutrosophic Off Automata generalize Fuzzy Off Automata.*

Proof. Let $\tilde{A}_{\text{off}} = (Q, \Sigma, \tilde{\delta}_{\text{off}}, q_0, F)$ be a Fuzzy Off Automaton, where $\tilde{\delta}_{\text{off}} : Q \times \Sigma \times Q \to [\Psi, \Omega]$.

We construct a Neutrosophic Off Automaton $\tilde{A}_{\text{NO}} = (Q, \Sigma, \tilde{\delta}_{\text{NO}}, q_0, F)$ by defining the neutrosophic transition function $\tilde{\delta}_{\text{NO}}$ as:

$$\tilde{\delta}_{\text{NO}}(q, a, q') = (\mu_T(q, a, q'), \mu_I(q, a, q'), \mu_F(q, a, q')),$$

where:

$$\mu_T(q, a, q') = \tilde{\delta}_{\text{off}}(q, a, q'), \quad \mu_I(q, a, q') = 0, \quad \mu_F(q, a, q') = 0.$$

Since $\tilde{\delta}_{\text{off}}$ maps to $[\Psi, \Omega]$, the truth-membership degree $\mu_T(q, a, q')$ also maps to $[\Psi, \Omega]$. The indeterminacy and falsity membership degrees are set to zero for all transitions.

Under this construction, the NOA $\tilde{A}_{\text{NO}}$ captures the behavior of $\tilde{A}_{\text{off}}$, with the fuzziness represented in the truth-membership degrees. Thus, every FOA can be represented as a NOA, demonstrating that NOA generalize FOA. □

Similarly, the Single-Valued Neutrosophic Over General Neutrosophic Automaton and the Single-Valued Neutrosophic Under General Neutrosophic Automaton can also be defined.

4.4. *Finite Hyperautomaton*

In recent years, the concept of a Finite Hyperautomaton has been introduced, known as a generalization of the Finite Automaton. By applying the principles of Superhypergraphs [65, 158, 159], it is anticipated that this framework can be extended to define a Finite SuperhyperAutomaton, and their relationships can be explored. Additionally, there is potential for defining structures such as General Fuzzy Hyperautomaton and General Neutrosophic Hyperautomaton, which merit further investigation. Below, we provide definitions and theorems, including conceptual-level formulations.

Definition 4.21 (Finite Hyperautomaton)**.** (cf. [25, 26]) A *Finite Hyperautomaton (FH)* is a tuple

$$H = (\Sigma, X, Q, Q_0, F, \delta, \alpha),$$

where:

- Σ is a finite alphabet of symbols.

- $X = \{x_1, x_2, \ldots, x_k\}$ is a finite set of word variables.
- Q is a finite set of states.
- $Q_0 \subseteq Q$ is the set of initial states.
- $F \subseteq Q$ is the set of accepting states.
- $\delta \subseteq Q \times (\Sigma \cup \{\#\})^k \times Q$ is the transition relation, where $(\Sigma \cup \{\#\})^k$ is the set of k-tuples formed by elements of Σ padded with a special symbol $\#$.
- α is a quantification condition over the variables in X, expressed as a sequence:

$$\alpha = Q_1 x_1 Q_2 x_2 \ldots Q_k x_k,$$

where $Q_i \in \{\forall, \exists\}$ indicates whether x_i is universally or existentially quantified.

Remark 4.22 (Key Components and Definitions)**.** (cf. [25, 26])

(1) *Hyperwords:* A *hyperword* over Σ is a finite set of finite words over Σ. A *hyperlanguage* is a set of hyperwords.

(2) *Zipping Function:* Let $s = (w_1, w_2, \ldots, w_k)$ be a tuple of words in a hyperword, where w_i is a finite word over Σ. The *zipping function* $\text{zip}(s)$ produces a word over $(\Sigma \cup \{\#\})^k$, defined as:

$$\text{zip}(s) = s_1 s_2 \ldots s_{\lceil s \rceil},$$

where $s_i[j] = w_j[i]$ for $i \leq |w_j|$, and $s_i[j] = \#$ otherwise. Here, $\#$ is the padding symbol, and $\lceil s \rceil$ is the length of the longest word in s.

(3) *Unzipping Function:* The *unzipping function* reverses the zipping process, mapping a word over $(\Sigma \cup \{\#\})^k$ back to a tuple of words.

(4) *Acceptance Condition:* Let S be a hyperword and $v : X \to S$ be an assignment of word variables to words in S. The hyperword S is *accepted* by H if it satisfies the quantification condition α and the underlying automaton $H^\wedge$ accepts $\text{zip}(v)$.

Definition 4.23 (Acceptance Condition)**.** (cf. [25, 26]) Let α be the quantification condition and $H^\wedge$ be the underlying automaton of H. The satisfaction relation $S \models (\alpha, H)$ is defined as follows:

- If $\alpha = \varepsilon$ (empty quantification), then $S \models (\alpha, H)$ if $H^\wedge$ accepts $\text{zip}(v)$ for some assignment v.
- If $\alpha = \exists x_i \alpha'$, then $S \models (\alpha, H)$ if there exists $w \in S$ such that $S \models (\alpha', H)$ for the assignment $v[x_i \to w]$.
- If $\alpha = \forall x_i \alpha'$, then $S \models (\alpha, H)$ if for all $w \in S$, it holds that $S \models (\alpha', H)$ for the assignment $v[x_i \to w]$.

Definition 4.24 (Accepted Hyperlanguage)**.** (cf. [25, 26]) The *hyperlanguage* of H, denoted $L(H)$, is the set of all hyperwords S such that $S \models H$.

Theorem 4.25. *A Finite Hyperautomaton (FH) is a generalization of a Finite Automaton (FA).*

Proof. Let $A_{\mathrm{FA}} = (\Sigma, Q, Q_0, F, \delta)$ be a Finite Automaton, where:

- Σ is a finite alphabet.
- Q is a finite set of states.
- $Q_0 \subseteq Q$ is the set of initial states.
- $F \subseteq Q$ is the set of accepting states.
- $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation.

We construct a Finite Hyperautomaton $H_{\mathrm{FH}} = (\Sigma, X, Q, Q_0, F, \delta', \alpha)$ that simulates A_{FA}, as follows:

- Set $X = \{x\}$, a singleton set containing one word variable.
- The set of states Q, initial states Q_0, accepting states F, and alphabet Σ are the same as in A_{FA}.
- The transition relation δ' is adapted to operate on tuples over $(\Sigma \cup \{\#\})^1$, which is effectively $\Sigma \cup \{\#\}$. Since we have only one word variable, we can set $\delta' = \delta$.
- The quantification condition is $\alpha = \exists x$, indicating an existential quantification over the word variable x.

In this configuration:

- The hyperwords S are sets of words over Σ.
- The hyperword S is accepted by H_{FH} if there exists a word $w \in S$ such that $H^{\wedge}$ accepts $\mathrm{zip}(w)$.
- Since $\mathrm{zip}(w) = w$, the underlying automaton $H^{\wedge}$ operates exactly like A_{FA}.

Therefore, H_{FH} accepts a hyperword S if and only if there exists a word $w \in S$ such that A_{FA} accepts w. This demonstrates that every Finite Automaton can be represented as a Finite Hyperautomaton with one word variable and an existential quantification condition.

Thus, the Finite Hyperautomaton generalizes the Finite Automaton. □

Definition 4.26 (Finite Superhyperautomaton)**.** A *Finite Superhyperautomaton (FSH)* is a tuple

$$H = (\Sigma, X, \mathcal{Q}, \mathcal{Q}_0, \mathcal{F}, \delta, \alpha),$$

where:

- Σ is a finite alphabet of symbols.
- $X = \{x_1, x_2, \ldots, x_k\}$ is a finite set of word variables.
- $\mathcal{Q} \subseteq \mathcal{P}(Q) \setminus \{\emptyset\}$ is the set of *superstates*, where Q is a finite universal set of states.
- $\mathcal{Q}_0 \subseteq \mathcal{Q}$ is the set of initial superstates.

- $\mathcal{F} \subseteq \mathcal{Q}$ is the set of accepting superstates.
- $\delta \subseteq \mathcal{Q} \times (\Sigma \cup \{\#\})^k \times \mathcal{Q}$ is the transition relation.
- α is a quantification condition over the variables in X, expressed as:

$$\alpha = Q_1 x_1 \; Q_2 x_2 \; \ldots \; Q_k x_k,$$

where $Q_i \in \{\forall, \exists\}$ indicates universal or existential quantification over variable x_i.

Remark 4.27 (Key Components and Definitions)**.** (1) *Superstates*:

- Each superstate $S \in \mathcal{Q}$ is a non-empty subset of the universal set of states Q.

(2) *Hyperwords*:

- A *hyperword* over Σ is a finite set of finite words over Σ.
- A *hyperlanguage* is a set of hyperwords.

(3) *Zipping Function*:

- For a tuple $s = (w_1, w_2, \ldots, w_k)$ of words in a hyperword, where each w_i is a finite word over Σ, the *zipping function* $\text{zip}(s)$ produces a word over $(\Sigma \cup \{\#\})^k$, defined as:

$$\text{zip}(s) = s_1 s_2 \ldots s_{\lceil s \rceil},$$

where $s_i[j] = w_j[i]$ if $i \leq |w_j|$, and $s_i[j] = \#$ otherwise. Here, $\#$ is a padding symbol, and $\lceil s \rceil$ is the length of the longest word in s.

(4) *Acceptance Condition*:

- Let S be a hyperword and $v : X \to S$ be an assignment of word variables to words in S.
- The hyperword S is *accepted* by H if it satisfies the quantification condition α and there exists a sequence of superstates in $\mathcal{Q}$ corresponding to the computation over $\text{zip}(v)$ according to the transition relation δ, starting from an initial superstate in $\mathcal{Q}_0$ and ending in an accepting superstate in $\mathcal{F}$.

Definition 4.28 (Acceptance Condition)**.** Let α be the quantification condition and H be the finite superhyperautomaton. The satisfaction relation $S \models (\alpha, H)$ is defined recursively as follows:

- If $\alpha = \varepsilon$ (empty quantification), then $S \models (\alpha, H)$ if there exists an assignment $v : X \to S$ and a computation in H over $\text{zip}(v)$ that leads from an initial superstate in $\mathcal{Q}_0$ to an accepting superstate in $\mathcal{F}$.
- If $\alpha = \exists x_i \alpha'$, then $S \models (\alpha, H)$ if there exists $w \in S$ such that $S \models (\alpha', H)$ with the assignment $v[x_i \to w]$.
- If $\alpha = \forall x_i \alpha'$, then $S \models (\alpha, H)$ if for all $w \in S$, it holds that $S \models (\alpha', H)$ with the assignment $v[x_i \to w]$.

Theorem 4.29. *The Finite Superhyperautomaton (FSH) generalizes both the Finite Hyperautomaton (FH) and the Finite Automaton (FA).*

Proof. To show that the FSH generalizes both FH and FA, we demonstrate that FH and FA are special cases of FSH.

1. FSH Generalizes FH. Let $H_{\mathrm{FH}} = (\Sigma, X, Q, Q_0, F, \delta, \alpha)$ be a Finite Hyperautomaton. We construct an FSH $H_{\mathrm{FSH}} = (\Sigma, X, \mathcal{Q}, \mathcal{Q}_0, \mathcal{F}, \delta', \alpha)$ as follows:

- Set the universal set of states Q of H_{FH} as the universal set for H_{FSH}.
- Define the set of superstates $\mathcal{Q}$ as the set of singleton subsets of Q:

$$\mathcal{Q} = \{\{q\} \mid q \in Q\}.$$

- Similarly, define the initial superstates and accepting superstates as singleton subsets:

$$\mathcal{Q}_0 = \{\{q_0\} \mid q_0 \in Q_0\}, \quad \mathcal{F} = \{\{f\} \mid f \in F\}.$$

- Define the transition relation δ' by lifting the transition relation δ of H_{FH} to operate on singleton superstates:

$$\delta' = \left\{(\{q\}, a, \{q'\}) \mid (q, a, q') \in \delta\right\}.$$

Under this construction, the behavior of H_{FH} is preserved in H_{FSH} because the superstates are singleton sets, and transitions correspond directly to those in H_{FH}. Therefore, FH is a special case of FSH when the superstates are singleton subsets of Q.

2. FSH Generalizes FA. Let $A_{\mathrm{FA}} = (Q, \Sigma, \delta, q_0, F)$ be a Finite Automaton. We construct an FSH $H_{\mathrm{FSH}} = (\Sigma, X, \mathcal{Q}, \mathcal{Q}_0, \mathcal{F}, \delta', \alpha)$ as follows:

- Let $X = \{x\}$ be a singleton set of word variables.
- Set the universal set of states Q of A_{FA} as the universal set for H_{FSH}.
- Define the set of superstates $\mathcal{Q}$ as the set of singleton subsets of Q:

$$\mathcal{Q} = \{\{q\} \mid q \in Q\}.$$

- Define the initial superstate and accepting superstates:

$$\mathcal{Q}_0 = \{\{q_0\}\}, \quad \mathcal{F} = \{\{f\} \mid f \in F\}.$$

- Define the transition relation δ' by lifting δ:

$$\delta' = \left\{(\{q\}, a, \{q'\}) \mid (q, a, q') \in \delta\right\}.$$

- Set the quantification condition $\alpha = \exists x$, indicating that we are interested in the existence of an accepting computation over some word.

In this construction, H_{FSH} accepts a hyperword S if there exists a word $w \in S$ such that the automaton A_{FA} accepts w. Since the superstates are singleton sets, the transitions and states correspond directly to those in A_{FA}. Therefore, FA is a special case of FSH with singleton superstates and existential quantification over a single word variable. □

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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Evaluation of Agronomic Performance of two Varieties of Faba Bean (Vicia faba L.), INIAP 440 - Quitumbe and INIAP 442 - Sultana, using Neutrosophic Superhypersoft sets

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Abstract. The yield and earliness of broad bean varieties have depended on key factors to enhance agricultural productivity in various regions. Therefore, the objective of the study focused on evaluating the agronomic yield and earliness of the INIAP 442-SULTANA and INIAP 440-QUITUMBE broad bean varieties under specific edaphoclimatic conditions to determine their feasibility and potential for improving regional crop productivity. For this purpose, a combination of varietal attributes and leaf stages was used to determine eight treatments through modeling with Neutrosophic SuperHyperSoft Sets. Among the results, INIAP 442-SULTANA demonstrated higher earliness and yield compared to INIAP 440-QUITUMBE. In conclusion, it was observed that the INIAP 442-SULTANA-green treatment is a more viable option for improving broad bean crop productivity due to its higher yield and faster harvest.

Keywords: Antimicrobial resistance, Hedyosmum cuatrecazanum Occhioni, essential oil, antibacterial, antifungal, Neutrosophic SuperHyperSoft Sets, Fuzzy Extension SuperHyperSoft Sets, Single-Valued Triangular Neutrosophic Number.

1 Introduction

Broad bean cultivation in Ecuador is paramount to nutrition due to its high protein, carbohydrate, vitamin, and mineral content, making it an essential food for the population's diet [1]. This crop represents a promising alternative to other traditional crops in the central region of the country, such as potatoes, beans, and maize [2]. Crop rotation, in which broad beans can play a strategic role, not only diversifies income sources for farmers but also contributes to soil sustainability by preventing overexploitation and promoting enrichment with essential nutrients.

However, broad bean production faces various challenges [3], including limited access to modern technologies and updated knowledge, which restrict the crop's optimization potential. Additionally, adverse climatic factors such as heavy rains, prolonged droughts, frosts, pests, and diseases negatively impact its yield. This limitation reduces opportunities for integration into broader and more efficient markets [4], thereby decreasing the crop's profitability and competitiveness at both local and international levels, despite demand from strategic external markets.

Consequently, strengthening the agroalimentary system through broad bean cultivation is an important step for achieving food sovereignty in Ecuador [5]. In fact, it not only generates employment and additional income in rural areas, where agriculture is vital for subsistence but also reinforces the economic resilience of farming communities [6]. To maximize this potential, it is necessary to train farmers in advanced management techniques and develop varieties that combine characteristics such as earliness, disease resistance [7], quality, and high yields.

Inter-institutional cooperation among entities such as the National Institute of Agricultural Research (INIAP), the Technical University of Cotopaxi (UTC), and other organizations has made progress

in developing new broad bean varieties possible [8]. Therefore, this study focuses on evaluating the agronomic yield and earliness of the INIAP 442-SULTANA and INIAP 440-QUITUMBE broad bean varieties under specific edaphoclimatic conditions to determine the best treatment for their viability in improving productivity in the crops of the San José de Chanchaló sector, in the Cotopaxi province. For this purpose, Neutrosophic SuperHyperSoft Sets modeling was used to calculate the attributes and potential treatments.

2 Preliminaries

For the evaluation of each attribute or parameter associated with the linguistic term used within the single-valued neutrosophic set (SVNS), the decision-maker is able to assign, for each of the criteria considered and for each treatment, a value within the neutrosophic set of choice. Therefore, the following guidelines are outlined to be considered in Table 1.

Table 1: Relationship between measurement ranges and neutrosophic scales. Source: Own elaboration.

Attributes(a_i)/treatments	**SVNN**	**Attributes (b_i)**
Very important (VI)	(0.95,0,0)	Extremely High
	(0.85,0.05,0.12)	Very Very High
Important (I)	(0.75,0.15,0.22)	Very High
	(0.65,0.25,0.32)	High
Medium (M)	(0.55,0.35,0.42)	Moderately Moderate
	(0.45,0.45,0.52)	Moderate
	(0.35,0.55,0.62)	Medium Low
Not important (NI)	(0.25,0.65,0.72)	Low
	(0.15,0.75,0.82)	Very Low
Very Not Important (VNI)	(0.05,0.85,0.92)	Very Very Low
	(0,0.95,1)	Extremely Low

2.1 Neutrosophic SuperHyperSoft Sets.

This section serves the purpose of remembering the basic notions of Fuzzy Extension SuperHyperSoft Sets and neutrosophic theory [9].

Definition 1 ([1, 9, 10]). Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set [10] [11].

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A, having the following:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, the family F is known, then the function μ_A can be reconstructed by using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 2 ([2, 3, 11, 12]). Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets

$A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(\mathrm{U})$ is called a *HyperSoft Set* over U.

Definition 3 ([2, 3, 11, 12]). Let U be a universe set, $\mathcal{P}(\mathrm{U})$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(\mathrm{U})$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([4, 5, 13-15]). Let U be a universe set, $\mathcal{P}(\mathrm{U})$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}\left(\mathrm{U}\big(x(d^0)\big)\right)$ is called a *Fuzzy Extension SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc [12].

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set N* is characterized by three membership functions [13], which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in \mathrm{U}$, $T_A(x), I_A(x), F_A(x) \subseteq]^{-}0, 1^{+}[$, and $^{-}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$ [14].

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^{-}0$ and 1^{+} belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$ and $F_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

SVNSs (Single-Valued Neutrosophic Sets) emerged with the idea of applying neutrosophic sets for practical purposes. Some operations between SVNSs are expressed below:

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the sum between A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \tag{1}$$

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNN, the multiplication between A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \tag{2}$$

The product of a positive scalar with an SVNN, A = (a, b, c) is defined as:

$$A = (1 - (1 - a), b, c) \tag{3}$$

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 7 ([8, 16, 17]). The *single-valued triangular neutrosophic number*, $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, and falsity membership functions are defined as follows [15]:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \le a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 8 ([8, 16, 17]). Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Inversion: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$
6. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t- conorm.

3 Results

3.1 Details of the experiment

Study area: The research was conducted in the Salcedo canton, specifically in San José de Chanchaló, at an altitude of 3,134 meters above sea level. The climate in the area is cold, with temperatures ranging from 12 to 18 °C, reaching highs of 20 °C and lows of 3 °C. However, annual precipitation varies between 500 and 750 mm, while the average relative humidity is 40%, exceeding 70% during the winter. The predominant soil type is Mollisol, characterized by its excellent properties for agricultural development.

Experimental design: The design included four replications, resulting in 16 experimental units, each with an area of 25 m². The Tukey test at 5% was applied to analyze the differences between means with confidence intervals.

Factors under study: The seeds used were provided by the National Institute of Agricultural Research (INIAP). The INIAP 440-QUITUMBE variety originated in San Isidro, Carchi province, and has been previously evaluated in various locations in the Ecuadorian highlands. Meanwhile, the INIAP 442-SULTANA variety resulted from a cross between the progenitors ECU 8395 and ECU 2522.

Field design and statistical analysis: The trial design included the distribution of treatments in split plots. For statistical analysis, a variance analysis scheme (ANOVA) was used, considering the sources of variation: replications, varieties, plant stages, and their interaction.

3.2 Modeling with Neutrosophic SuperHyperSoft Sets

The common bean (Phaseolus vulgaris L.) is one of the most significant crops in agriculture, both in terms of production and its impact on food security. In various agricultural regions, the yield and earliness of bean varieties are key factors for improving productivity, especially in environments with variable edaphoclimatic conditions. In this regard, evaluating local varieties is crucial for determining their adaptability and efficiency under specific conditions. This study addresses the comparative evaluation of two bean varieties, through the modeling of Neutrosophic SuperHyperSoft Sets, to analyze their yield and earliness in a specific region, and how these factors influence the improvement of agricultural productivity. To this end, the following bean varieties have been identified as viable alternatives for bean farmers: INIAP 442-SULTANA and INIAP 440-QUITUMBE. For this purpose, they were interrelated under the formation of the following attributes, as shown in Table 2.

Table 2: Attributes to define possible alternatives according to the varieties and the state of the leaves. Source: Own elaboration.

Attributes	Name	Items
a_1	Varieties	INIAP 442-Sultana, INIAP 440-Quitumbe
a_2	Leaf status	Green, dry

In Table 2, the possible alternatives are represented, where the Neutrosophic SuperHyperSoft Sets are defined for the following sets A_i with $i \in \{1,2\}$ corresponding to each a_i. Therefore, the following pairs are defined:

$$P(A_1) = \{\{INIAP\ 442 - Sultana\}, \{INIAP\ 440 - Quitumbe\}, \{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe\}\}$$

$$P(A_2) = \{\{\text{Green}\}, \{\text{Dry}\}, \{\text{Green, Dry}\}\}$$

As the study aims to determine the best bean variety, the analysis proceeds to define all the pairs to be analyzed from P(U):

$$F: P(A_1)\ x\ P(A_2) \rightarrow P(U)$$

$$= \{\{INIAP\ 442 - Sultana\}, \{INIAP\ 440 - Quitumbe\}, \{\text{Green}\}, dry, \{green, dry\}, \{INIAP\ 442 - Sultana, \text{Green}\}, \{INIAP\ 442 - Sultana, \text{dry}\}, \{INIAP\ 442 - Sultana, green, dry\}, \{INIAP\ 440 - Quitumbe, \text{Green}\}, \{INIAP\ 440 - Quitumbe, \text{dry}\}, \{NIAP\ 440 - Quitumbe, green, \text{dry}\}, \{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe\}, \{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe, verde\}, \{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe, dry\}, \{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe, green, dry\}\}$$

Once each element is defined, the pairs to be modeled are selected according to the sets defined as treatments to be evaluated in the study. Subsequently, Table 3 presents the proposed treatments (L_i, where $i \in \{1,2, \dots ,8\}$), along with the neutrosophic membership degree (see Table 1) and the corresponding weight for each alternative, based on the attributes involved.

Table 3: Alternatives depending on the variety and the state of the leaves. Source: Own elaboration.

Treatment (l_i)	Alternative set	SVNN ($x(d^0)$)
T1	$\{INIAP\ 442 - Sultana, green, dry\}$	(VNI, NI, M, I. VI)
T2	$\{NIAP\ 440 - Quitumbe, green,$ dry$\}$	(VNI, NI, M, I. VI)
T3	$\{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe, green\}$	(VNI, NI, M, I. VI)
T4	$\{INIAP\ 442 - Sultana, NIAP\ 440 - Quitumbe, dry\}$	(VNI, NI, M, I. VI)
T5	$\{INIAP\ 442 - Sultana,$ Green$\}$	(VNI, NI, M, I. VI)
T6	$\{INIAP\ 442 - Sultana,$ dry$\}$	(VNI, NI, M, I. VI)
T7	$\{INIAP\ 440 - Quitumbe,$ Green$\}$	(VNI, NI, M, I. VI)
T8	$\{INIAP\ 440 - Quitumbe,$ dry$\}$	(VNI, NI, M, I. VI)

To determine the best alternative, an evaluation is conducted based on the following attributes (b_i) or parameters represented in Table 4. Each alternative is assessed using the b_i, where $i \in \{1,2, \dots 9\}$. Additionally, each value can be represented as $\{[31.33 - 36.25], [(171.25 - 245)]\}$ for one or more sets (see Table 5).

Table 4: Attributes for evaluating each treatment. Source: Own elaboration.

Sub-attribute	Name	Description	Worth
b_1	Number of pods	Average obtained from 10-20 plants randomly selected per plot.	(31.33-36.25)
b_2	Days to harvest	Time elapsed from sowing until 50% of the plants reached maturity in the green and dry states.	(171.25-245)
b_3	Number of plants	Total number of plants counted in each plot.	(53.75-71)
b_4	Pod yield	Total weight of pods harvested in tender state, expressed in kg/ha.	(6.35-23.54)
b_5	Number of seeds	Average number of seeds obtained by shelling 10 randomly selected pods.	(2.9-3.75)
b_6	Pod length	Measurement in centimeters of 10 pods, excluding the tip.	(8.48-12.58)
b_7	Seed length	Dimensions in centimeters obtained from 10 selected seeds.	(1.92-2.85)
b_8	Wide seeds	Dimensions in centimeters obtained from 10 selected seeds.	(1.2-1.92)
b_9	Grain yield	Total weight of seeds per plot, converted to kg/ha.	(3.17-9.47)

Table 5: Evaluation of each treatment according to the attributes b_i. Source: Own elaboration.

Alternatives/attributes	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
T1	34.61	199	71	16.86	3.63	11.86	2.83	1.77	7.71
T2	33.41	217.25	53.75	7.14	3.08	8.64	2.09	1.29	3.35
T3	35.88	180.38	62.38	15.73	3.38	10.69	2.36	1.56	6.32
T4	32.15	235.88	62.38	8.27	3.33	9.81	2.56	1.5	4.74
T5	36.25	171.25	71	23.54	3.5	12.58	2.81	1.92	9.47
T6	32.98	226.75	71	10.19	3.75	11.15	2.85	1.62	5.95
T7	35.5	189.5	53.75	7.93	2.9	8.48	1.92	1.2	3.17
T8	31.33	245	53.75	6.35	3.25	8.8	2.26	1.38	3.52

To evaluate specific differences between the means of the analyzed variables, the Tukey Test at 5% was applied. The results, presented in Table 5, indicate that the INIAP 442-SULTANA variety (T1) achieved a significant range in most of the evaluated variables, surpassing the INIAP 440-QUITUMBE variety (T2). Regarding the plant state factor, the *green* state demonstrated a significant range in days to harvest, pod yield, and grain yield, indicating its superiority over the dry plant state (T3 and T4).

In the green state, the combination INIAP 442-SULTANA GREEN (T5) obtained the highest range, standing out for its superior performance in key attributes such as pod yield (b_4) (23.54 kg) and grain

yield (b_9) (9.47 kg). Conversely, the combination INIAP 440-QUITUMBE DRY (T8) achieved lower ranges, reflecting reduced performance in the analyzed attributes.

Comparison of agronomic parameters: In terms of agronomic parameters, the INIAP 442-SULTANA variety (T1) exhibited superior values in pod length (b_6) (11.86 cm), seed length (b_7) (2.83 cm), and seed width (b_8) (1.77 cm), significantly outperforming INIAP 440-QUITUMBE (T2), whose values were 8.64 cm, 2.09 cm, and 1.29 cm, respectively. Similarly, grain yield (b_9) was notably higher in the INIAP 442-SULTANA variety (7.71 kg/ha) compared to INIAP 440-QUITUMBE (3.35 kg/ha).

Effect of plant state (attribute a_2): The green state (T3) showed better performance in most of the evaluated variables, particularly in grain yield (b_9) (6.32 kg/ha compared to 4.74 kg/ha in the dry state (T4)). However, the interaction between state and variety revealed that the dry state had less impact on the INIAP 442-SULTANA variety, which exhibited intermediate ranges in the evaluations.

The results highlight the superiority of the INIAP 442-SULTANA variety, particularly in the green state, and its consistent performance under dry state conditions. The interaction between factors confirms that both variety and plant state significantly influence productive and agronomic parameters. This positions INIAP 442-SULTANA as a promising option to optimize productivity in crops under different environmental conditions. Table 6 presents the variance calculations for the attributes b_i.

Table 6: Analysis of variance (ANOVA) of each attributeb_i. Source: Own elaboration.

FV	Repetition	Variety	State	Variety*state	Error	Total	CV%
gl	3	1	1	1	9	15	
b1	14.89 ns	5.76 ns	55.5 ns	0.81 ns	15.6		11.61
b2	0.08 ns	1332.25 **	12321 **	0.00 ns	0.69		0.4
b3	31.58 ns	1190.25 **	0.00 ns	0.00 ns	26.97		8.33
b4	1.76 ns	378.40 **	222.83 **	138.71 **	0.67		6.8
b5	0.94 ns	1.21 ns	0.01 ns	0.36 ns	0.53		21.71
b6	0.89 ns	41.54 **	3.08 ns	1.22 ns	2.19		14.45
b7	0.01 ns	2.18 **	0.15 ns	0.09 ns	0.17		16.6
b8	0.06 ns	0.9 **	0.01 ns	0.23 ns	0.07		16.92
b9	0.52 ns	76.26 **	10.06 **	15 **	0.2		8.16

Analysis of Variance (ANOVA): The results of the analysis of variance for the variables evaluated showed statistical significance ($p < 0.05$) for the variety factor in the attributes days to harvest, number of plants, pod yield, pod length, seed length, seed width, and grain yield. These differences highlight the distinguishing effect of the varieties INIAP 442-SULTANA and INIAP 440-QUITUMBE. Additionally, the plant state attribute (a_2) exhibited statistical significance in days to harvest, pod yield, and grain yield, emphasizing the influence of green and dry conditions on these variables.

In the interaction of variety × state (attributes a_1 and a_2), significant differences were identified in pod yield and grain yield. The coefficient of variation (CV) ranged from a minimum of 0.4% for days to harvest to a maximum of 21.71% for the number of seeds, indicating acceptable variability in the data.

Neutrosophic Membership Degree: Once the evaluation was defined, the neutrosophic membership degree of each treatment was determined, as shown in Tables 7 and 8. Additionally, the most important treatment in the study was identified. To achieve this, the t_{norm} and t_{conorm} were used to determine the intersection $F(l_1, l_2, \dots, l_n) \cap G(b_1, b_2, \dots . b_n)$, through the aggregation operation $\left(min_j\{\mathrm{T}_{ij}\}, max_j\{\mathrm{I}_{ij}\}, max_j\{\mathrm{F}_{ij}\}\right)$. This value results in the single-valued neutrosophic number, representing the effectiveness and selection of the best treatment (see Table 9).

Table 7: Neutrosophic Membership degree of each alternative based on a_n. Source: Own elaboration.

l_n/a_n	b_1	b_2	b_3	b_4
T1	(0.55,0.35,0.42)	(0.55,0.35,0.42)	(0.95,0,0)	(0.45,0.45,0.52)
T2	(0.35,0.55,0.62)	(0.25,0.65,0.72)	(0,0.95,1)	(0,0.95,1)
T3	(0.85,0.05,0.12)	(0.75,0.15,0.22)	(0.35,0.55,0.62)	(0.45,0.45,0.52)
T4	(0,0.95,1)	(0,0.95,1)	(0.35,0.55,0.62)	(0,0.95,1)
T5	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)
T6	(0.25,0.65,0.72)	(0.15,0.75,0.82)	(0.95,0,0)	(0.15,0.75,0.82)
T7	(0.75,0.15,0.22)	(0.65,0.25,0.32)	(0,0.95,1)	(0,0.95,1)
T8	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)

Table 8: Neutrosophic Membership degree of each alternative based on a_n (continued). Source: Own elaboration.

l_n/a_n	b_5	b_6	b_7	b_8	b_9
T1	(0.75,0.15,0.22)	(0.75,0.15,0.22)	(0.95,0,0)	(0.65,0.25,0.32)	(0.65,0.25,0.32)
T2	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)
T3	(0.45,0.45,0.52)	(0.45,0.45,0.52)	(0.35,0.55,0.62)	(0.35,0.55,0.62)	(0.35,0.55,0.62)
T4	(0.35,0.55,0.62)	(0.25,0.65,0.72)	(0.55,0.35,0.42)	(0.25,0.65,0.72)	(0.15,0.75,0.82)
T5	(0.55,0.35,0.42)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)	(0.95,0,0)
T6	(0.95,0,0)	(0.55,0.35,0.42)	(0.95,0,0)	(0.45,0.45,0.52)	(0.35,0.55,0.62)
T7	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)	(0,0.95,1)
T8	(0.25,0.65,0.72)	(0,0.95,1)	(0.25,0.65,0.72)	(0.15,0.75,0.82)	(0,0.95,1)

Table 9: Degree of importance of each alternative. Source: Own elaboration.

l_n/a_n	$F(l_1, l_2, \dots, l_n) \cap G(b_1, b_2, \dots . b_n)$	Position
T1	(0.55,0.35,0.42)	M
T2	(0,0.95,1)	NVI
T3	(0.35,0.45,0.52)	M
T4	(0,0.95,1)	NVI
T5	(0.65,0.35,0.42)	I
T6	(0.15,0.75,0.82)	NI
T7	(0,0.99,1)	NVI
T8	(0,0.99,1)	NVI

The results indicate that the variety INIAP 442-SULTANA demonstrates superior agronomic performance (classified as *medium importance*) compared to INIAP 440-QUITUMBE (classified as *very not important*). This advantage is attributed to its earliness, with an average of 199 days to harvest, positioning it as an earlier option compared to other varieties cultivated in the country. Differences in days to harvest relative to previous reports highlight the influence of local edaphoclimatic factors, which affect plant growth and development.

In terms of yield, INIAP 442-SULTANA achieved the best results in green pod yield (classified as *important*, T5) and dry pod yield (classified as *not important*, T6). These outcomes reflect the productive capacity of the T5 treatment of this variety, as well as its adaptability to the experimental environment's conditions.

In contrast, INIAP 440-QUITUMBE did not outperform INIAP 442-SULTANA in any of the variables analyzed. This may be attributed to the fact that INIAP 440-QUITUMBE, being a variety with over three decades in the field, has surpassed the optimal productivity period typically associated with newly released resistant varieties. Consequently, the evaluated treatment T5 {INIAP 442-Sultana,

Green} has been classified as *important* (0.65, 0.35, 0.42), standing out for its earliness and adaptability to diverse edaphoclimatic conditions.

4 Conclusion

The modeling of Neutrosophic SuperHyperSoft Sets has enabled the evaluation and classification of eight combined treatments to optimize agricultural productivity under varying cultivation conditions. The INIAP 442-SULTANA variety (T1) stands out for its earliness and high yields compared to INIAP 440-QUITUMBE (T2). Additionally, the interaction between the attributes variety and plant state significantly influences yields, with INIAP 442-SULTANA demonstrating greater consistency in both states. Furthermore, the evaluation of neutrosophic membership degrees identifies treatment T5 ({INIAP 442-Sultana, Green}) as exhibiting resilient and adaptable performance, positioning it as a preferred option for optimizing agricultural productivity under different cultivation conditions. Finally, it is suggested to continue research to assess the impact of additional variables, such as soil type and pest resistance, to refine the selection of varieties in environments affected by climate change.

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A Neutrosophic Framework for Artificial Immune Systems

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Abstract: Artificial immune systems (AIS) draw inspiration from the mechanisms of the natural immune system. They extract ideas from the functioning of the natural immune system in order to use them to build computer models to solve real-world problems. While traditional AIS models mimic the biological immune system's capacity to distinguish between self and non-self entities, they often face challenges in environments where data may be incomplete or ambiguous. In this paper we introduce neutrosophic AIS that includes degrees of truthiness, falsehood and indeterminacy to better address ambiguity within pattern recognition tasks. Within this neutrosophic framework we discuss and redefine main AIS concepts, including self and non-self categorization, clonal selection, negative selection, and immunological memory. A numerical example, from the field of computer security, illustrates the application of the suggested approach in detecting non-self entities. With the addition of neutrosophic logic, the proposed model significantly enhances AIS's adaptability and pattern recognition capabilities, addressing uncertainties inherent in real-world applications.

Keywords: Neutrosophic logic; artificial immune system; self and non-self categorization; clonal selection; negative selection; immunological memory; pattern recognition.

1. Introduction

The human immune system is a complex system consisting of an intricate network of specialized tissues, organs, cells and chemical molecules [1]. The natural immune system recognizes, destroys and remembers an almost unlimited number of foreign particles and also protects the human body from cells that do not function normally in the body as it has the ability to distinguish (non-self or antigen) from self cells. When a pathogen (infectious foreign element) enters the body, it is detected and mobilized for elimination. The system is capable of "remembering" each infection, so that a second exposure to the same pathogen is dealt with more efficiently.

Artificial Immune System (AIS) is an area of research that bridges the disciplines of immunology, computer science and engineering [2-4]. During the last two decades, the field of AIS is progressing slowly and steadily as a branch of Computational Intelligence (CI) amongst other methods such as neural networks and evolutionary computing. Inspired by the biological immune system's ability to recognize and respond to foreign pathogens, AIS leverages these principles to develop algorithms capable of pattern recognition, anomaly detection, and adaptive learning.

The properties of the natural immune system that the artificial immune system exploits are [5]:

1. The natural immune system only needs to be aware of normal cells,
2. The natural immune system can distinguish between normal and foreign cells,
3. A foreign cell can be classified as harmful or non-harmful,
4. Lymphocytes are cloned and post-cloned in order to attach themselves to the foreign cells that the body encounters,
5. The natural immune system has a rapid response to antigens that the body has already encountered, which is due to memory cells.

Taking into account the characteristics and models of the natural immune system, several models of artificial immune systems have been developed. These models are as follows:

1. Artificial immune systems based on the classical view of the immune system are: the negative selection algorithm [6-9] and evolutionary approaches [10-14]
2. Artificial immune systems based on the clone selection theory are: dynamic clone selection [15-16] and multilayered artificial immune systems [17-18].
3. Artificial immune systems based on network theory are: artificial immune network [19], self-stabilizing artificial immune systems [20], augmented artificial immune network [21], dynamic weighted B-Cell artificial immune systems [22] and aiNet model artificial and immune network [23].
4. Artificial immune systems based on danger theory [24-27].

AIS are being used in many applications such as anomaly detection [28-29], pattern recognition [30], data mining [31], computer security [32-36], adaptive control [37] and fault detection [38-39]. These applications highlight the flexibility and robustness of AIS, positioning it as a valuable approach in environments that require adaptability, real-time response, and handling of uncertain or variable data.

In AIS related literature, it is observed that most methods utilize binary or fuzzy logic based to categorize between self and non-self entities. But in this case, the aforesaid methods are not always reliable when dealing with uncertain data or irregular conditions in real world applications, such as in cybersecurity or anomaly detection. From this point of view, we propose a new methodology that integrates neutrosophic logic in classic AIS concepts. Neutrosophic logic which is a new branch of logic is considered more flexible than other classical or fuzzy methods as it takes into account indeterminacy membership - the uncertainty and indeterminacy that frequently occur and characterize many (if not, all) real-world applications.

The main objective of current research is to present a more robust AIS framework by integrating neutrosophic logic. Rather than categorizing decisions as simply true or false, the main idea is to constitute AIS more adaptable and capable of handling uncertainty. To illustrate how this integration works, we provide a numerical example and demonstrate how our approach improves AIS performance in complex and unpredictable real-world scenarios. This research work marks a critical advancement in leveraging AIS for complex, dynamic environments, as demonstrated through its application to cybersecurity challenges.

The structure of this article is as follows: In Section 2, we define and explain the neutrosophic mathematical framework needed to formulate the proposed neutrosophic AIS (n-AIS) model. Next, in Section 3, we highlight the applicability of the suggested n-AIS model in an illustrative example from the field of cybersecurity. In Section 4, through this case study, we highlight the system's ability to assess and process various types of uncertainty, such as ambiguous threat classifications or incomplete attack vectors, leveraging the suggested n-AIS model. Lastly, the "Conclusions" section wraps up the key points of our study and proposes potential research work.

2. Materials and Methods

Definition 1 [40] Consider X to be a space of points (objects) and x to be a generic element in X. A truth membership function $\boldsymbol{T_A}$, an indeterminacy membership function $\boldsymbol{I_A}$, and a falsity membership function $\boldsymbol{F_A}$ characterize a single-valued neutrosophic set (SVNs) A in X. For each point x in X, $\boldsymbol{T_A(x)}$, $\boldsymbol{I_A(x)}$, $\boldsymbol{F_A(x)} \in [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by $A = \{\langle \boldsymbol{x}, \boldsymbol{T(x)}, \boldsymbol{I(x)}, \boldsymbol{F(x)} \rangle \mid x \in \boldsymbol{X}\}$ with $0 \leq \boldsymbol{T_A(x)} + \boldsymbol{I_A(x)} + \boldsymbol{F_A(x)} \leq 3$.

In traditional binary systems, T(x) and F(x) are binary (0 or 1), but in a neutrosophic system, each value can vary continuously, allowing for nuanced classifications.

In AIS, entities are classed as "self" (belonging to the system and harmless) or "non-self" (foreign and possibly harmful). Inspired by the biological immune system, this categorization allows AIS to recognize and respond to abnormal patterns, similar to finding diseases in a biological environment.

Definition 2 Let S denote the set of self-entities and N the set of non-self entities. An entity x is categorized based on thresholds for T, I, and F as follows:

Self: if T(x) > $T_{threshold}$ and F(x) < $F_{threshold}$ (1)
Non-Self: if F(x) > $F_{threshold}$ and T(x) < $T_{threshold}$ (2)
Indeterminate if I(x) > $I_{threshold}$ (3)
representing cases where it's unclear whether x is self or non-self.

The process of cloning is most commonly known as clone selection which is the proliferation of lymphocytes that recognize antigens [5]. Learning in the immune system occurs by a process known as affinity maturation.

Definition 3 Let P(x) be the population of cloned cells associated with entity x. Each clone $\boldsymbol{c_i}$∈P(x) has a neutrosophic vector (T($\boldsymbol{c_i}$),I($\boldsymbol{c_i}$),F($\boldsymbol{c_i}$)), where mutation adjusts these values based on exposure to new information. *Mutation* can be defined as:
T′($\boldsymbol{c_i}$)=T($\boldsymbol{c_i}$)+ΔT (4)
I′($\boldsymbol{c_i}$)=I($\boldsymbol{c_i}$)+ΔI (5)
F′($\boldsymbol{c_i}$)=F($\boldsymbol{c_i}$)+ΔF (6)
where ΔT, ΔI and ΔF are changes based on mutation rates influenced by exposure to uncertain patterns.

In AIS, the fitness function is a measure used to evaluate each clone's effectiveness or "fitness" in identifying and responding to patterns or threats.

Definition 4 The *fitness function* of a clone $\boldsymbol{c_i}$ can be mathematically defined using a neutrosophic similarity measure that considers the degrees of truth (T), indeterminacy (I), and falsity (F). The fitness score Fitness($\boldsymbol{c_i}$) can be represented as:
Fitness($\boldsymbol{c_i}$)=αT($\boldsymbol{c_i}$)−βF($\boldsymbol{c_i}$)−γI($\boldsymbol{c_i}$) (7)
where α,β,γ are weighting factors that determine the relative importance of truth, falsity, and indeterminacy in evaluating fitness.

A higher fitness score indicates that the clone c_i has a closer alignment with the desired characteristics and is thus prioritized in the AIS for further replication or retention.

For over 50 years immunologists have been based their thoughts, experiments, and clinical treatments on the idea that the immune system functions by making a distinction between self (related to belonging molecules in the organism) and non-self (related to foreign molecules in the organism). AIS are based on the human immune system's ability to recognize practically any pathogenic agent. According to the Self-Nonself hypothesis, the body recognizes itself by discriminating its own cells and molecules from alien ones.

Definition 5 *Negative Selection* in a n-AIS framework involves identifying and eliminating clones that are likely to react against self-entities by measuring their similarity to known self-patterns. Define the neutrosophic distance d_{self}(x,y) between a clone x and any self-entity y as:
d_{self}(x,y) = |T(x)−T(y)|+|I(x)−I(y)|+|F(x)−F(y)| (8)

If d_{self}(x,y) < $\boldsymbol{\epsilon}$ (where $\boldsymbol{\epsilon}$ is a threshold), and T(x)≈T(y), clone x is eliminated due to high likelihood of being self-reactive.

Immunological memory could be defined as a stimulus-specific change of immune reactivity that persists in the absence of the stimulus. In neutrosophic AIS, memory cells store the neutrosophic vector (T,I,F) of each recognized non-self entity.

Definition 6 *Immunological memory* in a n-AIS model refers to the system's ability to retain information about previously identified non-self entities (patterns or threats) by storing their neutrosophic values—truth (T), indeterminacy (I), and falsity (F)—to improve future responses. Each memory cell m corresponds to a previously identified non-self entity and is represented by a neutrosophic vector:
m = (T(m), I(m), F(m)) (9)
where T(m), I(m), and F(m) denote the truth, indeterminacy, and falsity values associated with the stored pattern.

When a new non-self entity (x) is detected with neutrosophic values (T(x), I(x), F(x)), the system compares it to existing memory cells. If x matches an existing memory cell based on a neutrosophic similarity measure, the memory cell m m is updated to integrate the new values of x as follows:

$T(m)=\lambda T(m)+(1-\lambda)T(x)$ (10)

$I(m)=\lambda I(m)+(1-\lambda)I(x)$ (11)

$F(m)=\lambda F(m)+(1-\lambda)F(x)$ (12)

where $\lambda \in [0,1]$ is a decay factor that controls the weight given to past values versus the new observation.

The memory cell m is updated only if the neutrosophic distance between x and m satisfies:

$d_{memory}(x,m) < \boldsymbol{\epsilon}$ (13)

where $\boldsymbol{\epsilon}$ is a predefined similarity threshold and it determines the maximum allowable "distance" between the neutrosophic values of the new input $\boldsymbol{x}$ and the existing memory cell $\boldsymbol{m}$ for them to be considered similar enough.

The neutrosophic distance $d_{memory}(x,m)$ is defined and calculated as in equation (8). This neutrosophic immunological memory architecture enables the AIS to dynamically adapt to changing patterns by updating memory cells with new data, hence improving the system's ability to recognize and respond to non-self entities in unpredictable circumstances.

3. Results

Network security systems face continuous streams of data from diverse sources, which may vary in nature and threat level. In this section we will examine a system equipped with n-AIS that needs to evaluate incoming network traffic to determine whether it represents a "self" (normal traffic) or "non-self" (potentially malicious traffic) entity. The goal is to enhance threat detection in a dynamic environment where traffic patterns might be unpredictable or unclear.

When the n-AIS system first encounters an incoming traffic pattern x, it evaluates its characteristics against predefined rules or thresholds.

A memory cell m has previously stored a recognized malicious traffic pattern with values (Eq. 9): $m = (0.2, 0.5, 0.7)$ (14)

We set a similarity threshold $\epsilon=0.3$ for updating the memory cell based on new incoming patterns. In this way, the system stays adaptive without over-reacting to every minor fluctuation, hence boosting its resilience and flexibility in tracking new harmful tendencies.

Suppose an incoming traffic sample x has the following neutrosophic values:

$x = (0.3, 0.4, 0.6)$ (15)

By applying Eq. (8), we get:

$d_{memory}(x,m) = |T(x) - T(m)| + |I(x) - I(m)| + |F(x) - F(m)| = 0.3$ (16)

The system considers incoming traffic similar enough to the recorded pattern in memory cell, as $d_{memory}(x,m) = 0.3$, which equals threshold value ϵ. The system can update m with values from, allowing it to adapt to variations in malicious traffic patterns.

The memory cell m is updated by merging old and new values from x. A weighted average is used to determine how much weight is given to new versus old data.

Next, we need to define a value for decay factor $\lambda \in [0,1]$. The choice of λ is based on the desired memory cell sensitivity and stability. Higher λ values (0.7 or 0.8) provide more weight to previous values in memory cells, making the system less susceptible to new inputs and allowing for longer retention of old knowledge. Lower λ levels (e.g., 0.4 or 0.5) increase the system's responsiveness to new data by influencing memory cells more substantially. In this case study the decay factor is set to $\lambda = 0.7$.

Now by applying Eq. (10) – (12), the memory cell m now has: $m = (0.23, 0.47, 0.67)$ (17)

In the case study explained previously, the n-AIS framework showcases its adaptability to new but similar patterns which occur over time, highlighting a steady learning process for identifying malicious traffic. By comparing each new incoming pattern against stored patterns in its memory cells, the n-AIS refines its understanding of what constitutes "malicious" activity in network traffic. This can be achieved with the usage of appropriate neutrosophic measures (distance measure) along with the similarity threshold which allows the n-AIS to selectively update memory cells only when a

new pattern closely aligns with a previously stored one. In this way unnecessary changes are minimized and system stability is preserved.

When an incoming traffic pattern fulfils the similarity requirement, the n-AIS makes use of a proper operator (neutrosophic immunological memory operator) that controls the weight given to past values versus the new observation to update the memory cell values. The latter procedure, which is regulated by a decay factor, enables the system to respond to modest behavioral changes without significantly affecting its memory cells. Within this framework, the n-AIS stays alert to new and changing threats in network traffic but also keeps its core ability to recognize harmful patterns it's already familiar with. During the course of time the n-AIS makes small and careful advancements so that it can better adapt more precisely to changes in network traffic patterns while lowering the likelihood of false positives and missed detections.

In order to test the efficiency of proposed model we conducted the following an experimental test. The objective of the test is to compare the n-AIS model with traditional AIS methods (e.g., clonal selection algorithm) using publicly available datasets that contain various features related to network intrusion for classification tasks. To achieve this, we utilized the KDD Cup 99 dataset [41] for evaluation and tested the performance of our model against the traditional AIS model based on the following metrics: 1) accuracy, 2) false positive rate, and 3) execution time. For more technical details on the environment used for the test, please refer to Appendix B.

The results obtained are depicted in the following table:

Table 1. Summary of results between n-AIS and traditional AIS model.

Metric	*n-AIS (%)*	*AIS (%)*	*Improvement (%)*
Accuracy	96.2	93.2	+3.0
False positive rate	3.2	5.1	-1.9
Execution time (seconds)	24.3	20.1	+4.2

From the above results it is shown that the n-AIS model outperforms AIS in all metrics used for the experiment indicating a favorable trade-off for practical applications. This improvement underlines the practical advantages of integrating neutrosophic logic into AIS, enabling improved adaptability and robustness in handling uncertainty and ambiguity within real-world applications.

5. Conclusions

The field of artificial immune systems (AIS) is one of the most recent natural computing approaches to emerge from engineering, computer science and theoretical immunology. Using the immune system as inspiration has proved very useful when trying to address many computational problems. These computational techniques have many potential applications, such as in distributed and adaptive control, machine learning, pattern recognition, fault and anomaly detection, computer security, optimization, and distributed system design. By suggesting a hybrid AIS model which integrates neutrosophic logic, we achieve a flexible framework to handle uncertain data. In this light, we introduce a novel n-AIS model which redefines the main concepts and operators of AIS in order to help the system deal better with ambiguous patterns. This capacity stems from the utilization of proper neutrosophic measures and concepts that "mimic" AIS operators such as negative selection and immunological memory, ensuring enhanced efficiency in proposed model. For example, cell memory updates only when this is necessary, keeping things stable while adapting to new patterns. The case study examined demonstrates that suggested method works effectively in dynamic contexts like cybersecurity, where data is subject to change and unpredictable. It shows its capacity in detecting malicious activity without raising many false alarms.

As this research serves as an initial step towards this research direction, future work could examine the use of the n-AIS model in other fields, such as fault detection in industrial systems, financial fraud detection, or medical diagnostics, where data uncertainty is also common.

Furthermore, different neutrosophic similarity measurements might be explored to increase the model's capacity to recognize subtle patterns.

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Appendix A

Since this paper is intended for the general public, in order for the paper to be self-contained, we provide below dictionary definitions of principal immune systems terms.

Immune system: the bodily system that protects the body from foreign substances, cells, and tissues by producing the immune response and that includes especially the thymus, spleen, lymph nodes, special deposits of lymphoid tissue (as in the gastrointestinal tract and bone marrow), macrophages, lymphocytes including the B cells and T cells, and antibodies; https://www.merriam-webster.com/dictionary/allele. Accessed 18/11/2024.

Antigen: any substance (such as an immunogen or a hapten) foreign to the body that evokes an immune response either alone or after forming a complex with a larger molecule (such as a protein) and that is capable of binding with a product (such as an antibody or T cell) of the immune response; https://www.merriam-webster.com/dictionary/allele. Accessed 18/11/2024.

Antibody: any of a large number of proteins of high molecular weight that are produced normally by specialized B cells after stimulation by an antigen and act specifically against the antigen in an immune response, that are produced abnormally by some cancer cells, and that typically consist of four subunits including two heavy chains and two light chain; https://www.merriam-webster.com/dictionary/allele. Accessed 18/11/2024.

Self and Non-Self: In immunology, "self" refers to the body's own cells and molecules, which are usually recognized and ignored by the immune system. "Non-self" refers to foreign substances (antigens) that the immune system recognizes as potentially harmful and targets for destruction;

Lymphocyte: any of the colorless weakly motile cells originating from stem cells and differentiating in lymphoid tissue (as of the thymus or bone marrow) that are the typical cellular elements of lymph, include the cellular mediators of immunity, and constitute 20 to 30 percent of the white blood cells of normal human blood; https://www.merriam-webster.com/dictionary/allele. Accessed 18/11/2024.

Clonal selection: a procedure whereby specialised immune cells (such T-cells or B-cells) that are able to identify a given antigen are chosen to proliferate, expanding the number of immune cells that are prepared to react to that antigen; https://en.wikipedia.org/wiki/Clonal_selection. Accessed 18/11/2024.

Negative Selection: the process of eliminating any *developing* T or B lymphocytes that are autoreactive, i.e. reactive to the body itself; https://en.wikipedia.org/wiki/Central_tolerance. Accessed 18/11/2024.

Immunological Memory: the ability of the immune system to quickly and specifically recognize an antigen that the body has previously encountered and initiate a corresponding immune response. https://en.wikipedia.org/wiki/Immunological_memory. Accessed 18/11/2024.

Appendix B

- Software configuration

Python Version: Python 3.8
Integrated Development Environment (IDE): Visual Studio Code (VS Code)
Package Manager: pip or conda (for package installation)
Libraries and Frameworks:
NumPy: Version 1.21.2 for numerical operations

Pandas: Version 1.3.3 for data manipulation and analysis
Scikit-learn: Version 0.24.2 for machine learning model training and evaluation

- Dataset Details

Dataset Used: KDD Cup 1999 Dataset (subset of 10% of the full dataset, used for experiment)
Number of Records: Approximately 494,000 records
Number of Features: 41 features, including categorical and continuous attributes
Target Variable: "Attack Type" (denoted as "class" in the dataset)

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Detection of Cardiovascular Diseases Using Predictive Models Based on Deep Learning Techniques: A Hybrid Neutrosophic AHP-TOPSIS Approach for Model Selection

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Abstract: In Ecuador and globally, cardiovascular diseases are the leading cause of mortality, accounting for a worrying 26.49% of deaths in 2019. An approach based on deep learning is applied to improve the capacity for early prediction and reduce its incidence. In this work, three different models were proposed and compared: deep neural networks (DNN), convolutional neural networks (CNN), and multilayer perceptron (MLP). Experiments were conducted in two scenarios: one using a dataset that included 12 variables, and another in which the variables were reduced to those most significantly correlated with cardiovascular disease, i.e., 4 variables; both scenarios with 918 clinical records per variable. Using the Neutrosophic AHP-TOPSIS method for model selection, the CNN model trained with the original dataset was identified as the best-performing model among the proposed options. In specific terms, the evaluation metrics of the CNN model were as follows: an accuracy of 92.17%, a sensitivity of 94.51%, a specificity of 90.78%, an F1-Score of 93.30%, and an area under the ROC curve of 90.03%.

Keywords: Heart Disease, Prediction, Convolutional Neural Network, Deep Neural Network, Multilayer Perceptron, Neutrosophic AHP-TOPSIS

1. Introduction

The concept of cardiovascular disease (CVD) refers to any condition that can affect the health of the heart and blood distribution in the human body. These conditions can result in lesions in the arterial vessels, which have the potential to cause difficulties in several essential organs [1]. According to the World Health Organisation (WHO), cardiovascular diseases constitute the most lethal set of conditions affecting the human population, being the leading cause of mortality globally. These pathologies account for more than 37% of all deaths worldwide, and this proportion is expected to continue to increase until 2030 [2]. In the context of Ecuador, statistics for 2019 revealed that these diseases alarmingly constituted 26.49% of all deaths registered in the country [3]. Data obtained from the 2018 STEPS survey sponsored by WHO provided an even more worrying perspective by showing that approximately one-quarter of the Ecuadorian population, i.e. around 25%, present three or more risk factors related to chronic non-communicable diseases [4].

Factors that increase the likelihood of developing cardiovascular disease include family history, smoking, elevated LDL cholesterol levels, hypertension, advanced age, and diabetes [5]. In addition, lifestyle behaviors and medical issues such as sedentary lifestyles, unhealthy diets, obesity, and excessive alcohol consumption also contribute significantly to risk [6]. Assessment of disease severity in patients is performed by a variety of methods, such as stress tests, chest X-rays, CT scans, cardiac MRI, coronary angiography, and electrocardiograms [7]. Accurate and early detection is essential in resource-limited settings to improve treatments and long-term outcomes [8]. Lack of specialists and errors in diagnosis and treatment pose risks, underlining the importance of early detection for effective care [1].

In this article, three different deep learning (DL) models were proposed and compared for assisting healthcare practitioners in the early detection of cardiovascular diseases: deep neural networks (DNN), convolutional neural networks (CNN), and multilayer perceptron (MLP). Datasets with patient medical history such as electrocardiogram information, and risk factors such as age, gender, blood pressure, and cholesterol level, among others, are analyzed. Through the process of training using historical patient data, the model can learn to recognize subtle patterns that may signal an individual's likelihood of developing cardiovascular disease in the future, which will support decision-making by doctors and specialists during the diagnosis and prescription of treatments of CVD.

The rest of the paper is organized as follows. Section 2 summarizes works related to this study. In section 3, a brief explanation of the machine learning models to be applied is presented. Section 4 shows the experimental evaluations and discussion of the results. Finally, Section 5 includes some final comments and directions for future work.

2. Related works

In recent years, there has been a growing interest in using predictive models based on deep learning techniques in the healthcare field [9]. These models offer a promising avenue for improving the accuracy of predictions, particularly in the context of cardiovascular diseases. Through their ability to decipher complex patterns in data, these models present a possible solution for improved diagnosis and prognosis.

Miao et al. [10] explored the application of DNNs in the detection and prediction of coronary heart disease (CHD). Their approach was to improve diagnostic accuracy, especially in regions with limited resources and limited cardiology expertise. By incorporating regularisation techniques and random elimination mechanisms into its DNN architecture, the study yielded a diagnostic accuracy of 83.67%, sensitivity of 93.51%, and specificity of 72.86%, highlighting the feasibility of DNN models in the accurate diagnosis of CHD.

A similar thread is presented in the work of Ibrahim et al. [11] where early identification of acute myocardial infarction (AMI) took center stage. Using a variety of DL models, including CNN, Recurrent Neural Networks (RNN), and XGBoost, the researchers achieved significant accuracy. Their results, with ROC curve values of 90.7%, 82.9%, and 96.5% respectively, underlined the efficacy of these models in the timely classification of AMI.

Joo et al. [12] ventured into the realm of machine learning (ML) and big data in the context of cardiovascular disease prediction. Their approach incorporated attributes such as age, gender, blood pressure, cholesterol levels, smoking habits, and medical history. Using a variety of ML models, including logistic regression, deep neural networks, random forest, and lightGBM, the authors demonstrated that their methodology outperformed conventional approaches, emphasizing the importance of considering medication data.

In a different approach, Zheng et al. [13] presented a pioneering approach based on DNN to predict major adverse cardiovascular events (MACE) in patients with non-ST-segment elevation myocardial infarction (NSTEMI). A comparative analysis with traditional ML algorithms supported

the superiority of the DNN model in terms of accuracy, sensitivity, F1 score, and area under the ROC curve was carried out.

These trends were continued by Priya and Thilagamani [14]who used support vector machines (SVM), gated recurrent units (GRU), and CNN, along with pulse wave velocity measurements and features derived from optimization algorithms. Their approach, with area under the ROC curve values of 87.4%, 91.2%, and 88.6%, offered a compelling avenue to identify risk factors for arterial stiffness in diabetic patients.

Sarra et al. [15] proposed an innovative ANN-based diagnostic system for cardiovascular diseases. By achieving a diagnostic accuracy of 93.44%, their ANN model outperformed conventional ML methods, pointing to the potential of ANN as an efficient tool for accurate disease diagnosis.

Similarly, Kishor and Jeberson [16] proposed an LSTMS-DBN approach for predicting cardiovascular events with deep learning using information collected by IoT devices. In particular, Long Short-Term Memory was used to identify and prevent arterial events and Deep Belief Network (DBN) to represent and select more efficient features of the recorded dataset. The LSTM-DBN approach shows an 88.42% mean accuracy compared to other deep learning algorithms including simple RNN, GRU, CNN and Ensemble.

In another work, Khanna et al. [17] presented an IoTDL-HDD model based on IoT and deep learning for the detection of cardiovascular diseases using biomedical electrocardiogram signals. The model uses the BiLSTM feature extraction technique, Artificial Flora optimization (AFO) as a hyperparameter optimizer and Fuzzy Deep Neural Network (FDNN) for disease classification. The IoTDL-HDD model achieves an accuracy of 9.452% in comparison with other models including Deep Neural Network (DNN), Fuzzy Support Vector Machine (FSVM), XGBoost, and others.

Along the same line, Venkatesan et al. [18] designed a cardiac disease diagnosis approach that relies on fuzzy c-means neural network (FNN) and a deep convolutional neural network for improving the accuracy of prediction of the diseases. The results revealed an accuracy of 86.4% and an F1-score of 97 in comparison with convolutional neural network (CNN) and ensemble machine learning (EML).

Likewise, Pan et al. [19] proposed an Enhanced Deep Learning approach assisted by a Convolutional Neural Network (EDCNN) to improve the diagnostics of heart disease. The model is implemented in an Internet of Medical Things Platform (IoMT) and achieved a precision rate of up to 99.1% in comparison with conventional approaches such as Artificial Neural Networks (ANN), DNN, and Recurrent Neural Networks (RNN) among others.

In the same way, Shekhar [20] presented an IoT-centered Deep Learning Modified Neural Network (DLMNN) to assist healthcare practitioners in the effective diagnosis of heart diseases by relying on wearable IoT devices attached to the patient body. The outcome of the conducted experiments shows an improvement in terms of accuracy of 92.59% and security of 95.87% in comparison with existing algorithms.

Notably, Hussain et al. [21] proposed a unique 1D CNN architecture to improve patient classification, obtaining an impressive training accuracy of 98.9% and testing accuracy of 90.32%, while Sharma et al. [22] moved into the realm of risk prediction for patients, achieving an accuracy of 71.4% when classifying patients with diabetes and hypertension, two key risk factors for heart disease.

In summary, these studies follow a logical sequence of advances in the use of ML and DL models for the detection and prediction of cardiovascular disease. Each research builds on the previous one, showing how these advanced techniques are transforming medical care towards a more precise and personalized approach. While challenges remain to be addressed, these advances demonstrate a promising path toward a more effective medical future.

3. Method

This section breaks down the process into two parts. The first subsection details the systematic procedure used to process the clinical information used during the training and testing phase of the cardiovascular disease predictive model. The second subsection outlines the proposed architectures under the predictive models by leveraging the scikit-learn and Keras libraries and describes the methods used for the collection and thorough evaluation of the performance of each model.

3.1. Experimental setup

3.1.1. Data

A dataset hosted on the Kaggle platform was used to train the model. This dataset consists of a total of 918 records and 12 columns corresponding to parameters related to cardiovascular disease risk factors [23]. These factors are used to assess heart diseases in patients and determine their presence and risk. As mentioned above, the dataset consists of 12 attributes and each of them has a different data type. Table 1 describes in detail the attributes of the dataset used.

Given that there were some categorical variables in the dataset used, it was decided to use the mapping process to obtain descriptive statistics for the categorical columns. In this process, a unique numerical value is assigned to each category or label present in an object-type variable.

In summary the features present in the dataset were as follows: the average age of the individuals is around 53.51 years; information on the gender of the individuals is included; the attribute "Chest Pain Type" is categorized into levels (ASY:0, ATA:1, NAP:2, TA:3); the resting blood pressure averages around 132.40 mmHg; the average cholesterol level is around 198.80 mg/dL; the variable "Fasting Glucose" is binary (0 or 1) and denotes if fasting blood glucose exceeds 120 mg/dL; the "Resting Electrocardiogram" results are recorded in three types (LVH:0, Normal:1 and ST:2); the maximum heart rate reached during exercise averages about 136.81 beats per minute; the presence of "Exercise Induced Angina" is indicated; the attribute "ST Segment Depression" is measured relative to rest, with an average of about 0.89; the "ST Segment Slope" reflects its variation during exercise (Up:0, Flat:1 and Down:2); finally, the target variable "Heart Disease" indicates whether there is presence or absence of heart disease.

Table 1. The twelve attributes and descriptions used for the development of the Deep Learning-based models.

Attribute	Type	Description
Age	Numeric	Patient's age in years
Sex	Categorical	Sex of patient (M: Male, F: Female)
ChestPainType	Categorical	Type of chest pain (TA: Typical Angina, ATA: Atypical Angina, NAP: Non-Anginal Pain, ASY: Asymptomatic)
RestingBP	Numeric	Resting blood pressure in mm Hg
Cholesterol	Numeric	Serum cholesterol level in mm/dl
FastingBS	Numeric	Fasting blood sugar level (1: if > 120 mg/dl, 0: otherwise)
RestingECG	Categorical	Resting electrocardiogram results (Normal, ST, LVH)
MaxHR	Numeric	Maximum heart rate reached (numerical value between 60 and 202)
ExerciseAngina	Categorical	Exercise-induced angina (Y: Yes, N: No)
Oldpeak	Numeric	ST-segment depression (numeric value)
ST_Slope	Categorical	ST-segment slope (Up: Ascending, Flat: Flat, Down: Descending)
HeartDisease	Numeric	Diagnosis of heart disease (1: Sick, 0: Healthy)

Based on the correlation matrix presented in Figure 1, it can be seen there are strongly correlated variables. On the one hand, variables such as ExerciseAngina, Oldpeak, and ST_Slope show moderate

to strong positive correlations concerning the presence of heart disease. This suggests that when these values increase, there is a greater likelihood that the patient has heart disease. On the other hand, variables such as MaxHR, ChestPainType, and Sex show moderate negative correlations with heart disease, indicating that higher values for these variables are associated with a lower likelihood of heart disease.

Based on the analysis of the correlation matrix [24], it was recommended to create a new dataset using the original dataset, named "Dataset B", including only the variables ExerciseAngina, Oldpeak, ST_Slope and the target variable (HeartDisease). This action sought to reduce dimensionality and focus on the most influential characteristics for the analysis. Comparing the performance of the models on both datasets provided a clearer perspective on whether dimensionality reduction negatively affected predictive ability, which was a crucial step in ensuring the robustness and effectiveness of the model in line with the objectives of the analysis.

In terms of the training, validation, and testing process, it was considered due to the importance of splitting the dataset into separate sets to avoid over-fitting the model and to assess its performance in real-life situations. This was achieved through a function called "train_val_test_split", which partitioned the data into training (70%), validation (15%), and testing (15%) [25]. The training set allowed the model to learn patterns, while the validation set was used to adjust hyperparameters and avoid overfitting. The test set, which contained previously unseen data, evaluated the final effectiveness of the model.

Another crucial aspect was featuring scaling, which was carried out using the technique of scaling by standardization [26]. Using the StandardScaler method, numerical features were given a mean of zero and a standard deviation of one. This process is applied to each attribute of the dataset.

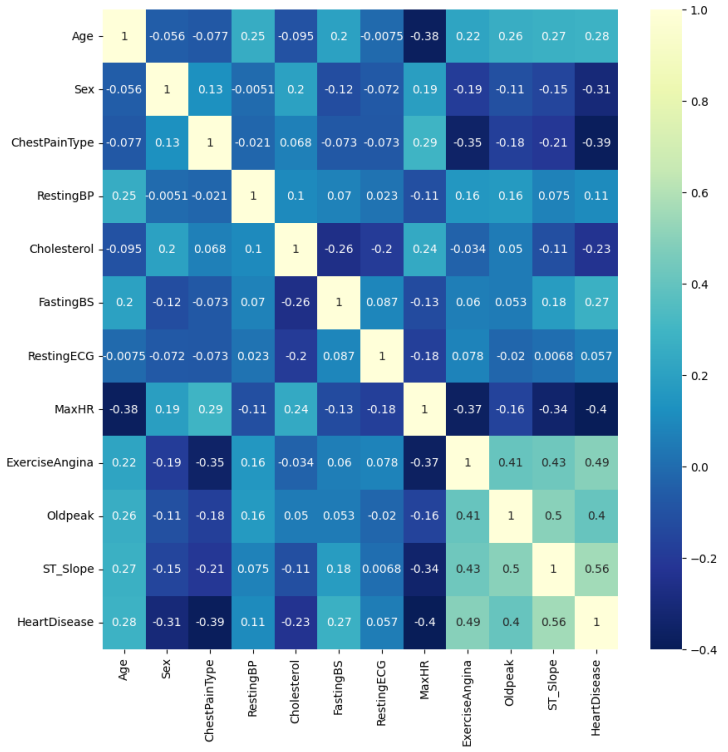

Figure 1. Correlation matrix of the variables of the dataset analyzed.

3.1.2. Model features and target

The experimental design addresses a multivariate issue in both cases using the original dataset and dataset B. The case of the original dataset is characterized by eleven input variables and one

output variable. The case of dataset B is characterized by three input variables and one output variable. In both cases, the output variable takes binary predictive values to classify the diagnosis of heart disease.

3.1.3. Problem definition

In the current scientific paper, the problem definition is the following: based on age, sex, chestPainType, restingBP, cholesterol, fastingBS, restingECG, maxHR, exerciseAngina, oldpeak, ST_Slope; can it be determined whether he/she has heart disease?

3.1.4. Hyperparameters

Refers to parameters external to the model itself, and their values typically can't be derived from the training data. These values are set by the model designer to fine-tune the learning algorithms. In this study, a grid search algorithm has been employed to optimize the models. This involved an iterative process where hyperparameters were adjusted to identify configurations that resulted in the best generalization on the test dataset. The grid search procedure explored all possible combinations of hyperparameters and selected the most effective subset. This procedure will be applied to the best model selected during the validation.

3.1.5. Model evaluation.

Evaluation models are applied using the following procedure:

Training procedure is applied to estimate model performance quickly.

K-fold cross-validation is applied to get a more robust estimate of how the model generalizes. K=5 partitions are considered.

The selected model resulting from the above processes is applied hyperparameters tuning to improve performance metrics.

3.1.6. Performance evaluation

In literature reviews focusing on binary classification performance, the evaluation of models often involves the use of several standard metrics. These metrics include the false negative rate (FNR), false positive rate (FPR), true negative rate (TNR), true positive rate (TPR), positive predictive value (PPV), and accuracy (ACC). These metrics are commonly employed to assess and select models with the lowest predictive errors [27]. To compute these metrics, the following equations are used:

$$FNR = \frac{\mathrm{FN}}{\mathrm{TP} + \mathrm{FN}} \tag{1}$$

$$FPR = \frac{\mathrm{FP}}{\mathrm{FP} + \mathrm{TN}} \tag{2}$$

$$specificity = TNR = \frac{\mathrm{TN}}{\mathrm{FP} + \mathrm{TN}} \tag{3}$$

$$recall = sensitivity = TPR = \frac{TP}{\mathrm{TP} + \mathrm{FN}} \tag{4}$$

$$precision = PPV = \frac{TP}{\mathrm{TP} + \mathrm{FP}} \tag{5}$$

$$F1 = 2 * \frac{\mathrm{precision} * \mathrm{recall}}{\mathrm{precision} + \mathrm{recall}} \tag{6}$$

$$acc = \frac{\mathrm{TP} + \mathrm{TN}}{\mathrm{TP} + \mathrm{TN} + \mathrm{FP} + \mathrm{FN}} \tag{7}$$

Where TP is the true positive, TN is the true negative, FP is the false positive, and FN is the false negative.

3.2. Model design

The definition of model architecture was a fundamental step in this work. Three main models were selected to be used and compared to obtain the best performance in cardiovascular disease prediction. Based on state-of-the-art information and analysis, the following models were chosen: Deep Neural Network (DNN), Multilayer Perceptron (MLP), and Convolutional Neural Network (CNN). These models are widely recognized for their effectiveness in classification problems and have distinctive characteristics.

3.2.1. Deep Neural Network (DNN)

The DNN is a DL architecture that incorporates multiple hidden layers. Each layer contains neurons that process and transmit information through the network [28], ultimately arriving at the output layer as shown in Figure 2(a).

The model structure starts with a flattening layer to transform the input data into a one-dimensional shape. Dropout layers are applied at a rate of 20% to avoid overfitting by randomly deactivating neurons during training. The model incorporates dense layers, also called fully connected layers, which employ the ReLU activation function, and a "he_normal" weight initialization designed for ReLU. The model consists of three dense layers, with different numbers of units (300, 100, and 10), allowing the network to learn more complex representations. Between each dense layer, a dropout layer is used to regularize the model and prevent overfitting. The last dense layer has a single unit and a sigmoid activation function, suitable for binary classification.

3.2.2. Multilayer Perceptron (MLP)

It is a neural network architecture that includes multiple hidden layers [29]commonly used in regression and classification problems in ML [30]. This model, designed as a sequential network is specifically adapted to binary classification. Its structure comprises a flattening layer, followed by two dense layers with ReLU activation, each preceded by dropout layers (20%). The last layer, dense and with sigmoid activation, consists of a single output unit as shown in Figure 2(b).

2.2.3. Convolutional Neural Network (CNN)

An initial two-dimensional architecture was chosen for a CNN, using 3x3 cores. Performance improvements were then sought through a more parameter and computationally efficient structure. The final architecture is shown in Figure 2(c).

The proposed architecture for the CNN includes several key layers: the first, a reshape layer, transforms the input data from (11,) to (11, 1), adding an extra dimension common in sequential data; then, two 1D convolutional layers follow this reshape layer, the first with 40 filters and size 5 convolutions along with ELU activation function, and the second with 8 filters and also ELU activation, allowing the network to learn important features in the sequential data; a flattening layer converts the output data from the convolutional layers to one-dimensional format, ready to be processed by the dense layers; the last two layers are dense, the first with 80 units and ELU activation, and the last one is an output dense layer with a single unit and sigmoidal activation, thus setting up the model for binary classification and prediction of probability of belonging to the positive or negative classes.

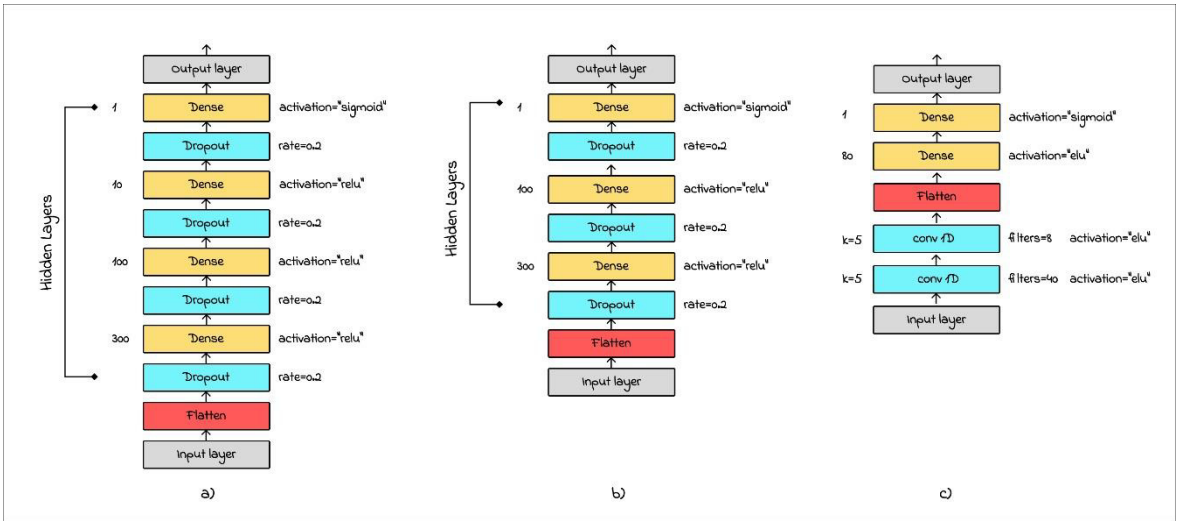

Figure 2. The architecture of the models considered: a) DNN model, b) MLP model, and c) CNN model.

4. Results and Discussion

This section presents a detailed analysis of the results obtained by validating and optimizing the proposed models for cardiovascular disease prediction. The model selection was based on a comprehensive comparison of the performance of three different models: DNN, MLP, and CNN. The models were evaluated on two different datasets, designated as the original dataset and B (reduced variable dataset).

4.1. DNN model

The compilation and training of the DNN models were carried out by configuring the Adam optimizer with a learning rate of 0.001. For the loss function "binary_crossentropy" was chosen, suitable for binary classification, a batch size of 32 (batch_size) and 50 epochs were determined to process the data.

The training results were evaluated using the confusion matrix, showing how the model classified the samples. The matrix reflected a higher number of correctly identified records in the original dataset compared to dataset B. A characteristic of the ROC curve is that if it is very close to the upper left corner, it implies a good performance of the model. Figure 3 shows the Classification DNN model's false positive and true positive rates using the ROC plot.

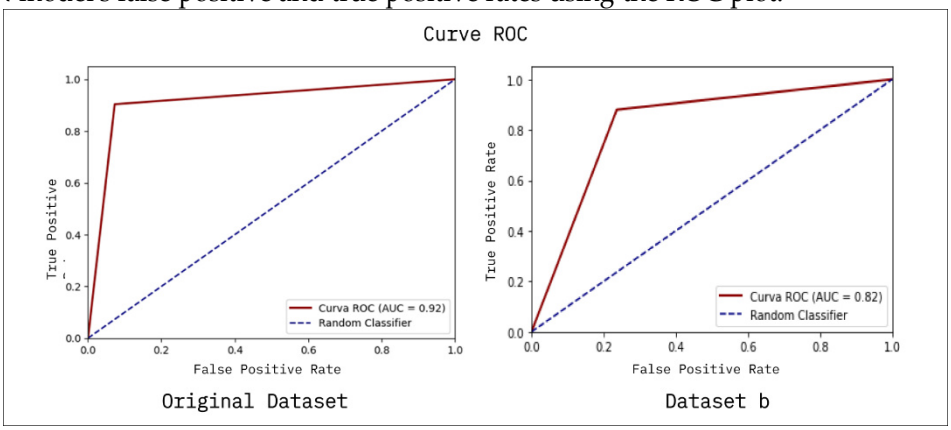

Figure 3. ROC of scores of DNN model

Once the metrics have been evaluated with the corresponding DNN model, the following Table 2 is obtained:

Table 2. DNN Evaluation Metrics

Metrics	Original Dataset [%]	Dataset B [%]
acc	91.30	83.33
precision	94.93	84.88
sensitivity	90.36	87.95
specificity	92.72	76.36
F1	92.59	86.39

The previous table shows that the DNN Classification model obtained an accuracy percentage of 91.30% by correctly classifying 75 data as negative cases, representing 92.72% specificity. As positive cases, 51 were obtained, representing 90.36% sensitivity. It is noteworthy that in this model analyzed, better results have been achieved using the original dataset.

4.2. MLP model

With a learning setting of 0.01, the model compiler was set up using the Adam optimizer. The loss function "binary_crossentropy" was chosen, which is suitable for the evaluation of predictions in this type of task. Following the same approach, a batch size of 32 samples and 50 epochs was set for the training process.

The training results of the MLP model were analyzed through the confusion matrix. In this MLP model, it is still maintained that better results are obtained with the original dataset. Figure 4 and Table 3 corroborate this observation, although the ROC curve and the performance metrics showed that the model had high accuracy.

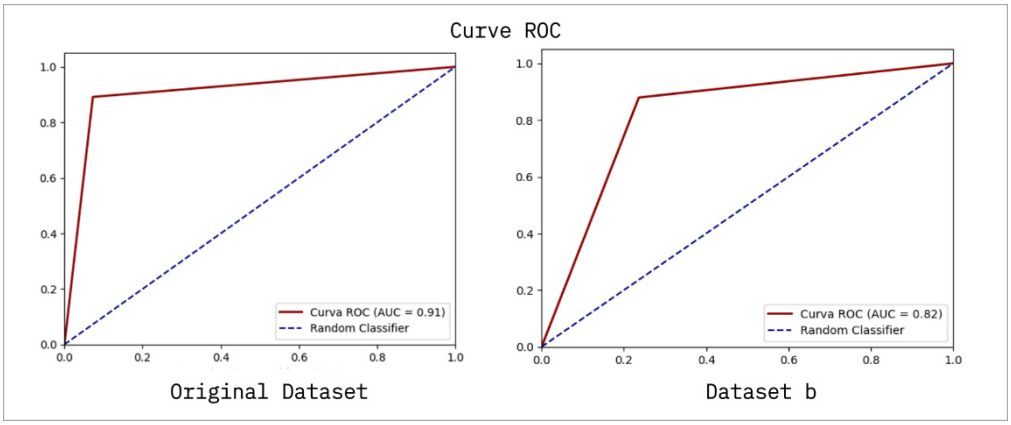

Figure 4. ROC of scores of MLP model

Table 3. MLP Evaluation Metrics

Metrics	Original Dataset [%]	Dataset B [%]
acc	90.5	83.33
precision	94.8	87.95
sensitivity	89.1	87.95
specificity	92.7	76.36
F1	91.92	86.39

The previous table shows that the MLP Classification model obtained an accuracy percentage of 90.50% by correctly classifying 74 data as negative cases, representing 92.70% specificity. As positive

cases, 51 were obtained, representing 89.1% sensitivity. However, it can be noted that if we compare the DNN model and the MLP model, slightly better results are achieved with the DNN model. That is, preliminarily some neural network complexity contributes to slightly better prediction results.

4.3 CNN model

The optimizer is defined using the Adam algorithm with a learning rate of 0.0001. The model is then compiled using the loss function "binary_crossentropy", which is effective in minimizing loss during training. The model is trained for 50 epochs with the training data and evaluated with the validation set to get a complete picture of its performance.

The training results of the CNN model were analyzed through the confusion matrix. In this CNN model, it is still corroborated that better results are obtained with the original dataset. Figure 5 and Table 4 confirmed this observation, although the ROC curve and the performance metrics showed that the model had high accuracy.

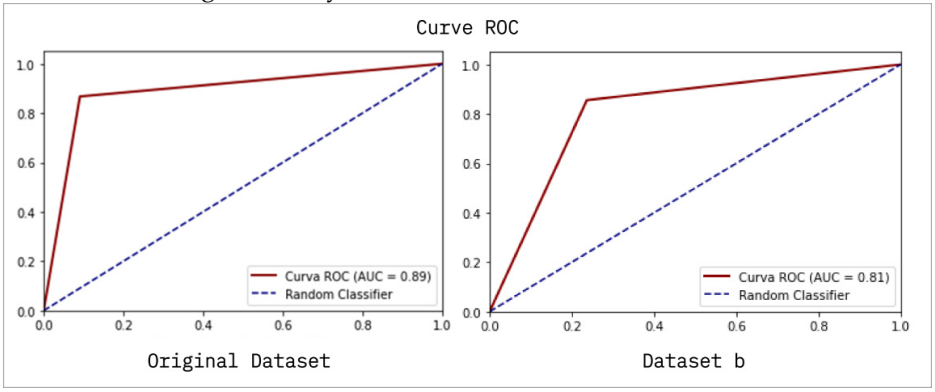

Figure 5. Confusion matrix of CNN training stage

Table 4. CNN Evaluation Metrics

Metrics	Original Dataset [%]	Dataset B [%]
acc	88.40	81.88
precision	93.50	84.52
sensitivity	86.74	85.54
specificity	90.90	76.36
F1	90.00	85.02

The previous table shows that the CNN classification model obtained an accuracy percentage of 88.40% by correctly classifying 72 data as negative cases, representing 90.90% specificity. As positive cases, 50 were obtained, representing 86.74% sensitivity. The evaluation metrics in Table 4 indicate good binary classification performance, with high precision, specificity, and F1 Score.

4.4. Selecting the best model

Table 5 provides a comprehensive overview of the performance of the three implemented models on two different datasets, designated as A and B, where A refers to the original dataset and B to the reduced dataset of variables. These results were obtained by applying K-fold cross-validation with K=5 partitions.

For each model and dataset, binary classification performance metrics are evaluated to measure its prediction ability with the heart disease variable. In terms of precision and accuracy, the best results with the CNN model are obtained using the original dataset (A), indicating its ability to predict positive cases with a low margin of error.

Table 5. Model selection using cross-validation

Model	Dataset	Accuracy	Precision	Recall	Specificity	F1-S	Roc-Auc
DNN	A	84.88%	84.09%	88.93%	80.21%	86.33%	**91.82%**
	B	85.17%	83.99%	90.69%	79.90%	87.17%	89.94%
MLP	A	83.80%	82.60%	88.35%	78.51%	85.32%	91.21%
	B	85.17%	83.66%	**91.27%**	79.24%	**87.26%**	90.07%
CNN	A	**85.20%**	**85.18%**	87.73%	**82.51%**	86.35%	88.82%
	B	83.32%	81.58%	89.14%	76.69%	85.10%	80.95%

The analysis highlights that dataset B benefits the sensitivity and F1-score of the MLP model, while the CNN model exhibits superior performance on most metrics compared to the DNN and MLP models. However, the choice of the optimal model must consider the context of the problem. A key point to note in Figure 6 is that the DNN and MLP models achieve a slightly higher value in true positives, with 51 positive values according to the confusion matrix, in contrast to the 50 positive values of the CNN model.

For selecting the model, the integration Neutrosophic Hierarchical Analytical Process and TOPSIS (NAHP-TOPSIS) [31,32] methods are used. In this case single valued neutrosophic trapezoidal numbers are uses the single-valued trapezoidal neutrosophic number,, is a neutrosophic set on, whose truth, indeterminacy and falsehood membership functions are defined as follows, respectively [33, 34]: $\tilde{a} = \langle(a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle\mathbb{R}$

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & a_2 \le x \le a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_3 \le x \le a_4 \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \tag{9}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \tag{10}$$

Where, and. $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$ $a_1, a_2, a_3, a_4 \in \mathbb{R} a_1 \le a_2 \le a_3 \le a_4$

For simplicity, we use the linguistic scale of trapezoidal neutrosophic numbers, see Table 1 and also compare with the scale defined in [35].

Table 6. Saaty's scale translated to a neutrosophic trapezoidal scale.

Saaty's scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$

7	Very strongly influential	$\tilde{7} = \langle(6,7,8);0.90,0.10,0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9);1.00,1.00,1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3);0.40,0.65,0.60\rangle$
		$\tilde{4} = \langle(3,4,5);0.60,0.35,0.40\rangle$
		$\tilde{6} = \langle(5,6,7);0.70,0.25,0.30\rangle$
		$\tilde{8} = \langle(7,8,9);0.85,0.10,0.15\rangle$

Each metric with the others in terms of relative importance in the context of healthcare pair-wise comparison matrix is obtained.

Table 7. Pairwise Comparison Matrix of Criteria

	Accuracy	**Precision**	**Recall**	**Specificity**	**F1-Score**	**ROC-AUC**
Accuracy	$\tilde{1}$	$1/\tilde{2}$	$\tilde{3}$	$\tilde{2}$	$1/4$	$\tilde{1}$
Precision	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$	$1/\tilde{3}$	$1/\tilde{4}$	2
Recall	$1/\tilde{3}$	$1/\tilde{2}$	$\tilde{1}$	$1/\tilde{2}$	$1/\tilde{2}$	$1/\tilde{3}$
Specificity	$1/\tilde{2}$	$\tilde{3}$	$\tilde{2}$	$\tilde{1}$	$1/\tilde{2}$	$1/\tilde{2}$
F1-Score	$\tilde{4}$	$\tilde{4}$	$\tilde{2}$	$\tilde{2}$	$\tilde{1}$	$\tilde{4}$
ROC-AUC	$\tilde{1}$	$1/\tilde{2}$	$\tilde{3}$	$\tilde{2}$	$1/\tilde{4}$	$\tilde{1}$

The weights derived from the AHP technique signify the significance of each parameter in the identification of cardiovascular disease. ROC-AUC (0.43) and F1-Score (0.35) possess the greatest significance, underscoring their essential functions in facilitating precise class differentiation and harmonizing precision with recall. Specificity (0.149) and Accuracy (0.143) underscore the necessity for dependable recognition of real negatives and overall precision. Meanwhile, Precision (0.141) and Recall (0.075) are of lesser significance yet are crucial for reducing diagnostic inaccuracies. These priorities correspond with the objective of attaining accurate and reliable forecasts in healthcare.

For selectin the model the original dataset result "A" is used then TOPSIS methods is applied. Based on the TOPSIS analysis using the updated AHP weights, the models are ranked as follows:

CNN (TOPSIS Score: 0.67),

DNN (TOPSIS Score: 0.62),

MLP (TOPSIS Score: 0.28).

The CNN model ranks highest due to its strong performance in key metrics such as ROC-AUC and F1-Score, which carry the most weight in this context. These metrics highlight the model's ability to balance precision and recall while maintaining robust class discrimination, crucial for accurate cardiovascular disease detection. The DNN model closely follows competitive performance, whereas the MLP model, with a lower score, demonstrates less alignment with the prioritized metrics.

Aditionaly in this work, hyperparameters were optimized to improve the forecasting metrics.

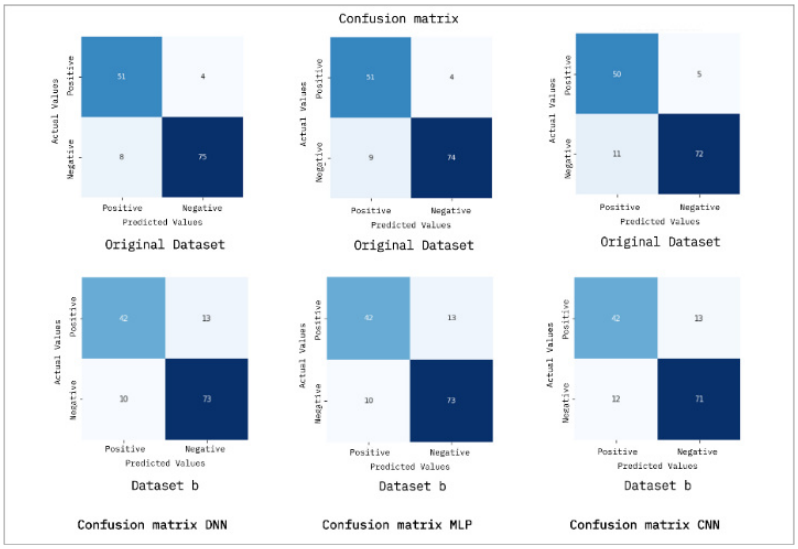

Figure 6. Confusion matrix DNN, MLP, and CNN models

4.5. Optimization of the best model

Hyperparameter optimization involves modifying different values in the model, such as the learning_rate, the number of hidden_layers, and the batch_size, among others. These additional settings ensure that the chosen model is the most suitable for achieving the work objectives and obtaining accurate and reliable results [36]. For this task, the Keras tuner library was used, recommended for its usefulness in the automated search of hyperparameters.

Table 6. Hyperparameters considered for the tuning of the CNN model

Hyperparameters	Values	Description
hp_units	min_value=8, max_value=256, step=8	Number of neurons
hp_layers	min_value=1, max_value=5, step=1	Number of hidden layers
hp_filters	min_value=2, max_value=40, step=2	Number of filters
hp_activation	['relu', 'sigmoid', 'tanh', 'softmax', 'elu'].	Activation function
hp_epochs	min_value=10, max_value=50, step=10	Number of epochs
hp_batch_size	min_value=4, max_value=40, step=4	Lot size used
hp_learning_rate	values=[$1e^{-1}$,$1e^{-2}$,$1e^{-3}$,$1e^{-4}$]	Learning rate

Table 6 shows the hyperparameters considered for the fitting and evaluation of the CNN model. These include the number of neurons, hidden layers, filters, activation function, training epochs, batch size, and learning rate. All these hyperparameters were implemented in a single function, called "model_builder", which defines the architecture, and compiles and trains the model.

During the configuration of the experimental design, CNN structures were executed, obtaining as the best topology the following model with hyperparameters: hp_units = 120, hp_layers = 2, hp_filters = 32, hp_activation = relu, hp_epochs = 50, hp_batch_size = 32 and hp_learning_rate = 0.0001.

Figure 7 and Table 7 show the contrast between the selected CNN model and the CNN model with its tuned hyperparameters. A considerable improvement is evident with the CNN model whose hyperparameters were adjusted to the original dataset. The confusion matrix shows an improvement in the prediction of negative cases of heart disease and a reduction of false negatives.

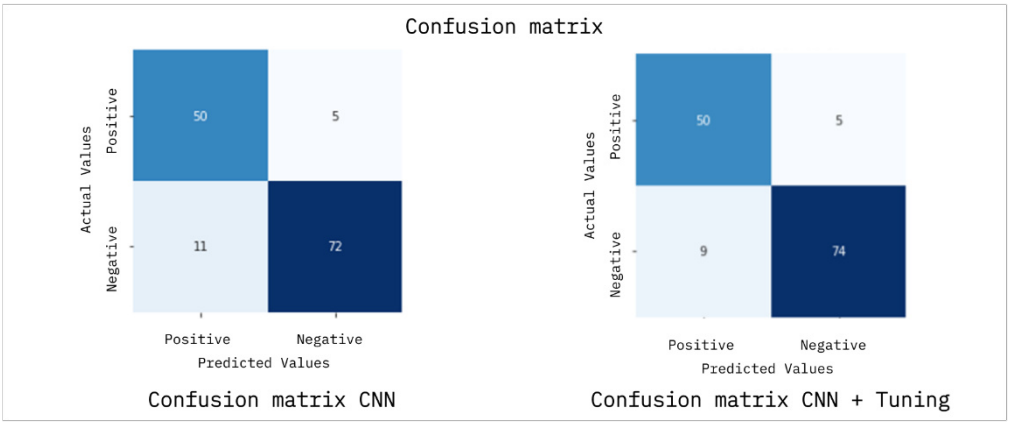

Figure 7. Comparison of the confusion matrix CNN and CNN Tuning

Table 7. Model selection by hyperparameter adjustment.

Model	Accuracy	Precision	Recall	Specificity	F1-Score	Roc-Auc
CNN	85.20%	85.18%	87.73%	82.51%	86.35%	88.82%
CNN + Tuning	92.85%	92.17%	94.51%	90.78%	93.30%	90.03%

The prediction of positive cases is maintained in both models. In the model with optimized hyperparameters all classification performance metrics exceed 90%, the highest of all with 94.51% being the sensitivity metric, i.e., the true positive rate.

5. Conclusions

This study applied deep learning models to predict whether a patient has heart disease. Experiments were conducted using two datasets with several cardiovascular disease risk factors. The original dataset with eleven risk factors and the reduced dataset with three risk factors considered statistical correlation were used. DNN, MLP, and CNN models were set up, trained, validated, and tested. Binary classification performance was used to select the best model for each type of architecture.

In all the models analyzed, the best results were obtained using the attributes and data from the original dataset, i.e., reducing the dataset by considering the statistical correlation of variables had no major influence on the results for this particular case study.

The best result in the general context of the binary forecasting metrics was achieved by the CNN model. This was ratified by applying an optimization with the hyperparameters of the model, achieving metrics that exceeded 90% performance.

The results of this study could serve as a baseline for other studies with machine learning models and recurrent neural networks, to further explore the binary forecasting performance of convolutional neural networks, which are generally used for image and video engineering problems. Another future study could consider the use of a dataset with a greater length and data density to test the incidence of the dataset size and number of variables considering reduction techniques applied in this area of health. Finally, future work would be to integrate the proposed model into a web or mobile app to support doctors in the diagnosis of cardiovascular. Moreover, future studies could explore the broader application of neutrosophic logic in the model selection process to handle uncertainty and indeterminacy more effectively, potentially enhancing decision-making in similar healthcare applications.

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Pre- and Post-harvest Application of Ethylene in Bulb Onion (Allium Cepa l.) Hybrid 'Burguesa' Using Plithogenic n-SuperHyperGraphs

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Abstract. Preserving the physical characteristics of onion bulbs has posed a challenge in post-harvest storage and conservation for the agricultural industry. This study analyzed factors influencing the preservation of these characteristics throughout the production cycle, with a focus on the impact of ethylene treatments during various storage and conservation stages. SuperHyperEdge relationships among the involved vertices were evaluated using plithogenic n-SuperHyperGraphs as the basis for statistical analysis. Results indicated that ethylene treatment applied during harvesting, with stems and roots intact, at a dose of 1.0 mL•L^{-1} improved bulb preservation by reducing weight and pH variations while maintaining higher moisture content. It was concluded that ethylene use is an effective tool for onion preservation, encouraging further research into its combination with other treatments.

Keywords: Agricultural product preservation, treatments, pests, sprouting, plithogenic n-SuperHyperGraphs.

1 Introduction

The quality of food results from the interaction between various intrinsic and extrinsic attributes, reflecting both the inherent characteristics of the product and the expectations and experiences of the consumer [1]. Among the intrinsic attributes, texture, sweetness, acidity, aroma, flavor, nutritional value, and the absence of hazardous compounds stand out. Extrinsic factors, on the other hand, include aspects related to production and distribution systems, such as the use of agricultural chemicals, packaging materials, and sustainability in harvest and post-harvest management practices [2].

It is essential to note that the quality of horticultural products cannot be improved after harvest but can only be preserved through the control of critical factors during handling [3]. Pre-harvest conditions, such as agronomic practices, climate, and genetic material, are decisive in developing crops with high-quality standards. Additionally, factors such as mineral nutrition, chemical treatments, pest management, and environmental control directly influence the internal and external attributes of the products, as well as their storage capacity [4]. Therefore, nutrients like nitrogen, phosphorus, potassium, calcium, and magnesium serve as key indicators for evaluating storage potential.

The application of growth regulators and chemical treatments allows the modification of horticultural product attributes by influencing their susceptibility to specific disorders during storage [5]. These strategies are crucial for extending the shelf life of food and minimizing its deterioration. In the case of onions, a non-climacteric crop, the low endogenous production of ethylene during storage necessitates specific conditions [6], such as low temperature and relative humidity, to reduce sprouting, rooting, and microbial growth.

Ethylene, an essential phytohormone in the plant life cycle, plays a key role in various physiological processes, from germination to development and maturation. Although traditionally studied in climacteric fruits, its relevance in crops such as onions has gained increasing interest [7]. The controlled application of ethylene at specific doses has proven to be an effective strategy for prolonging storage [8], reducing sprouting, and maintaining post-harvest quality [9] [10].

Onions, as a crop of historical and culinary significance, are characterized by their chemical composition, rich in water, carbohydrates, and sulfur compounds that determine their flavor and properties [11]. Proper management during production and post-harvest stages is essential to ensure quality [12], meet market demands, and contribute to the sustainability of their cultivation [13]. For this reason, the development of advanced and sustainable preservation techniques is a priority in horticultural research.

Consequently, the present study aims to analyze the factors influencing the preservation of the physical characteristics of onion bulbs throughout their production cycle, with an emphasis on the impact of ethylene treatments at different stages of storage and conservation. For this study, the use of plithogenic n-SuperHyperGraphs is proposed for evaluating and modeling the relationships among the elements involved, as well as conducting statistical analysis of each SuperHyperEdge to assess research outcomes and determine the most effective treatment.

2 Materials and Methods

The study was conducted using bulb onions (Allium cepa L.), hybrid Burguesa, organically cultivated in the province of Tungurahua, Ambato canton, Cunchibamba parish. After manual harvesting, the onions were transported to the laboratories of the Faculty of Agricultural Sciences and Natural Resources, where pre-classification and cleaning were performed to remove weeds, soil, and stones. Plithogenic n-SuperHyperGraphs were applied in the structural design of the statistical study [14].

The experiment was carried out under greenhouse conditions, where the onions were divided into beds according to the stages of ethylene application: pre-harvest (three days before harvest), harvest with intact stems and roots, and harvest with stems and roots cut. The ethylene doses used were 0.0, 1.0, and 1.25 $mL \bullet L^{-1}$, and sample evaluations were conducted every seven days during the storage period.

A product based on ethephon (Cerone 720), which releases ethylene upon being absorbed by plant tissues, was utilized. The first application took place during the pre-harvest stage when 60% of the bulbs exhibited neck fall. During the harvest stage, applications were performed both on bulbs piled with intact stems and roots and on bulbs with stems and roots cut.

Over a period of 99 days, random samples were taken every seven days across the 27 experimental units. Evaluations included weight, incidence of sprouting, and the presence of pests, diseases, and physiological disorders. Additionally, parameters such as pH, soluble solids content, and moisture were measured.

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in [15] [16]

First, an n-SuperHyperGraph is defined as follows:

Given $V = \{V_1, V_2, \cdots, V_{\mathrm{m}}\}$, where $1 \leq \mathrm{m} \leq \infty$ is a set of vertices, containing *Single Vertices* that are classical, *Indeterminate Vertices* which are unclear, vague, partially known, and the *Null Vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq \mathrm{n} \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq \mathrm{n} \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),

- *SuperVertex* (or *SubsetVertex)* contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1)- SuperVertex,* and the others can be *r- SuperVertex* for $r \leq \mathrm{n}-1$.

E_n contains the following types of edges:

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

Within the framework of the theory of Plithogenic n-SuperHyperGraphs, there are the following concepts:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervene,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights.

3 The Study

3.1 Analysis of the behavior of the frequencies of each variable.

For the development of the study, a Plithogenic n-SuperHyperGraph model was constructed to represent the interactions among the factors (vertices) that influence the preservation of the physical characteristics of onion bulbs. The effects of ethylene treatment on the relationships among vertices, attributes, and sub-attributes were considered. Table 1 presents the structure of the study.

Table 1: Structure of the vertices that affect the physical preservation of the onion bulb. Source: Own elaboration.

SuperVertex		**Vertices (V)**	**Main attributes**	**Sub-attributes**
Treatment (V_1)	Period (V_{11})	Pre-harvest (V_{111})	Days (V_{1111})	Range from 1 to 99 days (V_{11111})
		Harvest with stems and roots (V_{112})	Days (V_{1121})	Range from 1 to 99 days (V_{11211})
		Harvest without stems and roots (V_{113})	Days (V_{1131})	Range from 1 to 99 days (V_{11311})
		Ethylene (V_{114})	Dose (V_{1141})	0 ml/L (V_{11411}) 1.0 ml/L (V_{11412}) 1.25 ml/L (V_{11413})
Bulb preservation (V_2)	Physical characteristics(V_{21})	Weight (variation) (V_{212})	Initial (V_{2121})	grams (g) (V_{21211})
			Final (V_{2122})	grams (g) (V_{21221})
		pH (variation) (V_{213})	Initial (V_{2131})	hybrid, neutral, and acidic (V_{21311})
			Final (V_{2132})	alkaline, neutral, and acidic (V_{21321})
	Environmental conditions (V_{22})	Humidity (variation) (V_{221})	Initial (V_{2211})	% (V_{22111})
			Final (V_{2212})	% (V_{22121})
		Soluble solids (contents) (V_{223})	Initial (V_{2221})	ºBx (Brix degrees) (V_{22211})
			Final (V_{2222})	ºBx (Brix degrees) (V_{22221})
Affects on bulb preservation (V_3)		Sprouts(V_{31})	Sprout appearance (V_{311})	0;1 (V_{3111})
			Sprout growth rate (V_{312})	cm/day (V_{3121})
		Pests, diseases, and physiopathies (V_{32})	Pests (V_{321})	% of plants affected (V_{3211})
			Diseases (V_{322})	% of plants affected (V_{3221})
			Physiopathies (V_{323})	% of bulbs affected (V_{3231})

This Plithogenic n-SuperHyperGraph model provides an integrated representation of the complex interactions among factors influencing the preservation of the physical characteristics of onion bulbs, focusing on the use of ethylene and storage conditions. By understanding the relationships between vertices (factors) and their attributes/sub-attributes, the model evaluates how each factor affects the quality and durability of the bulb, contributing to the optimization of post-harvest management.

In the analysis of ethylene use for onion preservation, the input enveloping vertices include factors such as ethylene concentration and the initial conditions of the bulbs. Output vertices reflect the observed results, such as bulb quality, reduction in pests and diseases, and extended shelf life. Intermediate vertices represent the internal processes that mediate between inputs and outputs, such as the chemical reactions induced by ethylene and their impact on the bulb's physical and biological properties. This vertex-based model enhances the understanding of interactions within the preservation system and supports the optimization of ethylene treatment decisions.

In the study, the dominant super-enveloping vertex is the ethylene treatment, as it directly influences the physical properties and resistance of onion bulbs, impacting both quality and shelf life. Plithogenic connectors include ethylene concentration, which links the treatment to intermediate effects on bulb properties, and environmental storage conditions, which connect intermediate effects to the final conservation outcomes. These connectors facilitate the transition and analysis of interactions among the graph's various vertices.

3.2 Properties of the SuperHyperEdge (SHE_1): Treatment (V_1) and weight (variation) (V_{212}).

The weight variation based on the application period (pre-harvest, harvest with stems and roots, and harvest without stems and roots) does not exhibit statistical significance. However, on days 37, 92, and 99 (see Table 2), the application period influences weight variation, as significant and highly significant statistical differences are observed among the treatments. Regarding the dosage, a significant difference is determined on day 99, while the remaining days show non-significant results, indicating that the treatments behaved similarly in terms of weight variation over time in onion bulbs.

The coefficient of variation (CV) ranges from 16.82 to 258.63, reflecting the behavior of onions during post-harvest. The highest CV values are observed near the end of the experiment, influenced by the presence of diseases and physiopathies. In Tukey's test at 5% for the variation source "period" on days 37, 92, and 99, a marked difference in the ranges is observed. It is highlighted that the lowest weight variation occurs in the pre-harvest application period at 37 days and in the harvest without stems and roots periods at 92 and 99 days.

Table 2: Tukey test at 5% for three application periods in the weight variation variable. Source: authors.

Description	Average (g)					
Period (a)	37 days		92 days		99 days	
Pre-harvest	3.44	A	8.33	B	6.67	B
Harvest with stems and roots	4.56	AB	9.22	B	6.44	AB
Harvest without stems and roots	6.22	B	3.89	A	3.89	A

Previous studies have shown that onions harvested at a very mature stage, with 80% leaf tipping, exhibit lower weight, while those harvested with 60% leaf tipping show higher weights. For this reason, the onion bulbs used in this study were harvested when the onions reached a leaf tipping index of 60%, which was clearly reflected in the low weight variation, as the bulbs maintained high weights over time.

On the other hand, Tukey's test at 5% for the ethylene dosage at 99 days indicates that treatments with a 1.25 $mL \cdot L^{-1}$ dosage have a lower weight variation (4.11g) (see Table 3). In comparison with the 0.0 $mL \cdot L^{-1}$ ethylene dosage, where greater weight variation is observed, leading to product losses.

Table 3: Tukey 5% test against dosage on weight variation. Source: authors.

Description	Average (mL/L-1)	
Dose(b)	99 days	
0.0	7.11	B
1.0	5.78	AB
1.25	4.11	A

According to previous research, the most critical period for weight loss occurs between days 1 and 14, after which weight loss gradually decreases. In most treatments, onion bulbs show a weight loss of 10 to 20 grams per bulb during the first and second weeks. By the third week, weight loss is between 2 and 5 grams, and from the fifth week onwards, weight loss ranges from 1 to 2 grams until week 13.

3.3 Properties of SuperHyperEdge (SHE_2): Treatment (V_1) and pH (V_{213}).

For the variation source in the application periods at 7 and 59 days, two significant ranges are

established. The first range corresponds to treatments with pre-harvest application, where the pH values are slightly acidic. The second range corresponds to treatments with harvest application with stems and roots, showing pH values tending toward neutrality. According to the references consulted, the average pH values for red onion bulbs fluctuate between 5.3 and 5.8.

3.4 Properties of SuperHyperEdge (SHE_4): Treatment (V_1) and humidity (V_{222}).

For the application periods and different dosages at 37 days, two significant ranges were obtained. The first range corresponds to treatments with harvest application with stems and roots, with a dosage of 1.0 mL/L. In this case, a high humidity percentage above 90% is observed, meaning that over time, the onion maintained a high moisture content (see Figure 1). The second range corresponds to treatments with harvest application without stems and roots, with dosages of 1.0 mL·L^{-1} and 1.25 mL·L^{-1}, which reflect a lower humidity percentage.

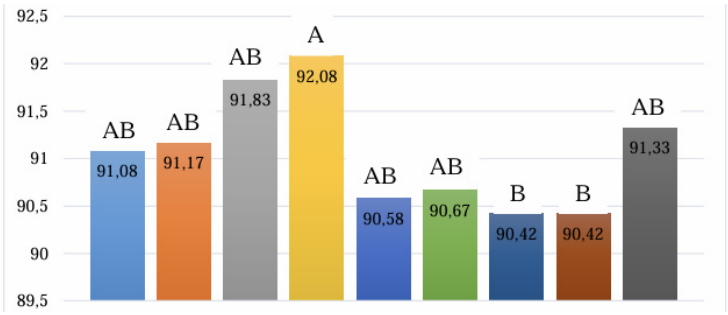

Figure 1: Averages for application periods and doses for the humidity variable (at 37 days). Source: Own elaboration.

Compared to previous studies, it is stated that the component found in the highest quantity in the bulbs is water, with variations ranging between 80% in varieties destined for the dehydration industry, and 94% in those for fresh consumption. As for the moisture content, it decreases by 10%, which corresponds to a 10% weight loss. The moisture percentage in onion bulbs across the treatments over 99 days fluctuates between 8% and 10%, indicating that the use of ethylene contributes to the preservation of the product.

3.5 Properties of SuperHyperEdge (SHE_5): Treatment(V_1) and soluble solids (V_{222}).

For the application periods and dosages at 94 days, two ranges were identified. The first range corresponds to the pre-harvest application period with a dosage of 1.0 mL/L, reflecting a high soluble solids content (ºBx). The second range corresponds to the pre-harvest application period with a dosage of 0.0 mL·L^{-1}, and to the harvest period with stems and roots, with a dosage of 1.0 mL·L^{-1}, showing lower values.

On the other hand, the sweet flavor of onion is provided by glucose, fructose, and sucrose, with these compounds varying in their degree of sweetness. In comparison, the literature has shown that varieties with a higher soluble solids content are more suitable for preservation, as they sprout less and are more resistant to diseases.

3.6 Properties of SuperHyperEdge (SHE_7): Treatment (V_1) and pests, diseases and physiopathies (V_{223}).

During the first days of the experiment, the samples did not show any incidence of pests, diseases, or physiological disorders from the start, so the recording of this variable began on day 58. However, starting from days 65, 72, 79, 92, and 99, the onion samples began to show diseases and physiological disorders, with highly significant differences between the treatments (see Table 4). Therefore, it is ob-

served that the application period and the concentration of the dosage considerably influence this variable.

Table 4: Tukey test at 5% for three periods on the incidence of pests, diseases and physiopathies. Source: authors.

Description	Average									
Period (a)	65 days		72 days		79 days		92 days		99 days	
Pre-harvest	4.44	A	4.44	A	6.67	AB	6.67	AB	6.67	AB
Harvest with stems and roots	2.22	A	2.22	A	2.22	A	2.22	A	2.22	A
Harvest without stems or roots	13.33	B	13.33	B	13.33	B	13.33	B	13.33	B

The coefficient of variation presents values between 82.16 and 133.82 because, during the early days of the experiment, no incidences of pests, diseases, or physiological disorders were recorded. Therefore, in this interval, values of zero were obtained, which do not correspond to a normal distribution of the Gaussian bell curve and the artificial adjustment of the square root of x plus 1 was applied.

For the source of variation related to the application period recorded on days 65, 72, 79, 92, and 99, two significant ranges were established. The first range corresponds to the application period during harvest with stems and roots, as these treatments presented a lower percentage of incidence of pests, diseases, and physiological disorders. The second range corresponds to the treatments applied during harvest without stems and roots, which showed higher percentages of incidence of pests, diseases, and physiological disorders.

According to the referenced literature, one of the main problems encountered in post-harvest is the sprouting of leaves, and roots, fungal and bacterial rot, and weight loss, among others. However, the treatments with ethylene doses showed completely closed stems and even the natural shedding of the outer cataphylls occurred. This led to a decrease in the moisture content of the onions, thereby reducing the likelihood of disease attacks, because of the effect of ethylene.

3.7 Properties of SuperHyperEdge (SHE_8): Treatment (V_1) and Sprouts (V_{32}).

For the variable sprout incidence, no sprouts were recorded during the first days up to day 79. However, for the vertices grouped by application period, the onions began to show incidences of sprouts on days 92 and 99. Therefore, the application period significantly influences sprout incidence (see Table 5).

Table 5: Analysis of variance for sprout incidence with three doses and three application periods. Source: authors.

FV	GL	92 DAYS			99 DAYS		
		SC	PV	NEXT	SC	PV	NEXT
Repetitions	2	562.96			800.00		
Application period (A)	2	2696.30	0.0158	*	2755.56	0.0186	*
Ethylene dose (B)	2	118.52	0.7903	ns	88.89	0.8479	ns
AxB	4	2014.81	0.1385	ns	2488.89	0.0999	ns
Error	16	3970.37			4266.67		
Total	26	9362.96			10400.00		
CV		92.46			81.65		
Average		17.04			20		

For the variation source of application periods, two significance ranges are established. The first range corresponds to treatments where the application period was during harvest with stems and roots, where the least number of sprouts were identified, along with the shortest time to sprout appearance. The second range, corresponding to the application period during harvest without stems and roots, shows high sprout incidence values, with greater presence in these treatments (see Table 6). On the other hand, previous studies indicate that the application of ethephon to onion plants 2 weeks before harvesting reduced sprout incidence by 5% after 32 weeks of storage, data that aligns with the present research.

Table 6: Tukey test at 5% for three application periods and three doses on the variable incidence of sprouts. Source: authors.

Description	Average			
Period (A)	92 days		99 days	
Pre-harvest	11.11	A	17.78	AB
Harvest with stems and roots	8.89	A	8.89	A
Harvest without stems or roots	31.11	B	33.33	B

3.8 Results of the interrelations between $SHE_1, SHE_2, SHE_3, SHE_4, SHE_5, SHE_6, SHE_7, SHE_8$.

The application of ethylene in the pre-harvest stage was identified as the most effective strategy for preserving the physical properties of the onion bulb. This treatment allowed for a lower weight variation (3.44%), a pH of 5.89, and a high moisture content (92.36%), ensuring better product quality during storage.

On the other hand, the application of ethylene during harvest, by preserving the stems and roots, showed additional benefits by minimizing the incidence of pests, diseases, and physiopathies (2.22%). Furthermore, this approach significantly reduced the development and appearance of shoots (8.89%), favoring both product stability and storage time.

The optimal ethylene dose determined was 1.0 $mL \bullet L^{-1}$, as it stood out for its ability to extend the shelf life of the bulb during storage. Under this condition, a reduction in weight loss (6.28%) and a lower incidence of pests and physiopathies (2.22%) were observed, preventing significant losses in quality and quantity of the stored product.

Finally, the interaction between the application stage and the dose used revealed that the most efficient treatment was carried out during harvest, with stems and roots, using a dose of 1.0 $mL \bullet L^{-1}$. This treatment allowed for a high moisture content (92.08%) and minimized the incidence of factors that compromise quality, such as pests and diseases, enhancing its effectiveness in preserving the onion bulb.

4. Conclusion

The application of ethylene during pre-harvest has been shown to be effective in preserving the physical characteristics of onion bulbs by reducing weight variation, maintaining an adequate pH, and a high percentage of humidity, which favors their conservation during storage. In addition, ethylene treatment at harvest, especially with intact stems and roots, has significantly reduced the incidence of pests, diseases and physiopathies, by improving the resistance of the bulbs and prolonging their shelf life. The optimal ethylene dose of 1.0 mL L^{-1} has been key to maintaining bulb quality by reducing weight loss and controlling damage. On the other hand, the integration with Plithogenic n-SuperHyperGraphs in the study has optimized the analysis of the effects of ethylene and its interaction with other factors, by opening new opportunities for the combination of treatments that improve efficiency in agricultural conservation.

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Evaluation of the Impact of Government Audit on Quality Management, Based on Neutroalgebra Generated by the PROSPECTOR Function

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Abstract. The present study focuses on the evaluation of the impact of government audit on quality management, using a novel approach based on the neutralalgebra generated by the PROSPECTOR function. The central question of the research lies in the need to improve the effectiveness of government audits to ensure more efficient and transparent quality management in public institutions. The relevance of this issue is accentuated in the current context, where the pressure to optimize government processes is crucial to guarantee efficiency and public responsibility. Although there are various studies on government auditing, a significant gap has been identified in the literature regarding the use of methods that integrate indeterminacy and uncertainty, essential factors in quality assessment in complex environments. To address this gap, the study applies neutralalgebra through the PROSPECTOR function, allowing for more robust analysis of data obtained from audits. The results reveal that this approach is highly effective in capturing complexity and ambiguity in quality management, offering more detailed insight than traditional methods. The contributions of the study are diverse: first, it introduces an innovative method that refines the analysis of government auditing; Second, it provides practical recommendations that can be implemented to improve the quality in the management of public institutions. Ultimately, this work not only advances the theoretical understanding of government auditing in quality management, but also offers practical tools for its effective application in public administration.

Keywords: Government Audit, Public Institutions, PROSPECTOR Function, Neutrofunction, Neutroalgebra

1. Introduction

Government auditing is a fundamental pillar to guarantee transparency and efficiency in the management of public institutions. Currently, the pressure to improve the quality of government management has increased significantly, demanding more precise and effective evaluation tools [1]. This study addresses the evaluation of the impact of government audit on quality management, using an approach based on neutralalgebra generated by the PROSPECTOR function. The relevance of this research lies in its potential to offer a new methodology that allows for a more comprehensive and accurate assessment of quality in public organizations, addressing the complexity and indeterminacy that are inherent to these processes [2]. Historically, government audits have been viewed as an essential mechanism for monitoring and improving public management. From the first fiscal control systems in ancient governments to complex contemporary audits, the objective has always been the same: to ensure that public resources are used efficiently and transparently [3]. In recent years, the focus of audits has expanded, going from the simple accounting review to the comprehensive evaluation of the quality of management, covering aspects such as the effectiveness, efficiency and sustainability of public policies [4]. However, the use of advanced mathematical tools to assess

management quality remains a developing area, with neutroalgebra emerging as a promising technique to address the limitations of traditional methods [5]. The problem that this study aims to solve is the lack of an approach that effectively integrates complexity and indeterminacy in the evaluation of quality management through government audit. Despite advances in audit techniques, a gap remains in the literature regarding the incorporation of uncertainty and ambiguity in quality assessment models [6]. How can the application of neutralalgebra, through the PROSPECTOR function [7,8], improve precision and effectiveness in evaluating the impact of government audits on quality management? This question is central to this research, which seeks to offer an innovative solution that addresses these complex dimensions comprehensively.

The objectives of this study are, firstly, to evaluate the impact of government audit on quality management using the neutralalgebra generated by the PROSPECTOR function [9], and secondly, to propose a methodological framework that allows improving the precision in the evaluation of quality in public institutions. Furthermore, the study aims to identify the critical areas in which this approach can be implemented to optimize transparency and efficiency in public administration. These objectives are aligned with the research question and will be developed in detail throughout the article, providing both a robust theoretical framework and practical applications for implementation in the government setting.

2. Preliminaries

2.1 Triangular single-value neutrosophic number.

Definition 1 [10,11] : The *neutrosophic set* N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership function I_A, and falsity membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A0, 1^+_A[$, and $\bar{}_A0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+_A$.

See that according to Definition 1, $T_A(x), I_A(x), F_A(x)$ they are standard or non-standard real subsets of $]\bar{}_A0, 1^+_A[$and therefore $T_A(x), I_A(x), F_A(x)$ can be subintervals of $[0, 1]$.

Definition 2 ([12, 13]): The *single-valued neutrosophic set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$ and $F_A: U \to [0,1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

The *single value neutrosophic The number* (SVNN) is symbolized by $N = (t, i, f)$, so that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 [14] : The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set in $\mathbb{R}$, whose membership functions of truth, indeterminacy and falsity are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x)=\begin{cases}\dfrac{\left(a_2-x+\gamma_{\tilde{a}}(x-a_1)\right)}{a_2-a_1}, & a_1\le x\le a_2\\ \gamma_{\tilde{a}}, & x=a_2\\ \dfrac{\left(x-a_2+\gamma_{\tilde{a}}(a_3-x)\right)}{a_3-a_2}, & a_2<x\le a_3\\ 1, & \text{otherwise}\end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\in[0,1]$and $a_1, a_2, a_3 \in \mathbb{R}. a_1\le a_2\le a_3$

Definition 4 ([15, 16]): Given $\tilde{a}=\langle(a_1,a_2,a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$two $\tilde{b}=\langle(b_1,b_2,b_3);\alpha_{\tilde{b}},\beta_{\tilde{b}},\gamma_{\tilde{b}}\rangle$single-valued triangular neutrosophic numbers and λany non-zero number on the real line. Then, the following operations are defined:

1. Addition:$\tilde{a}+\tilde{b}=\langle(a_1+b_1,a_2+b_2,a_3+b_3);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle$
2. Subtraction:$\tilde{a}-\tilde{b}=\langle(a_1-b_3,a_2-b_2,a_3-b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle$
3. Investment: $\tilde{a}^{-1}=\langle({a_3}^{-1},{a_2}^{-1},{a_1}^{-1});\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$, where $a_1,a_2,a_3\neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a}=\begin{cases}\langle(\lambda a_1,\lambda a_2,\lambda a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle, & \lambda>0\\ \langle(\lambda a_3,\lambda a_2,\lambda a_1);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle, & \lambda<0\end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}}=\begin{cases}\langle\left(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3>0\ and\ b_3>0\\ \langle\left(\frac{a_3}{b_3},\frac{a_2}{b_2},\frac{a_1}{b_1}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3<0\ and\ b_3>0\\ \langle\left(\frac{a_3}{b_1},\frac{a_2}{b_2},\frac{a_1}{b_3}\right);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, a_3<0\ and\ b_3<0\end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3>0\ and\ b_3>0\\ \langle(a_1b_3,a_2b_2,a_3b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3<0\ and\ b_3>0\\ \langle(a_3b_3,a_2b_2,a_1b_1);\alpha_{\tilde{a}}\wedge\alpha_{\tilde{b}},\beta_{\tilde{a}}\vee\beta_{\tilde{b}},\gamma_{\tilde{a}}\vee\gamma_{\tilde{b}}\rangle, & a_3<0\ and\ b_3<0\end{cases}$$

Where, ∧is a t-norm and ∨is a t-norm, [15] .

Let it be $\tilde{a}=\langle(a_1,a_2,a_3);\alpha_{\tilde{a}},\beta_{\tilde{a}},\gamma_{\tilde{a}}\rangle$a triangular neutrosophic number of a single value, then,

$$S(\tilde{a})=\frac{1}{8}[a_1+a_2+a_3](2+\alpha_{\tilde{a}}-\beta_{\tilde{a}}-\gamma_{\tilde{a}}) \quad (4)$$

$$\mathbf{A}(\tilde{a})=\frac{1}{8}[a_1+a_2+a_3](2+\alpha_{\tilde{a}}-\beta_{\tilde{a}}+\gamma_{\tilde{a}}) \quad (5)$$

They are called score and precision grades ã, respectively.

Let be $\{\widetilde{A}_1,\widetilde{A}_2,\cdots,\widetilde{A}_n\}$a set of n SVTNN, where $\widetilde{A}_j=\langle(a_j,b_j,c_j);\alpha_{\tilde{a}_j},\beta_{\tilde{a}_j},\gamma_{\tilde{a}_j}\rangle$(j = 1, 2,…, n), then the *weighted The average of the SVTNN* is calculated with the following equation:

$$\widetilde{A}=\sum_{j=1}^{n}\lambda_j\widetilde{A}_j \quad (6)$$

Where $\lambda_j$is the weight of A_j, $\lambda_j\in[0,1]$ and $\sum_{j=1}^{n}\lambda_j=1$.

2.3 Neutral Algebra and PROSPECTOR function

Definition 5 [17,18]: Let X be a given non-empty space (or simply a set) included in a universe of discourse U. Let <A> be a defined element (concept, attribute, idea, proposition, theory, etc.) in the set X. Then, through the process of neutrosification , we divide the set disjoint, depending on the application, but they are exhaustive (their union is equivalent to the entire space).

A *NeutralAlgebra* is an algebra with at least one *NeutralOperation* or a *NeutralAxiom* (an axiom that is true for some elements, indeterminate for other elements, and false for other elements).

NeutralAlgebra is a generalization of *Partial Algebra* , an algebra with at least one Partial *Operation* , while all its Axioms are true (classical axioms).

Definition 6 ([17,18,19]): A function f: X → Yis called *a Partial Function* if it is well-defined for some elements in X and is not defined for all other elements in X. Therefore, there exist some elements $a \in X$such that $f(a) \in Y$(well-defined) , and for all the other elements $b \in X$we have it $f(b)$is undefined.

Definition 7 ([17,18,19]): A function f: X → Yis called *NeutralFunction* if it has elements in X for which the function is well defined {degree of truth (T)}, elements in X for which the function is indeterminate {degree of indeterminacy (I)}, $T, I, F \in [0,1]$and $(T, I, F) \neq (0,0,1)$elements $(T, I, F) \neq (1,0,0)$in

Function classification

i. Function (Classical), which is a well-defined function for all elements in its domain of definition.

ii. NeutralFunction, which is a partially well-defined, partially indeterminate and partially externally defined function in its domain of definition.

iii. AntiFunction, which is an externally defined function for all elements in its domain of definition.

Definition 8 ([20,21]): A (classical) *algebraic structure* (or algebra) is a non-empty set A endowed with some operations (functions) (completely well defined) on A and satisfying some (classical) axioms (completely true) - according to Universal Algebra.

Definition 9 ([20,21]]): A *(classical) partial algebra* is an algebra defined on a non-empty set *PA* that is endowed with some partial operations (or partial functions: partially well-defined and partially undefined). While the axioms (laws) defined in a Partial Algebra are all totally (100%) true.

Definition 10 ([20,21]): A *NeutralAxiom* (or *Neutrosophic Axiom*) defined on a non-empty set is an axiom that is true for some set of elements {degree of truth (T)}, indeterminate for another set of elements {degree of indeterminacy (I)}, or false for the another set of elements {degree of falsity (F)}, where $T, I, F \in [0,1]$, with $(T, I, F) \neq (1,0,0)$which represents the Axiom (classical), and $(T, I, F) \neq (0,0,1)$which represents the AntiAxiom.

Classification of algebras

i) A *(classical) algebra* is a *non-empty set CA* that is endowed with total operations (or total functions, i.e. true for all elements of the set) and (classical) axioms (also true for all elements of the set).

ii) A *NeutralAlgebra* (or *NeutralAlgebraic Structure*) is a *non-empty NA* set that is endowed with: at least one *NeutralOperation* (or *NeutralFunction*), or a *NeutralAxiom* which refers to the set of operations (partial, neutral or total).

iii) An *AntiAlgebra* (or *AntiAlgebraic Structure*) is a *non-empty AA* set that is endowed with at least one *AntiOperation* (or *AntiFunction*) or at least an *AntiAxiom* .

Additionally, the PROSPECTOR function is defined in the MYCIN expert system as follows; is a mapping from $[-1,1]^2$within $[-1,1]$with the formula [22], :

$$P(x,y)=\frac{x+y}{1+xy} \quad (7)$$

This function is a uninorm with neutral element 0, so it complies with commutativity, associativity and monotonicity , see the different types of uninorms in , which include those defined for offsets . $P(-1,1)$and $P(1,-1)$they are not defined [23, 24].

3. Results.

First, we used a scale of linguistic terms so that participants could share their opinions on the topic at hand. This scale, along with the corresponding univalued triangular neutrosophic number, is summarized in the table below.

Table 1: Scale of linguistic terms and neutrosophic triangular scale associated with them.

Term linguistic	**SVTNN**
Very low (VL)	$\langle(0,0,1);0.00,1.00,1.00\rangle$
Medium- low (ML)	$\langle(0,1,3);0.17,0.85,0.83\rangle$
Low (L)	$\langle(1,3,5);0.33,0.75,0.67\rangle$
Medium(M)	$\langle(3,5,7);0.50,0.50,0.50\rangle$
High (High)	$\langle(5,7,9);0.67,0.25,0.33\rangle$
Medium-high (MH)	$\langle(7,9,10);0.83,0.15,0.17\rangle$
Very high (VH)	$\langle(9,10,10);1.00,0.00,0.00\rangle$

Let us note that the items in Table 1 correspond to the respondents' evaluations of agreement (positive). The disagreement scale is calculated based on the same items so that its SVTNN is multiplied by the scalar $\lambda=-1$. For example, the term "very little agreement" about meeting a certain criterion is associated with the SVTN $\langle(0,0,1);0.00,1.00,1.00\rangle$, while "very little disagreement" is calculated as $(-1)\langle(0,0,1);0.00,1.00,1.00\rangle=\langle(-1,0,0);0.00,1.00,1.00\rangle$.

On the other hand, to aggregate the survey values, the operator generated by the PROSPECTOR function [23,24] is used , which corresponds to Tables 2 and 3.

Table 2: Cayley table of $\odot$.

$x\odot y$	**-1**	**-0.9**	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0,2**	**-0,1**	
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-0,9	-1	-1	-1	-1	-1	-1	-1	-0,9	-0,9	-0,9	-0,9
-0,8	-1	-1	-1	-1	-0,9	-0,9	-0,9	-0,9	-0,9	-0,8	-0,8
-0,7	-1	-1	-1	-0,9	-0,9	-0,9	-0,9	-0,8	-0,8	-0,7	-0,7

$x \odot y$	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0,2	-0,1	
-0,6		-1	-0,9	-0,9	-0,9	-0,8	-0,8	-0,8	-0,7	-0,7	-0,6
-0,5		-1	-0,9	-0,9	-0,8	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5
-0,4		-1	-0,9	-0,9	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5	-0,4
-0,3		-0,9	-0,9	-0,8	-0,8	-0,7	-0,6	-0,6	-0,5	-0,4	-0,3
-0,2		-0.9	-0.9	-0.8	-0.7	-0.6	-0.6	-0.5	-0.4	-0.3	-0.2
-0.1		-0.9	-0.8	-0.7	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1
ndef.		-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0,1	0
0		-0,9	-0,8	-0,7	-0,6	-0,5	-0,4	-0,3	-0,2	-0,1	0
0.1		-0,9	-0,8	-0,6	-0,5	-0,4	-0,3	-0,2	-0,1	0	0.1
0,2		-0,9	-0,7	-0,6	-0,5	-0,3	-0,2	-0,1	0	0.1	0,2
0.3		-0,8	-0,7	-0,5	-0,4	-0,2	-0,1	0	0.1	0,2	0.3
0,4		-0,8	-0,6	-0,4	-0,3	-0,1	0	0.1	0,2	0.3	0,4
0,5		-0,7	-0,5	-0,3	-0,1	0	0.1	0,2	0.3	0,4	0,5
0,6		-0,7	-0,4	-0,2	0	0.1	0.3	0,4	0,5	0,5	0,6
0,7		-0,5	-0,2	0	0,2	0.3	0,4	0,5	0,6	0,6	0,7
0,8		-0,4	0	0.2	0.4	0.5	0.6	0.7	0.7	0.8	0.8
0.9		0	0.4	0.5	0.7	0.7	0.8	0.8	0.9	0.9	0.9
	def .										

Table 3: Cayley table of ⊙.

$x \odot y$	ndef.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
											def .
-0.9	-0.9	-0.9	-0.9	-0.8	-0,8	-0,7	-0,7	-0,5	-0,4	0	1
-0,8	-0,8	-0,8	-0,7	-0,7	-0,6	-0,5	-0,4	-0,2	0	0,4	1
-0,7	-0,7	-0,6	-0,6	-0,5	-0,4	-0,3	-0,2	0	0,2	0,5	1
-0,6	-0,6	-0,5	-0,5	-0,4	-0,3	-0,1	0	0,2	0,4	0,7	1
-0,5	-0,5	-0,4	-0,3	-0,2	-0,1	0	0.1	0.3	0,5	0,7	1
-0,4	-0,4	-0,3	-0,2	-0,1	0	0.1	0.3	0,4	0,6	0,8	1
-0,3	-0,3	-0,2	-0,1	0	0.1	0,2	0,4	0,5	0,7	0,8	1
-0,2	-0,2	-0,1	0	0.1	0,2	0.3	0,5	0,6	0,7	0,9	1
-0,1	-0,1	0	0.1	0,2	0.3	0,4	0,5	0,6	0,8	0.9	

ndef.	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .	ndef .
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0.1	ndef .	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.9	
0.2	ndef .	0.3	0.4	0.5	0.6	0.6	0.7	0.8	0.9	0.9	
0.3	ndef .	0.4	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	
0.4	ndef .	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	1	
0.5	ndef .	0.6	0.6	0.7	0.8	0.8	0.8	0.9	0.9	1	
0.6	ndef .	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	1	
0.7	ndef .	0.7	0.8	0.8	0.9	0.9	0.9	0.9	1	1	
0.8	ndef .	0.8	0.9	0.9	0.9	0.9	0.9	1	1	1	
0.9	ndef .	0.9	0.9	0.9	1	1	1	1	1	1	
1	ndef .	1	1	1	1	1	1	1	1	1	

Assessing the impact of government auditing on quality management requires the collaboration of a multidisciplinary team of experts, each addressing key aspects of the project. To evaluate the impact of government audit on quality management, the multidisciplinary team of experts is composed of the following specialists:

1. **Government Auditor:** Expert in specific auditing techniques and standards for the public sector.
2. Quality Manager: Specialist in quality management systems and the implementation of quality standards in government organizations.
3. **Data Analyst:** Professional in applied mathematics or statistics, in charge of analyzing quantitative data related to the impact of audits.
4. **Public Administration Consultant:** Expert in public policies and management, who provides knowledge about the context and implications of audits in the government sector.
5. **Organizational Psychologist:** Specialist in organizational behavior, who studies how audits affect the dynamics and work environment in public institutions.
6. **Information Technology Engineer:** Professional in information systems and technology, who evaluates the impact of digital tools used in auditing and quality management.
7. **Economist:** Economics expert who analyzes the economic effects of audits on the efficiency and effectiveness of public management.
8. **Administrative Law Attorney** : Administrative law specialist who advises on the legal implications of audits and ensures regulatory compliance.
9. **Organizational Communication Specialist:** Professional in charge of managing internal and external communication related to the results of audits and their impact on quality.
10. **Ethics and Social Responsibility Expert:** Ethics specialist who ensures that audits are conducted with integrity and that their findings are used to improve transparency and accountability in public management.

A group of 120 specialists in Ecuador were asked a series of questions to evaluate different aspects. Below, I share five important criteria along with four questions for each that were part of the survey

on how government auditing affects quality management:

1. Efficiency of the Audit Process

- How would you rate the efficiency of the audit process in terms of time and resources used?
- What improvements would you suggest to make the audit process more efficient?
- To what extent does the efficiency of audits impact the implementation of recommendations?
- What challenges does the audit process face in terms of efficiency?

2. Transparency and Communication

- How transparent do you consider the audit process to be in the disclosure of its findings?
- How would you evaluate the communication between the audit team and stakeholders?
- How effective are the audit recommendations in improving management quality?
- What suggestions do you have to improve transparency in the audit process?

3. Impact on Quality Management

- How has government auditing influenced quality management processes within institutions?
- What significant changes have you observed in quality management as a result of the audits?
- To what extent have audits contributed to improving the quality of the services offered?
- How would you rate the relationship between audit recommendations and improvement in quality management?

4. Regulatory Compliance

- How well do audit practices align with established regulations and standards?
- How would you rate the degree of regulatory compliance in the audited processes?
- What difficulties have arisen in relation to compliance during audits?
- How could regulatory compliance mechanisms be improved in the context of audits?

5. Stakeholder Satisfaction

- How satisfied are stakeholders with the audit results?
- How do you evaluate the response of stakeholders to audit recommendations?
- What aspects of audits do stakeholders consider most beneficial?
- What improvements would you suggest to increase stakeholder satisfaction with the audit process?

These criteria and questions are designed to provide a complete and detailed assessment of the impact of government auditing on quality management.

The survey is carried out based on the linguistic scale explained above. See Table 1 and :planation.

The variables x_{ijk} are designated according to SVTNN associated with the linguistic scale, which is e opinion of the ith respondent (i=1,2,..., 384), on the jth aspect to be evaluated (j = 1, 2, ..., j_k) within e kth criterion (k=1,2,..., 5).

For each i (i=1,2,...,384) and each k (k=1,2,...,5) is calculated $\bar{x}_{ik} = \sum_{j=1}^{j_k} \lambda_j x_{ijk}$, where $\lambda_j = \frac{1}{j_k}$. That is, $_k$it is the arithmetic mean of the SVTNN using formula 6, of each respondent for all aspects of each iterion.

Convert $\bar{x}_{ik}$to sharp values using precision degree formula 5, so that $\bar{y}_{ik} = \mathbf{A}(\bar{x}_{ik})$.

The values of $\bar{y}_{ik}$are rescaled to $\bar{\bar{y}}_{ik} = \frac{\bar{y}_{ik}}{10}$if $-10 \leq \bar{y}_{ik} \leq 10$, while $\bar{\bar{y}}_{ik} = -1$ or 1, if $\bar{y}_{ik} < -10$ or >).

The results are aggregated for all respondents as follows:

1. If <20% of respondents consider their opinions to be extreme, i.e. $\bar{\bar{y}}_{ik} = -1$ or 1, then the following formula applies:

$=\odot_{i=1}^{n} \frac{\text{round}(\bar{\bar{y}}_{ik}*10)}{10}$. Where n is the number of respondents with non-extreme opinions, that is $\bar{\bar{y}}_{ik} \neq$ -1 or 1, .

'here round is the rounding function and $\odot$is the NeutroOperator defined in Tables 2 and 3.

2. If > 10% of the pairs of respondents' opinions on criterion k are of type, $(-1, 1)$contradictions are considered to exist between the results of the kth aspect and the results are considered undefined, so this will need further analysis.
3. If > 20% of respondents have extreme opinions of the same type, either -1 or 1, and do not fall into the previous case, then $\tilde{y}_k = -1$ or 1it is considered, depending on the predominant opinion.

Each respondent provides an opinion on a linguistic scale for each aspect of the criterion, where the scale is transformed into numerical values. A scale of -5 to 5 is used for simplicity. The values will be random numbers between -5 and 5.

Table 4. Survey Data.

Respondent	Criterion	Aspect 1	Aspect 2	Aspect 3	Aspect 4
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				

Respondent	Criterion	Aspect 1	Aspect 2	Aspect 3	Aspect 4
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				
	Regulatory Compliance				
	Party Satisfaction				
	Process Efficiency				
	Transparency and Communication				
	Impact on Management				

Calculation of the Arithmetic Mean ($\bar{x}_{ik}$)

For each respondent, the arithmetic mean for each criterion is calculated, with the formula shown below:

$\bar{x}_{ik} = \sum_{j=1}^{j_k} \lambda_j x_{ijk}$,

Conversion to Sharp Values ($\bar{y}_{ik}$)

To convert these values to a crisp scale, the precision formula will be applied.

Let's assume that the function A($\bar{x}_{ik}$) rescales the values between -10 and 10. In this case, we will simply round the values from the mean:

- Criterion: Process Efficiency (Respondent 1) and $_{1k}$ =3.5
- Criterion: Transparency and Communication (Respondent 1) and $_{2k}$ =1.5

Scaling to Final Values ($\bar{\bar{y}}_{ik}$)

To obtain final values, the values are rescaled to be within the specified range. If the value is outside the range of -10 to 10, it is set to -1 or 1.

In this case, the values are already within the range, so:

- Process Efficiency: 3.5 → 3.5 (does not fit)
- Transparency and Communication: 1.5 → 1.5 (not adjusted)

Added Results

To calculate the aggregate results, the formula is applied if less than 20% of the opinions are extreme:

$\tilde{y}_k = \odot_{i=1}^{n} \frac{\text{round}(\bar{\bar{y}}_{ik} * 10)}{10}$.

By performing all the calculations, the following aggregate results are obtained:

Table 5: Aggregated Results

Criterion	Average Final
Process Efficiency	3.6
Transparency and Communication	1.5
Impact on Management	3.5
Regulatory Compliance	4.0
Party Satisfaction	3.7

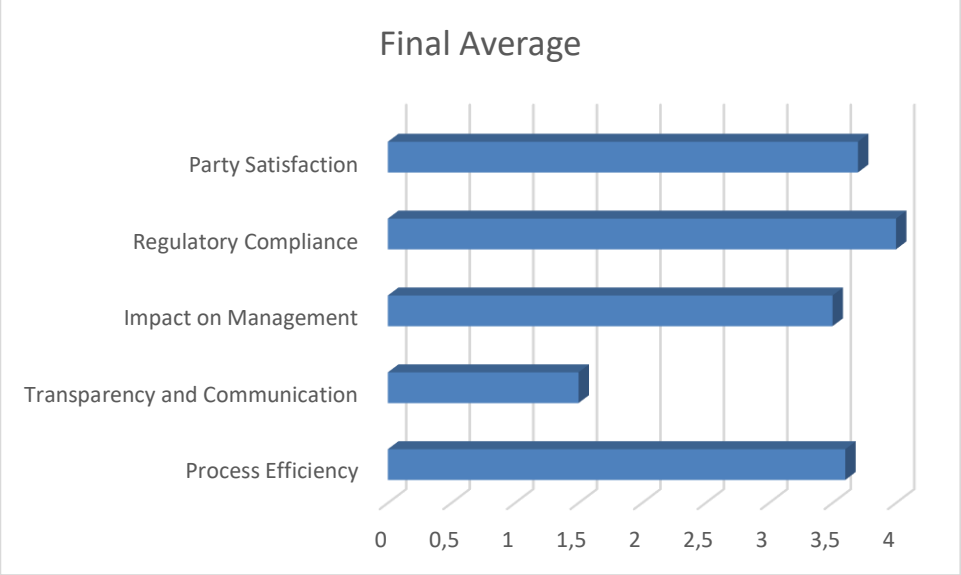

Chart 1: Aggregate Results

1. Process Efficiency: The final average of 3.6 suggests a positive assessment of the efficiency of the audit process, although there may be areas for improvement.
2. Transparency and Communication: With a mean of 1.5, the perception of transparency and communication is moderate, indicating a need to improve the dissemination of findings.
3. Impact on Management: A mean of 3.5 suggests that audits have a positive impact on quality management, but could have more influence.
4. Regulatory Compliance: With 4.0, a good level of alignment with regulations is observed, although there is always room to strengthen compliance.
5. Party Satisfaction: An average of 3.7 reflects favorable overall satisfaction, but it is crucial to address areas where deficiencies were identified.

The main results of this study suggest that, in general terms, government audit has a positive impact on quality management, with a final mean of 3.6 indicating a favorable evaluation of the efficiency of the process. However, the result of 1.5 in terms of transparency and communication indicates the need to make substantial improvements in these aspects. The interpretation of these findings indicates that, although the audit process can be considered efficient, the limited perception of transparency can affect the credibility and confidence in the audit results. This phenomenon could be explained by a lack of adequate channels to communicate the findings or by the complexity of the information presented, which could make it difficult for stakeholders to fully receive and understand the results. Thus, it is critical to establish mechanisms that promote more effective and accessible communication of audit findings. When comparing our results with previous research, we found that other studies have also reported similar levels of efficiency satisfaction (average of 3.5) and regulatory compliance (4.0), which reinforces the idea that government audits are aligned with regulatory expectations. and quality. However, some studies have highlighted the negative relationship between lack of transparency and public trust in the administration, which supports our recommendation to focus on the disclosure of audited information to build a higher level of credibility.

However, the study has certain limitations, such as possible subjectivity in scoring satisfaction and perceived transparency, which could vary depending on the individual experiences of the respondents. Additionally, the generalizability of these results could be restricted by the sample size

and demographics of respondents, who may not represent all of the stakeholders. These limitations should be carefully considered when interpreting the results and designing future research. The implications of these observations are significant for future research and practice in the field of government auditing. Additional studies are required that explore factors that influence perceptions of transparency and communication, as well as practical interventions that focus on improving audit disclosure. It is also essential to encourage stakeholder participation in the audit process, which could increase acceptance and positive perception of these processes. On the other hand, anomalous results were identified in the way in which stakeholders perceive the impact of audits on management (3.5), suggesting that, although there is a positive assessment, they may not be consistently perceived as transformative. These discrepancies may be due to lack of information on follow-up to audit recommendations or poor implementation of audit recommendations in specific areas. Recognizing these anomalies is crucial, as it points to an area of study that needs additional attention. These data indicate that, although government audit presents a solid foundation for quality management, it is imperative to address deficiencies in transparency and communication to maximize its impact and effectiveness. This study provides both a critical evaluation and a clear direction for future research and actions in the field of public audit.

4. Conclusion

The findings of this study reveal that government auditing is viewed generally positively, earning a process efficiency rating of 3.6 out of 5. This indicates that, although the process is considered to be working well, there is a clear need to improve aspects such as transparency and communication, which barely achieve an average of 1.5. These results underscore the importance of making changes that not only optimize the performance of the audit, but also strengthen the trust and understanding of all parties involved in the results. From a practical perspective, what we have found is key for those in charge of public management and government auditors. They can use this information to design policies and strategies that improve communication and information dissemination, which could foster greater stakeholder acceptance and participation in the audit process. Improving communication is essential to build credibility in public institutions and so that the quality of their management is perceived.

This study also makes important contributions to the field of public auditing. On the one hand, it offers a framework for evaluating the efficiency of the process and how it impacts quality management. Additionally, by highlighting the relationship between transparency, communication, and public trust, it opens doors for future research that delves into these connections. However, it is essential to recognize that this study has limitations. The subjectivity in the responses of the respondents and the diversity in their experiences could have influenced the evaluation of satisfaction and the perception of transparency. Likewise, the size and composition of the sample could restrict the possibility of generalizing the results to other situations. These limitations highlight the importance of conducting a more comprehensive analysis in future research. To investigate further, it would be useful to employ complementary methods, such as qualitative and mixed approaches, that offer deeper insights into how transparency and communication are perceived in government audit. Additionally, it would be beneficial to expand the study to various jurisdictions and contexts, which would help validate and generalize the findings. It is also crucial to pay attention to how audit recommendations are implemented and their impact on management, as this monitoring represents a key area of exploration for the advancement of research on this topic.

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Creative Exploration of Yagé in Intrapersonal Discovery through the Plithogenic IADOV Method

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Abstract. Post-Yage visions, influenced by ancestral Indigenous practices, have had a significant impact on the worldview and collective consciousness of participants. The objective of this study is to analyze post-Yage visions from an intercultural perspective, evaluating cultural depth, creativity, psychological impact, social relevance, and clarity in interpretation. The objective is to understand their influence on the Indigenous worldview and their impact on individual and collective consciousness through the modeling of the plithogenic IADOV. Correlations were identified between interpretive clarity, creativity, and psychological impact, highlighting their influence on the identity and social awareness of participants. In conclusion, this study provides deeper insights into the phenomenon of post-Yage visions by offering a comprehensive understanding of their cultural and psychological impact. Additionally, studying them from an intercultural perspective is recommended to enhance their understanding and applicability in the modern era.

Keywords: Indigenous worldview, Indigenous spirituality, cultural transcendence, Indigenous ritual, plithogenic IADOV.

1 Introduction

Post-Yagé visions have garnered increasing academic interest, particularly within the context of intercultural studies, due to their profound connection with the indigenous worldview and their influence on individual and collective consciousness [1]. Yagé, a ceremonial beverage used in various Indigenous cultures of the Americas, particularly in the Amazon region, induces visionary experiences that are interpreted not only as psychological phenomena but also as spiritual manifestations [2][3]. These visions are essential for understanding how ancestral practices impact communities, particularly in terms of cultural identity, spiritual values, and the intergenerational transmission of knowledge [4].

The present project arises as an extension of a previous study that addressed essential questions about how to artistically represent the trance experience and how to attribute meaning and significance to the images derived from it [5]. Thus, creativity assumes a leading role, especially within the field of design.

Visual arts are consolidated as essential tools for cultivating sensitivity, aesthetic expression, creative thinking, and communication [6]. Through their representations, they succeed in symbolizing complex experiences. From this perspective, the project explores the potential of visionary images as a medium to convey the richness of ancestral experiences and their relevance in the present [7].

Inspired by paradigmatic works of visionary art, this study investigates the connection between shamanic rituals and the symbolic representation of trance [8]. These artistic expressions not only capture a rich and profound visual language but also reflect the interactions between spiritual cosmology and heightened sensory perception [9].

With this approach, the primary purpose of the study is to analyze post-Yagé visions from an intercultural perspective by evaluating their cultural depth, creativity, psychological impact, social relevance, and clarity of interpretation. Indeed, it seeks to understand their influence on the indigenous worldview and their impact on the individual and collective consciousness of participants. To achieve this, the plithogenic IADOV method was employed to evaluate areas of satisfaction derived from the

experiences manifested during the Yagé ceremony.

2 Preliminaries

The study employed the plithogenic IADOV method to evaluate participants' experiences during the Yagé ceremony by integrating contradictory and subjective criteria. Through participant observation, interactions with the shaman, ingestion of Yagé, emotional manifestations, and the use of tobacco as a regulator were analyzed, along with a descriptive process of the ceremony (see Table 1).

Table 1: Descriptive process of the Yagé ceremony. Source: Own elaboration.

Item	Hour	Place	Activity
1	15H00	Sour Lake	Departure from Lago Agrio, towards Dureno
2	16H00	Dureno, Agrio Lake, Sucumbios	Crossing the Aguarico River by boat
3	16H30	Dureno, Agrio Lake, Sucumbios	Entrance to the A'i Cofán-Dureno community
4	16H45	A'i Cofán Community – Dureno, Lago Agrio	Walk into the jungle, towards the maloca
5	17H30	A'i Cofán Community – Dureno, Lago Agrio	Location of hammocks, green leaves in the maloca
6	19H00	A'i Cofán Community – Dureno, Lago Agrio	Contextualization and explanation of the ritual by the Shaman
7	20H00	A'i Cofán Community – Dureno, Lago Agrio	Ingestion of Yagé and chants by the shaman
8	21H00	A'i Cofán Community – Dureno, Lago Agrio	Intake by participants
9	21H30	A'i Cofán Community – Dureno, Lago Agrio	Visions
10	06H00	A'i Cofán Community – Dureno, Lago Agrio	Exit

2. Methods

To apply the plithogenic IADOV technique, experts must rely on an evaluation system based on a linguistic term that reflects the expert's opinion (see Table 2) [10]. Extending the IADOV method to plithogenic analysis through a neutrosophic scale allows for the structuring of group satisfaction [11][12], by separating the zones of indeterminacy and contradiction for a better interpretation of the results.

To understand the extension of the method, it must be defined that a plithogenic set is an extension of classical, fuzzy, intuitionistic fuzzy, and neutrosophic sets. Therefore, a plithogenic set is represented as (P, a, V, d, c), along with its characteristics and their intersection and union relationships (according to the consulted references [13][14]).

Table 2: Evaluation system for experts. Source: Own elaboration.

Plithogenic term	Code	Scale	Punctuation
Clearly satisfied	CS	(1,0,0)	3
More satisfied than dissatisfied	MSD	(1,0.35,0.35)	23
Not Defined	I	I	1.5
More dissatisfied than satisfied	MDS	(0.35,0.35,1)	1
Clearly dissatisfied	CD	(0,0,1)	0
Contradictory	C	(1,0,1)	2

In the previous table, the term I in neutrosophy is used, interpreted as a unit of undefined indeterminacy. On the other hand, to determine the neutrosophic plithogenic global satisfaction index (NPGSI) of the respondents, the aggregation operator H_N^P was applied, taking into account the evaluations of each element X within the plithogenic set P; $x \in Pd_F\ d_{IF}\ d_N$. This process yields the ISGPN from the sum of the elements analyzed within the evaluated plithogenic subset, where L_r^P is the frequency, and w_j is the assigned weight (see Equation 1).

$$H_N^P\left(S_1^P, S_2^P, \ldots, S_n^P\right) = \sum_{i=1}^{n}\left[w_j, L_r^P\right] \tag{1}$$

4 Case Study

This study explores the cultural and spiritual phenomenon of the A'i Cofán community of Dureño, focusing on the ingestion of Yagé and the resulting visions. The relevance of interpreting these experiences within their cultural context is emphasized, considering Yagé as a link between the physical and spiritual worlds. The visions described are not mere hallucinations, but symbolic representations that reflect profound aspects of the individual and collective psyche, allowing for a deeper connection with nature, ancestry, and the cosmos. The results highlight four visionary passages (see Figures 1 to 4):
The Welcome, which represents spiritual entrance;

Figure 1: Scene 1: The Welcome. Prepared by: Arleth Márquez.

- The River, symbolizes decisions accompanied by family support;

Figure 2: Scene 2: The river. Prepared by: Arleth Márquez.

- The Island, as a safe haven; and

Figure 3: Scene 3: The island. Prepared by: Arleth Márquez.

- The Tiger, an emblem of wisdom and leadership according to the A'i Cofán worldview.

Figure 4: Scene 4: The tiger. Prepared by: Arleth Márquez.

These experiences are artistically captured by combining visual symbolism and iconological analysis to reinterpret emotions, traditions, and spirituality. However, despite advances in the study of entheogens, few have addressed the cultural significance of visionary images in Indigenous communities.

The qualitative methodology, supported by participant observation, has been essential for capturing the richness of visionary experiences, recognizing their individual and contextual variability. Therefore, the application of the plithogenic IADOV method is necessary to evaluate the satisfaction of participants during the Yagé ceremony, by analyzing four fundamental elements observed in the case study:

- Creativity in post-Yagé visionary experiences.
- Psychological impact of post-Yagé visions.
- Social relevance of post-Yagé visions.
- Clarity in the interpretation of post-Yagé visions.

4.1 Creativity in post-Yagé visionary experiences.

Post-Yagé visionary experiences reflect a complex creativity that integrates artistic expression and deep cognitive processes. Through interviews and observations, three key aspects were identified. First, the visualization of landscapes and forms includes surreal scenes, geometric figures, and cultural symbols with profound spiritual meanings. Second, symbolic narratives highlight transformations and harmonies connected to Indigenous traditions and mythologies. Third, emotional expression manifests in visual and artistic metaphors that reflect feelings of healing and renewal.

To evaluate this creativity, the Neutrosophic IADOV method was used, allowing for the analysis

of artistic expression, visual and symbolic creativity, and emotional impact (see Table 3). This approach provides a comprehensive perspective on how post-Yagé visions contribute to the spiritual and emotional understanding of participants.

Table 3: Group creativity index evaluation. Source: Own elaboration.

Plithogenic criterion	Scale	Score (P)	Frequency (F)	$P \cdot F$	$\frac{F \cdot S}{n}$
Clearly satisfied	(1,0,0)	5	16	80	3.20
More satisfied than dissatisfied	(1,0.35,0.35)	4	3	12	0.48
Not defined	I	3	2	6	0.24
More dissatisfied than satisfied	(0.35, 0.35, 1)	2	1	2	0.08
Clearly dissatisfied	(0,0,1)	1	1	1	0.04
Contradictory	(1,0,1)	0	2	0	0.00
Creativity Index (CI) = 4.04					

The creativity expressed in post-Yagé visionary experiences is a multidimensional phenomenon that encompasses not only artistic and visual production but also emotional transformation and the spiritual connection to cultural traditions. The analysis using the plithogenic IADOV method highlights how participants positively value the symbolic and visual creativity generated by Yagé experiences, as it contributes to their understanding of the spiritual and cultural world. This creativity index, with a value of 4.04, suggests a collective perception of creativity as a key element in the interpretation and transmission of spiritual experiences within the Indigenous community.

4.2 Psychological impact of post-Yagé visions.

The analysis of the psychological impact of post-Yagé visions reveals profound changes in the emotional well-being and consciousness of participants. In the short term, the release of repressed emotions, depersonalization, and confrontation with deep fears are highlighted, generating emotional relief and renewal.

In the long term, these experiences foster significant personal transformations, with greater self-understanding, reduced anxiety and depression, and a stronger connection with the community and nature. These visions reaffirm cultural values and promote a sense of belonging and inner balance.

The evaluation, conducted using the plithogenic IADOV method, allowed for the measurement of satisfaction and psychological impact (see Table 4). It thus provided a comprehensive understanding of the emotional and spiritual effects associated with post-Yagé visions.

Table 4: Calculation of the overall psychological impact index. Source: Own elaboration.

Plithogenic criterion	Scale	Score (P)	Frequency (F)	$P \cdot F$	$\frac{F \cdot S}{n}$
Clearly satisfied	(1,0,0)	5	14	70	2.80
More satisfied than dissatisfied	(1,0.35,0.35)	4	7	28	1.12
Not defined	I	3	1	3	0.12
More dissatisfied than satisfied	(0.35, 0.35, 1)	2	1	2	0.08
Clearly dissatisfied	(0,0,1)	1	1	1	0.04
Contradictory	(1,0,1)	0	1	0	0.00
Psychological Impact Index (PII) = 4.16					

The psychological impact of post-Yagé visions on participants is profound and multifaceted, affecting both their emotional well-being and spiritual consciousness. In the short term, participants experience significant emotional release, while in the long term, many report personal transformations that contribute to greater inner peace, community connection, and harmony with nature. The results

obtained through the plithogenic IADOV method (psychological impact index = 4.16) reflect that visionary experiences are perceived positively in terms of psychological satisfaction, suggesting that Yagé plays a significant role in emotional healing and in the consolidation of a new worldview.

4.3 Social relevance of post-Yagé visions.

Post-Yagé visions, deeply linked to Indigenous traditions, strengthen cultural identity and community cohesion. These experiences connect participants with their spiritual heritage, revitalizing ancestral values and reinforcing the intergenerational transmission of knowledge.

Furthermore, they function as a mechanism for cultural preservation by legitimizing Indigenous spiritual practices in the face of globalization. The visions promote solidarity and mutual respect by fostering values such as the connection with nature and a sense of community belonging.

Together, these experiences not only ensure the continuity of traditional worldviews but also consolidate the relevance of spiritual practices as tools of cultural resistance and social renewal. The result of the plithogenic IADOV evaluation yielded a global social relevance index (IRS) of 4.20, indicating that participants perceive the social and cultural impact of post-Yagé visions very positively (see Table 5). This high index reflects the importance of the visions in preserving traditions and consolidating cultural values within Indigenous communities.

Table 5: Calculation of the global social relevance index (SRI). Source: Own elaboration.

Plithogenic criterion	Scale	Score (P)	Frequency (F)	$P \cdot F$	$\frac{F \cdot S}{n}$
Clearly satisfied	(1,0,0)	5	17	85	3.40
More satisfied than dissatisfied	(1,0.35,0.35)	4	3	12	0.48
Not defined	I	3	1	3	0.12
More dissatisfied than satisfied	(0.35, 0.35, 1)	2	2	4	0.16
Clearly dissatisfied	(0,0,1)	1	1	1	0.04
Contradictory	(1,0,1)	0	1	0	0.00
Social Relevance Index (SRI) = 4.20					

Post-Yagé visions have a profound impact on the social relevance of cultural traditions and values within Indigenous communities. Through these visionary experiences, cultural identity and ancestral knowledge are reaffirmed and preserved, contributing to the strengthening of community cohesion. Furthermore, post-Yagé visions play a crucial role in the transmission of cultural values and the validation of spiritual practices in the face of external influences. The results obtained through the plithogenic IADOV method indicate that these visions are considered highly relevant for communities in terms of their social impact and ability to strengthen cultural traditions.

4.4 Clarity in the Interpretation of post-Yagé Visions.

The interpretation of post-Yagé visions is essential for their understanding and preservation within Indigenous communities. These experiences, filled with symbolism and cultural metaphors, rely on oral tradition to transmit their meanings. The shared worldview facilitates collective understanding, while the shaman acts as a key guide to interpret and contextualize the visions in both spiritual and everyday terms. This process strengthens cultural identity and ensures the transmission of ancestral knowledge. The plithogenic IADOV method allows for the evaluation and quantification of clarity in interpretation, ensuring its value as part of the spiritual and cultural heritage of the communities (see Table 6).

Table 6: Evaluation of the clarity in the interpretation of post-Yagé visions. Source: Own elaboration.

Plithogenic criterion	Scale	Score (P)	Frequency (F)	$P \cdot F$	$\frac{F \cdot S}{n}$
Clearly satisfied	(1,0,0)	5	0	0	0.00
More satisfied than dissatisfied	(1,0.35,0.35)	4	1	4	0.16

Plithogenic criterion	Scale	Score (P)	Frequency (F)	$P \cdot F$	$\frac{F \cdot S}{n}$
Not defined	I	3	13	39	1.56
More dissatisfied than satisfied	(0.35, 0.35, 1)	2	10	20	0.80
Clearly dissatisfied	(0,0,1)	1	0	0	0.00
Contradictory	(1,0,1)	0	1	0	0.00
Interpretation Clarity Index (ICI) = 2.52					

The interpretation clarity index (ICI) is 2.52, indicating that, in general, post-Yagé visions are perceived as relatively clear in terms of their interpretation within the community. However, some participants face difficulties fully communicating their experiences, which is reflected in a moderate number of responses related to terms such as "Undefined" or "More dissatisfied."

Post-Yagé visions are clearly understood and communicated within Indigenous communities, especially by those who have a deep connection with the worldview and spiritual traditions. However, due to the metaphorical and abstract nature of the visions, some participants find challenges in fully conveying the meaning of their experiences, particularly when these meanings rely on complex symbolic language. Therefore, spiritual leaders play a crucial role in facilitating the interpretation of the visions, helping participants understand and explain their experiences.

4.5 Contributions of the interaction of the elements.

This analysis provides a detailed structure for understanding how the elements interact, their potential contradictions, and how their relative importance influences their collective effectiveness. Among them, the following can be observed:

- Creativity in visionary experiences: It fosters innovation and new perspectives, creating opportunities for disruptive changes. Its interaction with psychological impact triggers an emotional transformation in individuals and communities.
- Psychological impact: The emotional and cognitive changes resulting from psychological impact affect how people perceive and respond to experiences. Its interaction with social relevance ensures that the effects are not only personal but also collective, promoting social adaptation.
- Social relevance: It facilitates the social acceptance of innovative ideas, as well as interaction with creativity, ensuring that new proposals are meaningful in the social and cultural context. Meanwhile, clarity in interpretation ensures that these ideas are understood and applied by society.
- Clarity in interpretation: It enhances the communication of ideas and concepts by facilitating their adoption at both individual and collective levels. Its interaction with the other elements ensures that ideas are accessible, effective, and understood in the appropriate context.

On the other hand, plithogenic contradiction refers to the potential tension or conflict that arises when the interacting elements are not fully aligned. The evaluation of the degrees of contradiction between the elements is presented below (see Table 7).

Table 7: Contradiction of non-aligned elements. Source: Own elaboration.

Items	Plithogenic contradiction (Scale: low, medium, high)
Creativity in visionary experiences vs. psychological impact.	Medium: Creativity challenges psychological impact when novelty is not emotionally well-received.
Creativity in visionary experiences vs. social relevance.	Low: Although creativity can initially be challenging for social acceptance if it is relevant, it adapts easily.
Creativity in visionary experiences vs. Clarity in interpretation.	Low: Clarity facilitates the understanding of innovative ideas but increases the likelihood of contradictions if creativity is complex.

Items	Plithogenic contradiction (Scale: low, medium, high)
Psychological impact vs. social relevance.	Medium: Psychological impact can be intense, but if not aligned with social norms, there may be a disconnect.
Psychological impact vs. Clarity in interpretation.	Low: Clarity facilitates the understanding of psychological impact, although it increases the likelihood of confusion if not adequately explained.
Social relevance vs. Clarity in interpretation.	Low: Clarity is key to ensuring that social relevance is understood and applied correctly.

Psychological impact and clarity in interpretation are the most significant elements, as they are fundamental for the acceptance and understanding of ideas. Creativity and social relevance are crucial, but they depend on the other two elements to be effective in a social and psychological environment.

4.6 Discussion.

This study highlighted the complexity and richness of post-Yagé visionary experiences by emphasizing the need for illustrations as a means of capturing the essence of these lived moments. The visual representations, of a psychedelic nature, present themselves as a window into the inner world of individuals, revealing the diversity and depth of the visions experienced after the ingestion of Yagé.

Through a cultural approach, the project contributes to the understanding of the intersection between culture, spirituality, and cognition by interpreting the visions within an environment of ancestral traditions and beliefs. The research emphasizes the need to connect with the cultural roots of the Indigenous community, which is essential for appreciating the practices that still endure today.

Graphic representation has become an effective tool for communicating the experiences perceived during the trance, facilitating their interpretation, and understanding. Additionally, the study has highlighted the need to establish bridges between ancestral traditions and modernity, recognizing the cultural value of Indigenous communities as a step toward mutual understanding and respect in a globalized world.

The modeling of the IADOV plithogenic method, which included field visits and direct interviews with community members, provided an authentic and enriched perspective on the Yagé ceremony, enhancing the quality and validity of the results. Furthermore, an interesting question has been raised regarding the relationship between Yagé experience and creativity, opening a debate on how these practices influence individuals' artistic and creative expression.

However, it is crucial to expand the research to other Amazonian communities practicing Yagé rituals and other entheogens in order to gain a broader understanding of these practices. Therefore, an interdisciplinary approach is also recommended, integrating anthropology, psychology, and neuroscience for a better understanding of the effects and meanings of these rituals. Finally, it is suggested to delve deeper into the study of the indigenous worldview to better understand the philosophical and spiritual foundations of entheogen rituals in Amazonian communities.

5 Conclusion

The results obtained through the analysis conducted using the plithogenic IADOV have highlighted the interaction of key elements such as creativity in visionary experiences, psychological impact, social relevance, and clarity in interpretation. It has proven to be crucial in generating transformations both at the individual and collective levels. Despite the plithogenic contradictions between creativity and psychological impact, it has been observed that clarity in interpretation and an appropriately adjusted social relevance help resolve these tensions by optimizing the understanding of the experiences manifested during the Yagé ceremony. Furthermore, the study has emphasized the need to establish bridges between ancestral traditions and modernity, recognizing the cultural value of Indigenous communities as a step toward mutual understanding and respect in a globalized world. It has also highlighted the importance of preserving, documenting, and understanding Indigenous traditions, empha-

sizing the interaction between these traditions and contemporary society as a fundamental axis for cultural exchange and mutual enrichment. Additionally, the study suggests exploring, through longitudinal studies, the long-term effects of these experiences on different demographic groups and the impact of integration rituals on participants' emotional well-being.

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Neutrosophic Analysis of Strategies to Improve Productivity in an Restaurant Chain

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Abstract. The study analyzed productivity in an Ecuadorian restaurant chain using a multicriteria approach within a neutrosophic framework. The research focused on identifying effective strategies to optimize processes and addressing specific challenges such as staff turnover and compensation management. The TOPSIS method, combined with bipolar neutrosophic numbers, was employed to evaluate a group of strategies based on high-impact criteria. The results highlighted the implementation of economic incentives, training and retention programs, and performance management systems as the most effective alternatives. In contrast, internal communication campaigns and flexible schedules were found to be less prioritized. The research demonstrated the utility of neutrosophic logic in handling uncertainty and ambiguity, offering a replicable model for decision-making in complex business sectors.

Keywords: neutrosophic logic, multicriteria decision, business strategy, operational optimization, TOPSIS, bipolar neutrosophic sets.

1 Introduction

Productivity in the restaurant industry is a critical aspect that directly influences the sustainability and competitiveness of businesses. In Ecuador, restaurant chains face growing challenges due to the need to adapt to changing consumer demands, intense competition, and economic fluctuations. In this context, identifying and prioritizing strategies that optimize processes, reduce costs, and increase operational efficiency has become a priority for managers of these chains [1]. However, designing effective strategies requires considering multiple interrelated variables, which poses a significant challenge in decision-making.

In scientific literature, multicriteria decision-making (MCDM) methods have proven to be robust and versatile tools for addressing complex problems that involve diverse and often conflicting criteria [2]. These methods allow decision-makers to evaluate alternatives by considering both quantitative and qualitative factors, providing a systematic framework for making informed decisions [3]. One of the most widely adopted methods is TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), developed by Hwang and Yoon, recognized for its ability to identify the best alternative by comparing its proximity to an ideal solution. Its popularity stems from its simplicity, effectiveness, and applicability in various fields, such as operations management, strategic planning, and policy analysis [4].

However, classical multicriteria decision-making methods have limitations when addressing the uncertainty and indeterminacy inherent in many real-world situations [5], [6]. Decisions in business environments, such as in the case of restaurant chains, are influenced not only by precise data but also by subjective factors, imprecision, and dynamic contexts that are not always reflected in traditional models. This challenge has driven the development of more advanced approaches that integrate concepts from fuzzy and neutrosophic logic, offering new perspectives to handle the complexities of uncertainty [7], [8].

Neutrosophy, introduced by Florentin Smarandache, is an extension of classical logic that recognizes and models the coexistence of truth, falsehood, and indeterminacy within the same framework.

This approach represents a significant advancement by allowing the incorporation of degrees of uncertainty and ambiguity in decision-making processes [9]. In the evolution of this theory, tools such as bipolar neutrosophic numbers have been developed, which integrate the polarity of linguistic information, enabling a richer and more detailed representation of phenomena [10]. These tools have been applied in diverse fields such as supplier evaluation, technology selection, and educational quality management, demonstrating their relevance in addressing complex problems [11].

In this sense, the introduction of neutrosophic logic in the analysis of business strategies offers a promising approach for restaurant chains. This theoretical framework can model the uncertainty inherent in various factors within the system. In particular, the combination of the TOPSIS method with a neutrosophic environment can provide an innovative means to evaluate and prioritize strategies, considering multiple criteria and adapting to the complexities of the sector.

In this context, the present study aims to determine and prioritize effective strategies to increase productivity in an Ecuadorian restaurant chain by applying a multicriteria decision-making approach within a neutrosophic framework. This objective responds to the need to provide practical tools that allow managers to face current challenges with a more comprehensive and flexible perspective. The research emphasizes the importance of integrating advanced analysis methods to address the multidimensional nature of productivity problems, offering a significant contribution to the field of strategic management.

The purpose of this work lies not only in identifying key strategies but also in demonstrating how advancements in neutrosophic theory can enrich decision-making processes in complex business environments. Through this approach, the goal is not only to improve the productivity of restaurant chains but also to establish a replicable model in other industries, promoting more effective and adaptive management. The relevance of this study transcends the academic realm, offering practical solutions that contribute to the sustainable development of the restaurant sector in Ecuador.

2 Preliminaries

2.1 Definition of the Bipolar Neutrosophic Set

According to [4], C is considered a nonempty set, and a bipolar neutrosophic set (BNS) $\tilde{B}$ in C is defined as: [12]

$$\tilde{B} = \{c, \langle T_{\tilde{B}}^{+}(c), I_{\tilde{B}}^{+}(c), F_{\tilde{B}}^{+}(c), T_{\tilde{B}}^{-}(c), I_{\tilde{B}}^{-}(c), F_{\tilde{B}}^{-}(c)\rangle \big| c \in C\},$$

where the functions $T_{\tilde{B}}^{+}(c), I_{\tilde{B}}^{+}(c), F_{\tilde{B}}^{+}(c): C \to [0,1]$ represent positive degrees of membership of truth, indeterminacy, and falsity, respectively. On the other hand, the functions $T_{\tilde{B}}^{-}(c), I_{\tilde{B}}^{-}(c), F_{\tilde{B}}^{-}(c): C \to [-1,0]$ describe the corresponding degrees for the negative domain. This model allows to simultaneously capture the positive and negative characteristics of the elements in complex situations.

2.2 Bipolar TOPSIS method

The TOPSIS method, extended to the bipolar neutrosophic environment, constitutes a robust tool for multicriteria decision-making. The key steps are described below:

1. Let A be a set of alternatives $S = \{S_1, S_2, \cdots, S_m\}$ evaluated against a set of attributes $T = \{T_1, T_2, \cdots, T_n\}$. Let $W = \left[w_{1w_2} \cdots w_n\right]^T$ be a vector of weights, where $W = \left[w_{1w_2} \cdots w_n\right]^T$ and $\sum j = 1nwj = 1$. Each alternative Si, (i=1,2,⋯,m) is scored in terms of the attributes Tj, (j=1,2,⋯,n) using bipolar neutrosophic sets (BNSs). The steps of the bipolar neutrosophic TOPSIS method are described as follows [13]:

1. Construction of the Decision Matrix

Each value of the alternative is estimated with respect to n criteria. The value of each alternative under each criterion is provided in the form of BNSs organized in a decision matrix[14]:

$$K = [k_{ij}]_{m\times n} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix}$$

where $k_{ij} =< T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} >$. The values satisfy the constraints $T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+} \in [0,1]$, $T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} \in [-1,0]$, and $0 \leq T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+}, T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} \leq 6$, where i =1,2,3,…,m and j=1,2,3,…,n

2. Calculation of attribute weights

In the absence of predetermined weights, the deviation maximization method is used to determine the w_j values, defined as [15, 16]:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|\right)^2}}, \tag{1}$$

followed by normalization:

$$w_j^{*} = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|}{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m}|k_{ij}-k_{lj}|\right)}. \tag{2}$$

3. Weighted decision matrix

The weighted matrix is constructed by multiplying the calculated weights by each entry in the decision matrix [16, 17]:

$$K * W = [k_{ij}^{w_j}]_{m\times n} = \begin{bmatrix} k_{11}^{w_1} & k_{12}^{w_2} & \cdots & k_{1n}^{w_n} \\ k_{21}^{w_1} & k_{22}^{w_2} & \cdots & k_{2n}^{w_n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ k_{m1}^{w_1} & k_{m2}^{w_2} & \cdots & k_{mn}^{w_n} \end{bmatrix}$$

where each weighted element $k_{ij}^{w_j}$ is defined by transformations that adjust the membership degrees of truth, indeterminacy, and falsity for both positive and negative domains, as shown in equation (3)

$$\begin{gathered} k_{ij}^{w_j} =< T_{ij}^{w_j+}, I_{ij}^{w_j+}, F_{ij}^{w_j+}, T_{ij}^{w_j-}, I_{ij}^{w_j-}, F_{ij}^{w_j-} > \\ =< 1-(1-T_{ij}^{+})^{w_j}, (I_{ij}^{+})^{w_j}, (F_{ij}^{+})^{w_j}, -(-T_{ij}^{-})^{w_j}, -(-I_{ij}^{-})^{w_j}, -(1-(1-(-F_{ij}^{-}))^{w_j}) >, \end{gathered} \tag{3}$$

4. Determination of ideal solutions

In practical scenarios, attributes are classified as either benefit or cost types. For both cases, the positive ideal solutions (BNRPIS) and negative ideal solutions (BNRNIS) are determined by considering the bipolar neutrosophic characteristics [18,19, 20].

$$\begin{aligned} BNRPIS &= (\langle {}^{+}T_1^{w_1+}, {}^{+}I_1^{w_1+}, {}^{+}F_1^{w_1+}, {}^{+}T_1^{w_1-}, {}^{+}I_1^{w_1-}, {}^{+}F_1^{w_1-}\rangle, \langle {}^{+}T_2^{w_2+}, {}^{+}I_2^{w_2+}, {}^{+}F_2^{w_2+}, {}^{+}T_2^{w_2-}, \\ &\quad {}^{+}I_2^{w_2-}, {}^{+}F_2^{w_2-}\rangle, \ldots, \langle {}^{+}T_n^{w_n+}, {}^{+}I_n^{w_n+}, {}^{+}F_n^{w_n+}, {}^{+}T_n^{w_n-}, {}^{+}I_n^{w_n-}, {}^{+}F_n^{w_n-}\rangle), \\ BNRNIS &= (\langle {}^{-}T_1^{w_1+}, {}^{-}I_1^{w_1+}, {}^{-}F_1^{w_1+}, {}^{-}T_1^{w_1-}, {}^{-}I_1^{w_1-}, {}^{-}F_1^{w_1-}\rangle, \langle {}^{-}T_2^{w_2+}, {}^{-}I_2^{w_2+}, {}^{-}F_2^{w_2+}, {}^{-}T_2^{w_2-}, \\ &\quad {}^{-}I_2^{w_2-}, {}^{-}F_2^{w_2-}\rangle, \ldots, \langle {}^{-}T_n^{w_n+}, {}^{-}I_n^{w_n+}, {}^{-}F_n^{w_n+}, {}^{-}T_n^{w_n-}, {}^{-}I_n^{w_n-}, {}^{-}F_n^{w_n-}\rangle), \end{aligned} \tag{4}$$

Thus, for benefit type criteria, j=1, 2, …,n

$$\begin{aligned}\langle {}^{+}T_j^{w_j+},{}^{+}I_j^{w_j+},{}^{+}F_j^{w_j+},{}^{+}T_j^{w_j-},{}^{+}I_j^{w_j-},{}^{+}F_j^{w_j-}\rangle &= \langle max(T_{ij}^{w_j+}), min(I_{ij}^{w_j+}), min(F_{ij}^{w_j+}),\\ &\quad min(T_{ij}^{w_j-}), max(I_{ij}^{w_j-}), max(F_{ij}^{w_j-})\rangle,\\ \langle {}^{-}T_j^{w_j+},{}^{-}I_j^{w_j+},{}^{-}F_j^{w_j+},{}^{-}T_j^{w_j-},{}^{-}I_j^{w_j-},{}^{-}F_j^{w_j-}\rangle &= \langle min(T_{ij}^{w_j+}), max(I_{ij}^{w_j+}), max(F_{ij}^{w_j+}),\\ &\quad max(T_{ij}^{w_j-}), min(I_{ij}^{w_j-}), min(F_{ij}^{w_j-})\rangle.\end{aligned} \tag{5}$$

Similarly, for cost type criteria, j=1, 2, …,n

$$\begin{aligned}\langle {}^{+}T_j^{w_j+},{}^{+}I_j^{w_j+},{}^{+}F_j^{w_j+},{}^{+}T_j^{w_j-},{}^{+}I_j^{w_j-},{}^{+}F_j^{w_j-}\rangle &= \langle min(T_{ij}^{w_j+}), max(I_{ij}^{w_j+}), max(F_{ij}^{w_j+}),\\ &\quad max(T_{ij}^{w_j-}), min(I_{ij}^{w_j-}), min(F_{ij}^{w_j-})\rangle,\\ \langle {}^{-}T_j^{w_j+},{}^{-}I_j^{w_j+},{}^{-}F_j^{w_j+},{}^{-}T_j^{w_j-},{}^{-}I_j^{w_j-},{}^{-}F_j^{w_j-}\rangle &= \langle max(T_{ij}^{w_j+}), min(I_{ij}^{w_j+}), min(F_{ij}^{w_j+}),\\ &\quad min(T_{ij}^{w_j-}), max(I_{ij}^{w_j-}), max(F_{ij}^{w_j-})\rangle.\end{aligned} \tag{6}$$

5. Calculation of the normalized Euclidean distance with respect to BNRPIS and BNRNIS

The normalized Euclidean distance for each alternative represented by the set $\langle T_{ij}^{w_j+},I_{ij}^{w_j+},F_{ij}^{w_j+},T_{ij}^{w_j-},I_{ij}^{w_j-},F_{ij}^{w_j-}\rangle$ with respect to BNRPIS, denoted as $\langle {}^{+}T_j^{w_j+},{}^{+}I_j^{w_j+},{}^{+}F_j^{w_j+},{}^{+}T_j^{w_j-},{}^{+}I_j^{w_j-},{}^{+}F_j^{w_j-}\rangle$ is calculated as follows[21, 22]:

$$d_N(S_i,BNRPIS)=\sqrt{\frac{1}{6n}\sum_{j=1}^{n}\left\{\begin{matrix}(T_{ij}^{w_j+}-{}^{+}T_j^{w_j+})^2+(I_{ij}^{w_j+}-{}^{+}I_j^{w_j+})^2+(F_{ij}^{w_j+}-{}^{+}F_j^{w_j+})^2+\\ (T_{ij}^{w_j-}-{}^{+}T_j^{w_j-})^2+(I_{ij}^{w_j-}-{}^{+}I_j^{w_j-})^2+(F_{ij}^{w_j-}-{}^{+}F_j^{w_j-})^2\end{matrix}\right\}}, \tag{7}$$

Similarly, the normalized Euclidean distance of each alternative from the bipolar neutrosophic negative relative ideal solution $\langle T_{ij}^{w_j+},I_{ij}^{w_j+},F_{ij}^{w_j+},T_{ij}^{w_j-},I_{ij}^{w_j-},F_{ij}^{w_j-}\rangle$ is determined by the formula:

$$d_N(S_i,BNRNIS)=\sqrt{\frac{1}{6n}\sum_{j=1}^{n}\left\{\begin{matrix}(T_{ij}^{w_j+}-{}^{-}T_j^{w_j+})^2+(I_{ij}^{w_j+}-{}^{-}I_j^{w_j+})^2+(F_{ij}^{w_j+}-{}^{-}F_j^{w_j+})^2+\\ (T_{ij}^{w_j-}-{}^{-}T_j^{w_j-})^2+(I_{ij}^{w_j-}-{}^{-}I_j^{w_j-})^2+(F_{ij}^{w_j-}-{}^{-}F_j^{w_j-})^2\end{matrix}\right\}} \tag{8}$$

6. Calculation of the revised degree of proximity

The revised degree of proximity of each alternative to the BNRPIS, represented as $\rho(S_i)$, is calculated using the following formula[23]:

$$\rho(S_i)=\frac{d_N(S_i,BNRNIS)}{max\{d_N(S_i,BNRNIS)\}}-\frac{d_N(S_i,BNRPIS)}{min\{d_N(S_i,BNRPIS)\}},\ i=1,2,\dots,m. \tag{9}$$

7. Determination of the lower relationship

Based on the reviewed degrees of proximity, the lower ratio of each alternative $IR(i)$ is calculated as follows[23]:

$$IR(i)=\frac{\rho(S_i)}{\min\limits_{1\le i\le m}(\rho(S_i))} \tag{10}$$

It is evident that every value of $IR(i)$ lies in the closed unit interval [0,1].

8. Classification of alternatives

Finally, the alternatives are ordered according to the ascending values of the relationship $IR(i)$. The optimal alternative is selected as the one with the lowest choice value in this order.

3 Results

Based on the historical analysis carried out, a detailed evaluation of productivity was conducted in relation to key variables such as employee turnover, remuneration, inflation, and unemployment in four branches of a restaurant chain in Ecuador. The results highlighted the urgency of implementing strategies that address employee turnover and optimize remuneration management in order to improve productivity in the analyzed work centers.

In this context, a set of strategies was proposed to improve productivity and efficiency indicators in the branches studied. The proposed strategies include: (1) restructuring salary policies; (2) training and employee retention programs; (3) automation of operational processes to reduce reliance on human capital and improve efficiency; (4) adjustment of product prices; (5) internal communication campaigns; (6) implementation of economic incentives linked to individual and group productivity; (7) implementation of performance management systems; and (8) introduction of flexible schedules that promote work-life balance. See Figure 1.

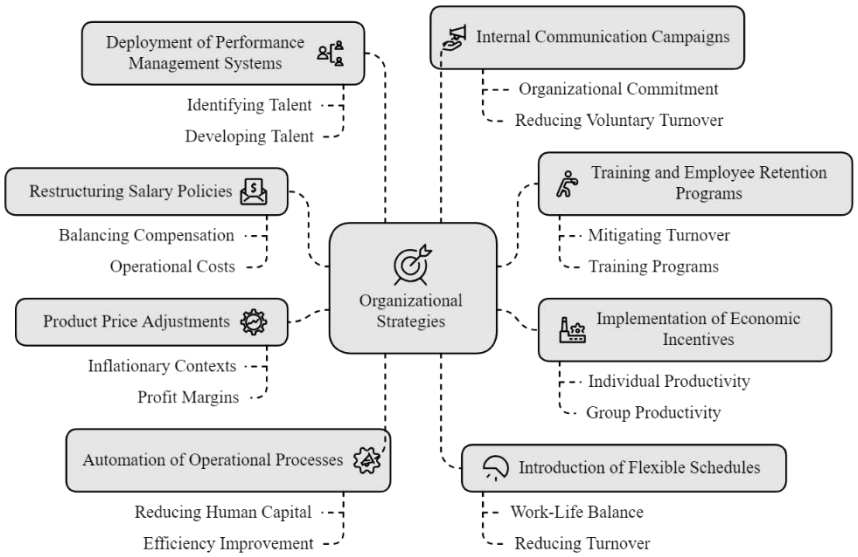

Figure 1: Strategies to be evaluated proposed by experts

To assess the effectiveness of these strategies, a series of criteria were selected, based on expert opinions, which were considered essential for the current context of the company. The evaluated criteria were: impact on productivity (C1), implementation costs (C2), operational feasibility (C3), and long-term sustainability (C4), to identify the most effective and feasible strategies for the specific business context. For each criterion, a specific level of importance was considered. For this study, the evaluation criteria weight vector was $w_j = [0.3; 0.2; 0.25; 0.25]$.

The strategies were assessed by the experts considering each of the evaluation criteria. The resulting initial evaluation matrix is presented in Table 1.

Table 1: Initial decision matrix

	C1	C2	C3	C4
S1	(0.8, 0.4, 0.7, -0.6, -0.4, -0.4)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.2, 0.7, 0.5, -0.4, -0.4, -0.3)	(0.4, 0.6, 0.5, -0.3, -0.7, -0.4)
S2	(0.3, 0.6, 0.1, -0.5, -0.7, -0.5)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.4, 0.2, 0.5, -0.6, -0.3, -0.1)	(0.2, 0.7, 0.5, -0.5, -0.3, -0.2)
S3	(0.3, 0.5, 0.2, -0.4, -0.3, -0.7)	(0.4, 0.5, 0.2, -0.3, -0.8, -0.5)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.3, 0.7, 0.6, -0.5, -0.5, -0.4)
S4	(0.6, 0.7, 0.5, -0.2, -0.1, -0.3)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.6, 0.3, 0.6, -0.1, -0.4, -0.2)	(0.8, 0.3, 0.2, -0.1, -0.3, -0.1)
S5	(0.4, 0.5, 0.2, -0.3, -0.8, -0.5)	(0.9, 0.5, 0.7, -0.3, -0.4, -0.3)	(0.3, 0.7, 0.6, -0.5, -0.5, -0.4)	(0.8, 0.4, 0.6, -0.1, -0.3, -0.4)

	C1	C2	C3	C4
S6	(0.5, 0.3, 0.3, -0.7, -0.2, -0.4)	(0.8, 0.4, 0.6, -0.1, -0.3, -0.4)	(0.6, 0.3, 0.6, -0.1, -0.4, -0.2)	(0.4, 0.2, 0.5, -0.6, -0.4, -0.4)
S7	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.4, 0.2, 0.5, -0.6, -0.4, -0.4)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)
S8	(0.4, 0.6, 0.5, -0.3, -0.7, -0.4)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)	(0.4, 0.7, 0.5, -0.2, -0.1, -0.3)	(0.2, 0.6, 0.1, -0.5, -0.3, -0.7)

From this initial matrix, a weighted matrix can be obtained, which incorporates the weights initially assigned to the criteria. Using equation (3), the following is obtained:

Table 2: Weighted decision matrix

	C1	C2	C3	C4
S1	(0.383, 0.76, 0.899, -0.858, -0.76, -0.142)	(0.044, 0.903, 0.631, -0.871, -0.786, -0.214)	(0.054, 0.915, 0.841, -0.795, -0.795, -0.085)	(0.12, 0.88, 0.841, -0.74, -0.915, -0.12)
S2	(0.101, 0.858, 0.501, -0.812, -0.899, -0.188)	(0.044, 0.903, 0.631, -0.871, -0.786, -0.214)	(0.12, 0.669, 0.841, -0.88, -0.74, -0.026)	(0.054, 0.915, 0.841, -0.841, -0.74, -0.054)
S3	(0.101, 0.812, 0.617, -0.76, -0.697, -0.303)	(0.097, 0.871, 0.725, -0.786, -0.956, -0.129)	(0.054, 0.88, 0.562, -0.841, -0.74, -0.26)	(0.085, 0.915, 0.88, -0.841, -0.841, -0.12)
S4	(0.24, 0.899, 0.812, -0.617, -0.501, -0.101)	(0.044, 0.903, 0.631, -0.871, -0.786, -0.214)	(0.205, 0.74, 0.88, -0.562, -0.795, -0.054)	(0.331, 0.74, 0.669, -0.562, -0.74, -0.026)
S5	(0.142, 0.812, 0.617, -0.697, -0.935, -0.188)	(0.369, 0.871, 0.931, -0.786, -0.833, -0.069)	(0.085, 0.915, 0.88, -0.841, -0.841, -0.12)	(0.331, 0.795, 0.88, -0.562, -0.74, -0.12)
S6	(0.188, 0.697, 0.697, -0.899, -0.617, -0.142)	(0.275, 0.833, 0.903, -0.631, -0.786, -0.097)	(0.205, 0.74, 0.88, -0.562, -0.795, -0.054)	(0.12, 0.669, 0.841, -0.88, -0.795, -0.12)
S7	(0.065, 0.858, 0.501, -0.812, -0.697, -0.303)	(0.097, 0.725, 0.871, -0.903, -0.833, -0.097)	(0.054, 0.88, 0.562, -0.841, -0.74, -0.26)	(0.054, 0.88, 0.562, -0.841, -0.74, -0.26)
S8	(0.142, 0.858, 0.812, -0.697, -0.899, -0.142)	(0.044, 0.903, 0.631, -0.871, -0.786, -0.214)	(0.12, 0.915, 0.841, -0.669, -0.562, -0.085)	(0.054, 0.88, 0.562, -0.841, -0.74, -0.26)

This matrix serves as the basis for obtaining the BNRPIS and BNRNIS, from which equations (7) - (10) are applied to obtain the following table:

Table 3: Strategies evaluated. Relationship index calculated for each alternative.

Proposed strategies	**ϱ**	$\boldsymbol{IR(i)}$
(1) Restructuring salary policies;	-0.381	0.74733
(2) Staff training and retention programs;	-0.178	0.34866
(3) Automation of operational processes;	-0.195	0.38344
(4) Adjustment of product prices;	-0.179	0.35014
(5) Internal communication campaigns	-0.510	1
(6) Implementation of economic incentives	-0.012	0.02307
(7) Implementation of performance management systems;	-0.080	0.15631
(8) Introduction of flexible schedules;	-0.425	0.83379

The results obtained showed that the strategy of implementing economic incentives had the lowest value of the lower relationship, indicating that this was the alternative closest to the positive ideal and, therefore, the most prioritized for improving productivity in the restaurant chain. This was followed by the strategies of implementing performance management systems (IR = 0.15631) and training and employee retention programs (IR = 0.34866), which also demonstrated a high proximity to the positive ideal.

In contrast, strategies such as internal communication campaigns and the introduction of flexible schedules had the highest IR values, positioning them as the least effective under the evaluated criteria. These results allow for the prioritization of resource allocation towards strategies with lower IR values, maximizing their impact on productivity.

4 Discussion

The implementation of the TOPSIS method in its neutrosophic variant was fundamental in addressing the inherent complexity of prioritizing strategies aimed at increasing productivity. This methodology allowed for the evaluation of alternatives by considering not only the direct relationships between criteria but also the uncertainties and contradictions typical of the business context. The ability to integrate neutrosophic values expanded the scope of traditional analysis by incorporating degrees of truth, falsehood, and indeterminacy, enriching the interpretation of results and increasing their accuracy.

The use of this variant of TOPSIS offered clear advantages. First, it allowed for the handling of imprecise and ambiguous data with a robust approach, overcoming the limitations of classical methods. Additionally, it provided a flexible framework for comparing strategies across multiple criteria, offering a more comprehensive view of the potential impact of each alternative. In this case, it was possible to prioritize those strategies with greater proximity to the positive ideal, thus ensuring informed decisions aimed at achieving specific organizational goals.

The methodology employed was not only applicable to the current context but also demonstrated its potential for use in other sectors. In industries with similar challenges, where evaluation criteria are diverse and operating conditions are subject to constant variations, the neutrosophic approach could facilitate strategic decision-making with a high level of confidence.

In conclusion, the neutrosophic TOPSIS method provided an analytical framework that transcended the limits of subjectivity, becoming a versatile tool for multicriteria evaluation. Its ability to adapt to different environments and disciplines positions it as an indispensable methodology for strategic analysis in high-complexity scenarios.

5 Conclusion

The present study focused on determining and prioritizing effective strategies to increase productivity in a restaurant chain in Ecuador, applying a multicriteria decision-making approach within a neutrosophic framework. A comprehensive analysis was conducted that allowed for the identification of the most relevant strategies by considering key criteria. The application of the TOPSIS method in its neutrosophic variant and the use of bipolar neutrosophic sets allowed handling the uncertainty inherent in the business context, effectively modeling the interaction between criteria and alternatives. This contributed to prioritizing strategies such as the implementation of economic incentives and performance management systems, which showed greater proximity to the positive ideal. In this way, the study provided a robust tool for strategically allocating resources, maximizing the impact on productivity, and adapting to the complexities of the studied environment.

This study provided empirical evidence of the potential of multicriteria decision-making methods with neutrosophic logic in business management. Additionally, it established a replicable framework that can be applied to other industries facing similar challenges. Future research could focus on expanding the use of this methodology to dynamic contexts or exploring its combination with advanced technological approaches, promoting more adaptive and sustainable decision-making.

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Assessment of Academic Integrity in University Students Using a Hybrid Fuzzy-Neutrosophic Model under Uncertainty

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Abstract: This study presents a model based on a fuzzy-neutrosophic approach to evaluate the academic integrity of university students in the context of uncertainty. The central issue of the research lies in the abrupt transition from in-person education to virtual learning caused by the pandemic, which increased fraudulent behaviors in teaching and learning environments, making it difficult to determine the level of academic integrity among students in higher education institutions. To address this issue, variables related to dishonesty—motivation, justification, and opportunities—were analyzed. Three evaluation scenarios were conducted, yielding positive results. The proposed approach, based on fuzzy logic and neutrosophy, effectively measured the academic integrity of students, providing a valuable tool for decision-making by experts in the educational field.

Keywords: Fuzzy logic, neutrosophic, academic dishonesty, uncertainty, academic integrity, COVID-19 pandemic, university students.

1. Introduction

The COVID-19 virus rapidly escalated into a pandemic with devastating effects on society. Almost overnight, schools and universities worldwide closed their doors, impacting 1.57 billion students across 191 countries. The abrupt and sweeping changes to daily life created an unprecedented situation for all organizations, including educational institutions, which were forced to suspend academic activities [1]. This suspension marked the beginning of a shift toward virtual learning, placing significant pressure on students to adapt to the new teaching and learning environment [2]. This transition introduced unique uncertainties and challenges, particularly in assessing students' academic integrity. To address this complex landscape, concepts such as fuzzy logic and neutrosophic models have emerged as innovative approaches capable of handling the ambiguity and imprecision inherent in uncertain situations. These methodologies offer a more accurate and context-sensitive framework for evaluating students' integrity within this evolving educational paradigm.

2. Preliminaries

Fuzzy logic is a reasoning approach that applies multiple truth or confidence values to restrictive categories during problem-solving. [10]. A set is a collection of objects that can be classified based on

their common characteristics. It is defined in two ways: by extension ({a, e, i, o, u}) or by comprehension.

A boolean set A is a map of a reference set S to the set {0, 1}, $A{:}S\rightarrow$ {0,1}, and is defined with a characteristic function.

$$\mu_A = \begin{cases} 1\ if\ x\ \in A \\ 0\ if\ x\ \notin A \end{cases} \quad (1)$$

Fuzzy sets give a quantitative value to each element, which represents the degree of membership to the set [3]. A fuzzy set A is an application of a reference set S in the interval [0, 1]

A: $S\rightarrow$ [0,1], and is defined using a membership function: $0 \leq \mu A\ (x) \leq 1$.

The closer the value is to 0, the less we can ensure the membership of an element to a set. On the contrary, the closer the value is to 1, the more we can ensure the membership of the element to the set [4]. It can be represented as a set of ordered pairs of a generic element x, x ∈A and its degree of membership $\mu A(x)$:

$$A = \{(x, \mu^A(x)), \mu^A(x) \in [0,1]\} \quad (2)$$

Working with fuzzy logic can be represented using linguistic variables to enhance the interpretability of data. Linguistic variables are those from natural language, characterized by fuzzy sets defined within the universe of discourse where they are established [3]. To define a set of linguistic terms, the granularity of the uncertainty for the set of linguistic labels to be used must first be determined [4]. The granularity of uncertainty refers to the cardinal representation of the set of linguistic labels employed to convey information. The degree of membership of an element M(x) to a fuzzy set will be determined by membership functions. The membership functions used for the fuzzy system are as follows.

Triangular Function:

This function is defined by its lower limit a, upper limit b, and modal value mmm, where a < m < b [5].

$$\mu(x) = \begin{cases} 0 & \\ \dfrac{x-a}{m-a} & si\ x \leq a \\ \dfrac{b-x}{b-m} & si\ a < x \leq m \\ 0 & si\ m < x \leq b \\ & si\ x \geq b \end{cases} \quad (3)$$

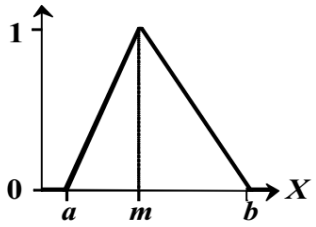


Figure 1. Triangular function

Trapezoidal Function:

This function is defined by its lower limit (**a)** and upper limit (**d)**, along with the intermediate limits (**b)** and (**c)**, which correspond to the lower and upper bounds of the plateau, respectively [5]

$$\mu(x) = \begin{cases} 0 & si\ x \leq a \\ \dfrac{x-a}{b-a} & si\ a < x \leq b \\ 1 & si\ b < x \leq c \\ \dfrac{d-x}{b-c} & si\ c < x \leq d \\ 0 & si\ x > d \end{cases} \quad \textbf{(4)}$$

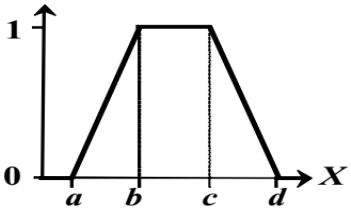


Figure 2. trapezoidal function

On fuzzy sets, logical operations such as intersection (conjunction), union (disjunction), and complement (negation) can be performed. These operations can be carried out using **T-norms and S-Norms**.

T-norms specify the conditions that operations must meet to intersect sets.

S-Norms define the conditions for performing unions [3].

Intersections occur in conjunctions and contributions, while unions are present in disjunctions and the global outcome. These operations are performed in expert systems to calculate the certainty factors of production rules. According to **T-Norms and S-Norms**, these operations satisfy the following conditions:

A T-norm operation satisfies the following properties:

Commutative	$T(x,y) = T(y,x)$	(5)
Associative	$T(x,T(y,z)=T(T(x,y),Z).$	(6)
Monotonicity	$T(x,y)>T(x,y)$ si $x\geq x' \cap y\geq y'$	(7)
Boundary Condition	$T(x,1)=x$	(8)

In a fuzzy system, linguistic variables, their corresponding linguistic labels, fuzzy sets, membership functions, inference rules, and certainty factors associated with the system's rules are key components. The system's input consists of qualitative values assigned to the linguistic variables.

These input values are transformed into membership values for fuzzy labels, which correspond to certainty factors. This process is called fuzzification, as it converts numerical values into fuzzy ones.

Once the values are fuzzified, production rules are applied using the certainty values. This step is known as fuzzy inference, during which T-Norms and S-Norms are employed to handle intersections and unions.

After obtaining the membership results for the output fuzzy sets, a numerical value must be derived from these results. This process is called defuzzification, and it is commonly performed using the centroid method, the most widely utilized approach in fuzzy systems.

2.1 Fuzzy Logic Method for Measuring University Students' Integrity

To measure the academic integrity of university students, the inference process based on the **Centroid or Center of Gravity (COG)** is applied in the numerical defuzzification of the value of formative variables. COG-based inference ensures that no coefficients need to be adjusted; it only requires knowledge of the membership functions for each defined label [3]. To infer using COG, the process begins with the membership values for each of the labels associated with the variable to be defuzzified. For each fuzzy output variable, the maximum value of the membership function for each label is truncated based on the value obtained during the inference process [4].

$$GOC = \int \frac{x * \mu(x)dx}{\mu(x)dx} \tag{9}$$

Where μ(x) represents the degree of membership of the element x, which will take values within the universe of discourse, using a defined step size. The smaller the step size, the more accurate the COG result will be.

To measure the academic integrity of university students during the COVID-19 pandemic, a fuzzy system is employed. The input variables are motivation, justification, and opportunity within an academic dishonesty framework, while the output variable is academic integrity. [9]. Each of these input and output variables is composed of the linguistic labels low, medium, and high.

- For the low label, a trapezoidal function is used, with its points on the x-axis at (0,0,0.2,0.5).
- For the medium label, a triangular function is used, with its points on the x-axis at (0.2,0.5,0.8).
- For the high label, a trapezoidal function is used, with its points on the x-axis at (0.5,0.8,1,1).

3 Neutrosophic Method for Measuring the Academic Integrity of University Students

Information aggregation, or information fusion, is the process of combining different data to produce a single output. Aggregation operators are a type of mathematical function used to merge information. They combine **n** values within a domain **D** and return a single value in the same domain.

Table 1. Scale of Linguistic Terms. Source: [6]

Linguistic Term	SVN Numbers
Extremely Good (EG)	(1,0,0)
Very Very Good (VVG)	(0.9, 0.1, 0.1)
Very Good (VG)	(0.8,0,15,0.20)
Good (G)	(0.70,0.25,0.30)
Moderately Good (MG)	(0.60,0.35,0.40)
Neutral (N)	(0.50,0.50,0.50)
Moderately Bad (MB)	(0.40,0.65,0.60)
Bad (B)	(0.30,0.75,0.70)
Very Bad (VB)	(0.20,0.85,0.80)
Very Very Bad (VVB)	(0.10,0.90,0.90)
Extremely Bad (EB)	(0,1,1)

Linguistic terms and indeterminacy are represented using SVN numbers and based on information aggregation [6].

Establishing the evaluation framework:
Criteria and alternatives to be evaluated are selected to prioritize the latter. The framework is defined as follows [7]:

- C= $\{c_1, c_2 \ldots, c_n\}$ with $n \geq 2$, a set of criteria
- E= $\{e_1, e_2 \ldots, e_k\}$ with $k \geq 1$, a set of experts
- X= $\{x_1, x_2 \ldots, x_m\}$ with $m \geq 2$, a finite set of alternatives

Information collection:
Information about the preferences of decision-makers is gathered. The utility vector [11] is represented as follows [6]:

$\mathbf{P_j = \{p_{j1}, p_{j2}, \ldots, p_{jk}\}}$, where $\mathbf{p_{jk}}$ is the preference regarding criterion c_k for alternative $\mathbf{x_j}$.
Evaluations will be given using SVN numbers.[11]

To rank alternatives, a scoring function is used:

$$s(Vj)=2+Tj-Fj-Ij$$

Additionally, the precision function is defined as follows[12]:

$$a(Vj)=Tj-Fj$$

Then:

If $s(Vj) < s(Vi)$, Then Vj is less than Vi, denoted as $Vj < Vi$
If $s(Vj) = s(Vi)$
If $a(Vj) < a(Vi)$, Then Vj is less than Vi, denoted as $Vj < Vi$

If $a(Vj)= a(Vi)$, Then Vj and Vi are equal, denoted as $Vj= Vi$

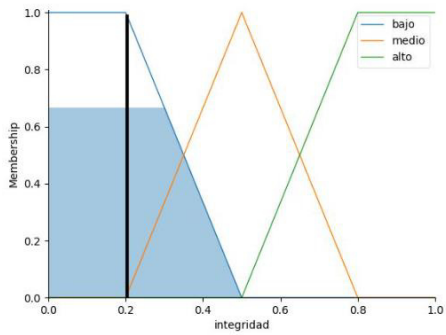
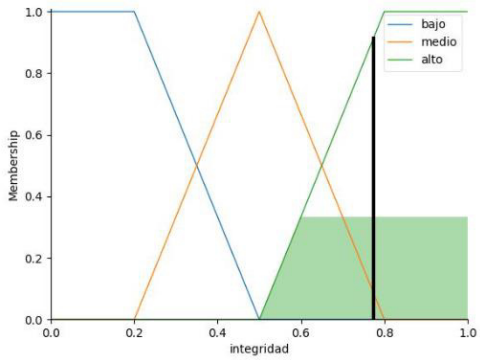
4. Results.

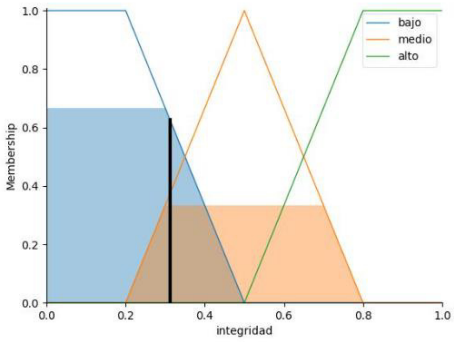
4.1. Fuzzy Logic Approach to Measure the Academic Integrity of University Students

Three scenarios are defined (scenario 1, scenario 2, and scenario 3), considering the input values of the linguistic variables, as shown in Table 1:

Table 2. Scenarios for Simulation

Scenarios	Motivation	Opportunity	Opportunity
Scenario 1	0.9	0.8	0.7
Scenario 2	0.2	0.1	0.4
Scenario3	0.7	0.5	0.7

Variable	Scientific Term	Analysis
Scenario 1		The academic integrity of the university student in scenario 1 is 20.417%, considered to be at a low level.
Scenario 2		The academic integrity of the university student in scenario 2 is 77.407%, considered to be at a high level.

Scenario 3		The academic integrity of the university student in scenario 3 is 31.212%, considered as low to medium level.

3.2 Neutrosophic Approach to Measure the Academic Integrity of University Students

Once the fuzzy measurements are obtained, a conversion is performed from the fuzzy value to the neutrosophic value using the linguistic terms of neutrosophy, resulting in Table 3.

Table 3. Scenarios According to SVN Numbers and Their Respective Linguistic Terms

Scenarios	**SVN Number**	**Linguistic Term**
Scenario 1	(0.20,0.85,0.80)	Very Bad (VB)
Scenario 2	(0.70,0.25,0.30)	Medium (M)
Sscenario 3	(0.30,0.75,0.70)	Bad (B)

The vector used to measure the academic integrity of university students during the COVID-19 pandemic has the following weights: **W= (0.57, 0.26, 0.19).** Subsequently, the opinions of the decision-makers are aggregated using the SVNWA aggregation operator. The results of the neutrosophic measurements according to the previously defined scenarios are shown in Table 4.

Table 4. Results of the Information Aggregation Method.

Scenario	**Aggregation**	**Score**	**Ranking**
Scenario 1	(0.12,0.91,0.88)	0.33	3
Scenario 2	(0.32,0.68,0.68)	0.96	1
Scenario 3	(0.18,0.85,0.82)	0.51	2

According to the scores, the priority will be as follows: **Scenario 2** > **Scenario 3** > **Scenario 1**. As a result of the aggregation and scoring, we find that the academic integrity of Scenario 1 ranks 3rd, the last position. Therefore, the measurement of the university student in this scenario was significantly impacted during the COVID-19 pandemic. It is up to the experts to decide what academic actions to take.

4. Conclusions (authors also should add some future directions points related to her/his research)

The fuzzy-neutrosophic approach under uncertainty, implemented to measure the academic integrity of university students during the COVID-19 pandemic, is based on the following: utilizing the factors of opportunity, justification, and motivation within an academic dishonesty context, and conducting fuzzy and neutrosophic measurements according to the established scenarios.

The theory of fuzzy logic applied to analyze and evaluate the academic integrity of university students during the COVID-19 pandemic enables the acquisition of accurate data compared to other qualitative methods.

Neutrosophic explicitly expresses the truth, uncertainty, and falsehood in the measurements of university students' academic integrity.

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Evaluation of Sustainable Agricultural Practices Using Organic Fertilizers for Soil Recovery in Salache: A Neutrosophic Superhypersoft Set Approach

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Abstract. Modern agriculture faces critical challenges due to soil degradation and the urgent need for sustainable practices to preserve the environment. This study analyzed the impact of organic fertilizers, including vermicompost, compost, manure, and control, on the development of Lupinus mutabilis and Pisum sativum. Different groups were evaluated based on chemical and physical soil properties, plant attributes, and crop cycles using Neutrosophic SuperHyperSoft Sets. The results demonstrated that vermicompost is the most effective option for optimizing soil properties and improving crop yields. In conclusion, the use of vermicompost and other organic sources is proposed as an effective strategy to enhance soil fertility and crop productivity, particularly in degraded lands, while fostering the adoption of more sustainable agricultural practices.

Keywords: Vermicompost, soil fertility, agricultural yield, agricultural sustainability, Neutrosophic SuperHyperSoft Sets.

1 Introduction

Soil desertification due to erosion is a critical environmental challenge that affects the productive capacity of land, particularly in areas with slopes exceeding 40% [1]. This process, driven by inadequate agricultural practices, limits food production and triggers economic and social issues such as poverty, migration, and family disintegration, perpetuating the vulnerability of communities [2]. Therefore, it is urgent to adopt comprehensive measures that combine soil restoration with sustainable development to ensure the well-being of affected populations [3].

In Ecuador, soil erosion is especially critical in areas with slopes greater than 12%, where soil losses range between 5 and 50 tons per hectare per year. Approximately half of the agricultural soils exhibit some degree of degradation, affecting soil fertility and productive capacity [4], which negatively impacts rural communities [5]. Furthermore, the expansion of agricultural frontiers, extensive livestock farming, and unplanned urbanization have increased the rate of degradation, affecting nearly 47% of the country's soils [6].

Hence, this study analyzes the impact of organic fertilizer sources on the development of Lupinus mutabilis and Pisum sativum in the Salache region [7]. The objective is to evaluate their effect on soil fertility properties, crop yields, and the efficiency of the applied treatments. The study proposes the use of tools such as Neutrosophic SuperHyperSoft Sets (NSHSS) to assess multiple attributes in modeling the complexity and uncertainty inherent in sustainable agricultural practices, as well as statistical analysis of the treatments involving organic fertilizers [8], such as vermicompost, to restore eroded soils [9].

2 Preliminaries

2.1 Neutrosophic SuperHyperSoft Sets.

This section serves the purpose of remembering the basic notions of Fuzzy Extension SuperHyperSoft Sets and neutrosophic theory [10].

Definition 1 ([1, 9, 10]**).** Given U is the initial universe set and E is the set of parameters. A pair

(F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set [11] [12].

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A using the following formula:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, if the family F is known, the function μ_A can be reconstructed using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ Soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 2 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *HyperSoft Set* over U.

Definition 3 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([4, 5, 13-15])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}\left(U\big(x(d^0)\big)\right)$ is called a *Fuzzy_Extension SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc [13].

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set N* is characterized by three membership functions [14], which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [15].

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^-0$ and 1^+ belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$ and $F_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Single-Valued Neutrosophic Sets (SVNS) were introduced with the idea of applying neutrosophic

sets for practical purposes. Some operations between SVNSs are expressed below:

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the sum of A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1 + a_2 - a_1a_2, b_1b_2, c_1c_2) \quad (1)$$

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the multiplication of A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1a_2, b_1+b_2 - b_1b_2, c_1 + c_2 - c_1c_2) \quad (2)$$

The product of a positive scalar SVNN, A = (a, b, c) is defined as:

$$A = (1 - (1 - a), b, c) \quad (3)$$

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

3 Results

3.1 Description of the experiment.

The study employed essential tools such as labels, drip tape, an A-frame level, stakes, string, a shovel, a hoe, a rake, and a measuring tape, complemented by native forest plants, an auger, and organic fertilizers (vermicompost, poultry manure, and bovine manure). The experimental material included bench terraces cultivated with Lupinus mutabilis and Pisum sativum. These resources and practices were applied to plots designed to evaluate the recovery of eroded soils and sustainable agricultural yields.

The experimental design was conducted at the Technical University of Cotopaxi following a split-plot scheme in a completely randomized block design with three replications, resulting in 24 experimental treatments. The analyzed factors included terrace management (single crop of Lupinus mutabilis and associated crop with Pisum sativum) and organic nutritional sources (no fertilizer, vermicompost, poultry manure, and bovine manure).

During the trial, three soil samplings were performed to assess changes in initial conditions, after the first crop cycle, and at the end of the second. The indicators measured included germination, plant height, number of branches, flowers, and pods, as well as yield, demonstrating the effectiveness of the applied practices in improving soil fertility and agricultural productivity.

3.2 Modeling with Neutrosophic SuperHyperSoft Sets

The study proposes determining the elements of the study universe for the types of fertilizers and crops to be applied, as outlined in Table 1. Additionally, it establishes the structure of the attribute groups (super-attributes) of the Neutrosophic SuperHyperSoft Sets (see Table 2).

Table 1: Types of fertilizers and crops in the experimental design. Source: Own elaboration.

Fertilizers and crops	Code	Items
Organic sources	FO1	Vermicompost
	FO2	Compost (bovine)
	FO3	Manure (poultry)
Soil texture	S1	Sandy loam (sand (62%), silt (32%) and clay (6%))
Terraces	Z1	Cultivation with Lupinus Mutabilis
	Z2	Growing with Pisum sativum

Table 2: Structure of the attributes of the neutrosophic SuperHyperSoft sets. Source: Own elaboration.

Super-attributes	**Code**	**attributes**	**Sub-attributes**	$(x(d^0))$
Chemical and physical properties (a_i) for the $P(A_i)$	a1	pH	Fertilizer extract: water 1:2.5	SVNN
	a2	Electrical conductivity (EC)	Fertilizer extract: water 1:2.5	
	a3	MO	%	
	a4	Nitrogen (N)	%	
	a5	Phosphorus (P)	%	
	a6	Potassium (K)	%	
	a7	Calcium (Ca)	%	
	a8	Magnesium (Mg)	%	
	a9	Copper (Cu)	%	
	a10	Iron (Fe)	Ppm	
	a11	Manganese (Mn)	Ppm	
	a12	Zinc (Zn)	Ppm	
Plant indicators (b_i) for the $P(B_i)$	b1	Germination	%	
	b2	Number of floors	Average	
	b3	Height	Cm	
	b4	Number of branches	Average	
	b5	Number of flowers	Average	
	b6	Number of Pods	Average	
Crop cycle (c_i) for the $P(C_i)$	c1	First cycle	04/18/2017 to 04/02/2018	
	c2	Second cycle	days	

It is observed in Table 2 that there are 3 Neutrosophic SuperHyperSoft Sets, defined by the attributes a_i, b_i, and c_i. Each super-attribute is composed of the following pairs:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_{12}) \rightarrow \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right)$$

$$G: \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_6) \rightarrow \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right)$$

$$H: \mathcal{P}(C_1) \times \mathcal{P}(C_2) \rightarrow \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right)$$

The following treatments are proposed based on the combinations of each element in the set P(Fi) = {{FO1}, {FO2}, {FO3}}, P(Si) = {{S1}}, and P(Ti) = {{T1}, {T2}}. Thus, a new classification or grouping of elements is sought by considering the fertilizer and the crop type of P(Ti). Another important point is that the soil type has been restricted to sandy loam, based on the experimental conditions of the study. Therefore, the following elements and treatments (T) are proposed over the universe U, as well as each subsection of the study development (see Table 3).

Table 3: Complexity of the set of elements and $P(T_i)$ treatments in the study. Source: Own elaboration.

Subsection	**Description**	**Code**	$\boldsymbol{P(T_i)}$
▪ **3.3** Physical and chemical analysis of treatments.	Without subscription	S1	$\{S_1, A_i\}$
	Vermicompost	FO1	$\{FO1, A_i\}$
▪ **3.4** Yield of the treatments.	Bovine manure	FO2	$\{FO2, A_i\}$
	Poultry manure fertilizer	FO3	$\{FO3, A_i\}$
▪ **3.5** Yield of the treatments in relation to crop development.	Lupinus Mutabilis Cultivation		$\left\{\{\{S1, A_i\}, C_i\}, \{B_i, Z_1\}\right\}$
			$\left\{\{\{FO1, A_i\}, C_i\}, \{B_i, Z_1\}\right\}$
			$\left\{\{\{FO2, A_i\}, C_i\}, \{B_i, Z_1\}\right\}$
			$\left\{\{\{FO3, A_i\}, C_i\}, \{B_i, Z_1\}\right\}$
	Pisum sativum cultivation		$\left\{\{\{S1, A_i\}, C_i\}, \{B_i, Z_2\}\right\}$
			$\left\{\{\{FO1, A_i\}, C_i\}, \{B_i, Z_2\}\right\}$

Subsection	Description	Code	$P(T_i)$
	Crop cultivation of Pisum sativum and Lupinus mutabilis.		$\{\{\{FO2, A_i\}, C_i\}, \{B_i, Z_2\}\}$
			$\{\{\{FO3, A_i\}, C_i\}, \{B_i, Z_2\}\}$
			$\{\{\{S1, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$
			$\{\{\{FO1, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$
			$\{\{\{FO2, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$
			$\{\{\{FO3, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$

3.3 Physical and chemical analysis of treatments

The physicochemical analysis of the organic fertilizers evaluated (vermicompost, bovine manure, and poultry manure) revealed significant differences in their attributes a_i (see Table 4). The vermicompost exhibited a pH of 7.26, lower than that of the other fertilizers, with 7.39 and 8.84 for bovine manure and poultry manure, respectively. In terms of electrical conductivity, bovine manure reached the highest value at 15.56 ms/cm, followed by vermicompost and poultry manure. Organic matter ranged from 18.4% in poultry manure to 20% in vermicompost. Regarding macronutrients, vermicompost stood out in potassium (0.53%) and iron (400 ppm), surpassing the other fertilizers. These results highlight the advantages of vermicompost as a viable option to enhance soil fertility and provide essential nutrients.

Table 4: Results of physical and chemical analysis of organic fertilizers. Source: Own elaboration.

a_i	Unit	Vermicompost	Bovine manure	Poultry manure
pH		7.26	7.39	8.84
EC	ms/cm	10.47	15.56	10,10
MO	%	20,00	20,31	18.40
Total N	%	0.77	1.03	0.90
P	%	0.59	0.27	0.80
K	%	0.53	0.19	0.12
AC	%	0.75	0.65	2.30
Mg	%	0.37	0.43	0.94
Cu	Ppm	8	8	2
Faith	Ppm	400	120	200
Mn	Ppm	40	95	20
Zn	Ppm	18	10	32

3.4 Yield of the treatments

The soil analysis in the control treatment showed an initially high pH (9.59 in M1 and 9.05 in M2), with a slight decrease by the end of the second cycle (9.09 and 8.90, respectively). Electrical conductivity was low, with slight increases in both treatments. The sandy loam texture remained constant, while organic matter increased at the end of the first cycle but decreased by the end of the second, reflecting the dynamics of input and decomposition (see Table 5). Macronutrients such as nitrogen, phosphorus, and potassium decreased over time, attributed to crop uptake. Meanwhile, calcium and magnesium showed significant declines. These results underscore the need for sustainable practices to preserve soil fertility and optimize its management in agricultural systems.

Table 5: Yield of the control treatment from the soil analysis over two crop cycles. Source: Own elaboration.

S1: Control treatment		Start	End of the 1st Cycle	End of the 2nd Cycle	Start	End of the 1st Cycle	End of the 2nd Cycle
Analysis	Unit	Level	Level	Level	Level	Level	Level
pH		AL 9.5	AL 8.87	AL 9.0	Al 9.0	AL 9.02	LAL 8.90
EC.	mmhos/cm	NS 0.2	NS 0.3	NS 0.3	NS 0.1	NS 0.5	NS 0.1
MO	%	M 4.9	A 6.7	M 4.7	B 2.8	A 5.8	M 4.1
TOTAL N	ppm	M 36.7	M 50.2	M 35.6	B 21.3	M 43.3	M 31.1

S1: Control treatment		Start		End of the 1st Cycle		End of the 2nd Cycle		Start		End of the 1st Cycle		End of the 2nd Cycle	
P	ppm	A	89	A	85	A	53	A	103	A	88	A	26
K	meq/100 g	A	0.5	A	1.1	A	1.5	A	1.4	A	0.6	A	0.5
AC	meq/100 g	A	22	A	17	A	16	A	22	A	8	A	10
Mg	meq/100 g	A	1	A	3.7	A	1.7	A	2	A	2.1	A	1.9
Cu	ppm	A	8	M	2	M	1	A	8	M	2	M	1
Mn	ppm	M	5	M	5	B	4	M	5	B	2	B	4
Zn	ppm	M	3	B	1	B	1	M	3	B	1	B	1
Ca/Mg	meq/100 g	A	21	0	5	A	10	A	11	O	4	O	5
Mg/K	meq/100 g	B	2	0	3	B	1	B	1	O	3	O	4
Ca+Mg/K	meq/100 g	A	46	0	18	0	12	O	17	O	16	O	26

The use of vermicompost improved the chemical and physical properties of the soil by reducing pH and stabilizing electrical conductivity. Organic matter increased significantly, enhancing cation exchange capacity and the availability of essential nutrients. These structural improvements favored germination, growth, and crop resilience.

Agricultural yield also increased, reaching 0.98 kg of Lupinus mutabilis and 1.19 kg of Pisum sativum in dry grain at the end of two cycles. Overall, vermicompost emerges as an effective treatment to optimize soil quality and crop productivity (see Table 6).

Table 6: Yield of vermicompost from the soil analysis over two crop cycles. Source: Own elaboration.

FO1: Vermicompost		Start		End of the 1st Cycle		End of the 2nd Cycle		Start		End of the 1st Cycle		End of the 2nd Cycle	
Analysis	Unit	Level		Level		Level		Level		Level		Level	
pH		Al	9.23	AL	9.3	AL	8.9	AL	9.41	AL	8.76	Me AL	8.41
EC.	mmhos/cm	NS	0.42	NS	0.6	NS	0.3	NS	0.28	NS	0.3	NS	0.3
MO	%	M	4.2	A	9.2	M	4.8	B	2.8	A	7.7	M	4.2
TOTAL N	Ppm	M	31.1	A	69.3	M	36.1	B	20.7	M	57.8	M	3 117
P	Ppm	A	137	A	205	A	99	A	84	A	171	A	75
K	meq/100 g	A	1.5	A	1.3	A	1.8	M	2.5	A	1	A	1.9
AC	meq/100 g	A	10	A	14	A	17	A	22	A	12	A	14
Mg	meq/100 g	A	1.5	A	3.3	A	2.4	A	1.5	A	3	A	2.6
Cu	Ppm	A	5	M	2	M	1	A	10	M	2	M	2
Mn	Ppm	M	10	M	5	B	4	M	8	B	3	B	4
Zn	Ppm	M	5	B	1	B	2	M	3	M	4	B	2
Ca/Mg	meq/100 g	A	7	O	4	A	7	A	15	O	4	O	5
Mg/K	meq/100 g	B	1	O	3	B	1	B	1	O	3	B	1
Ca+Mg/K	meq/100 g	B	7	O	14	O	11	B	9	O	15	B	9

The application of poultry manure reduced the soil pH from 9.18 to 8.5 and decreased the electrical conductivity from 0.75 to 0.2 mmhos/cm at the end of the second cycle. It also increased organic matter and improved the availability of macro and micronutrients by optimizing the structure, stability, and fertility of the soil. These changes favored cation exchange capacity and biological development, boosting agricultural productivity (see Table 7).

Table 7: Yield of poultry manure from the soil analysis over two crop cycles. Source: Own elaboration.

FO2: Poultry manure		Start		End of the 1st Cycle		End of the 2nd Cycle		Start		End of the 1st Cycle		End of the 2nd Cycle	
Analysis	Unit	Level		Level		Level		Level		Level		Level	
pH		AL	9.18	AL	8.85	Me AL	8.41	Al	9.03	AL	8.41	Me AL	8.25
EC.	mmhos/cm	NS	0.75	NS	1	NS	0.3	NS	0.52	NS	0.8	NS	0.2
MO	%	M	3.3	A	7.4	M	4.2	B	2.8	A	5.7	M	3.8
TOTAL N	Ppm	B	24.5	M	55.5	M	31.8	B	21.2	M	43.1	B	28.8
P	Ppm	A	50	A	330	A	231	A	94	A	418	A	183
K	meq/100 g	A	1.8	A	1.9	A	1.9	A	3.7	A	1.7	A	1.5
AC	meq/100 g	A	10	A	14	A	19	A	20	A	12	A	14

FO2: Poultry manure		Start		End of the 1st Cycle		End of the 2nd Cycle		Start		End of the 1st Cycle		End of the 2nd Cycle	
Mg	meq/100 g	A	1.1	A	2.7	A	3.9	A	1.6	A	4	A	2.4
Cu	Ppm	A	10	M	2	M	3	A	13	M	2	M	2
Mn	Ppm	M	5	M	5	M	9	M	10	M	10	M	5
Zn	Ppm	M	3	A	7	M	6	M	3	A	11	M	3
Ca/Mg	meq/100 g	TO	9	O	5	O	5	TO	12	O	3	TO	6
Mg/K	meq/100 g	B	1	B	1	B	2	B	0.4	B	2	B	2
Ca+Mg/K	meq/100 g	B	6	B	9	O	13	B	6	O	10	O	11

On the other hand, bovine manure reduced the soil pH from 9.6 to 9.04 and the electrical conductivity from 0.34 to 0.2 mmhos/cm after two cycles, by increasing organic matter and available nutrients. Therefore, this fertilizer improves soil properties, reduces the need for chemical fertilizers, and contributes to a more sustainable and efficient management of agricultural resources (see Table 8).

Table 8: Yield of bovine manure from the soil analysis over two crop cycles. Source: Own elaboration.

FO3: Bovine manure		Start		End of the 1st Cycle		End of the 2nd Cycle		Start		End of the 1st Cycle		End of the 2nd Cycle	
Analysis	Unit	Level		Level		Level		Level		Level		Level	
pH		AL	9.6	AL	9.39	Me AL	8.34	A	9.57	AL	9.44	AL	9.04
EC.	mmhos/cm	NS	0.34	NS	0.6	NS	0.2	AL	0.67	NS	0.7	NS	0.5
MO	%	B	2.9	A	6.1	M	4.2	A	7.8	A	7.4	A	5.2
TOTAL N	Ppm	B	21.4	M	45.7	M	31.4	M	58.7	M	55.5	M	39
P	Ppm	A	83	A	134	A	65	A	99	A	125	A	99
K	meq/100 g	A	0.6	A	1.4	A	2.1	A	1	A	1.7	A	2.2
AC	meq/100 g	A	22	A	13	A	17	A	17	A	13	A	16
Mg	meq/100 g	A	1.1	A	1.9	A	3.4	A	1.3	A	2.1	A	2.1
Cu	Ppm	A	10	M	2	M	1	A	10	M	2	M	3
Mn	Ppm	M	5	B	3	B	3	B	3	B	1	B	4
Zn	Ppm	M	3	B	1	B	1	M	3	B	1	B	1
Ca/Mg	meq/100 g	A	20	A	7	O	5	A	13	A	6	A	7
Mg/K	meq/100 g	B	2	B	1	B	2	B	1	B	1	B	1
Ca+Mg/K	meq/100 g	0	37	O	11	O	10	O	18	B	9	B	8

3.5 Yield of the treatments in relation to crop development.

The use of organic fertilizers significantly influenced the variables evaluated in two crop cycles of Lupinus mutabilis. According to the results obtained, the humus treatment stood out with the highest germination percentage (89%), followed by bovine manure (83.62%), poultry manure (78.98%), and the control treatment (73.39%). On average, the plant height reached 86.81 cm, with 11.63 branches, 21.59 flowers, and 21.75 pods per plant. These values reflect that treatments with organic matter, especially humus, improved yield compared to the untreated soil (see Table 9).

Table 9: Yield of the treatments over two cycles of Lupinus mutabilis cultivation. Source: Own elaboration.

Treatments	Germination (%)	Number of plants	Height (cm)	Number of branches	Number of Flowers	Number of Pods
	CH average	CH average	CH average	CH average	CH average	CH average
FO1: Humus	89	73	87	12	22	22
FO2: Bovine manure	83.62	67.63	87.28	11.83	21.55	20.90
FO3: Poultry manure	78.98	63.79	85.54	11.48	21.55	21.32
S1: Control treatment	73.39	59.88	87.08	11.25	21.30	22.40

Treatments	Germination (%)	Number of plants	Height (cm)	Number of branches	Number of Flowers	Number of Pods
Total	325.47	263.83	347.23	46.52	86.37	87.00
Average	**81.37**	**65.96**	**86.81**	**11.63**	**21.59**	**21.75**

For its part, the yield in Pisum sativum also showed a favorable response to the use of organic fertilizers. Humus was the most efficient treatment, with an average germination of 84.47%, while the other treatments (bovine manure, poultry manure, and control) showed decreasing values. The average plant height was 99.78 cm, with 11.64 branches, 21.19 flowers, and 18.79 pods per plant. The observed improvements are associated with the ability of organic fertilizers to stabilize the chemical and physical properties of the soil by optimizing fertility (see Table 10).

Table 10: Yield of the treatments over two cycles of Pisum sativum cultivation. Source: Own elaboration.

Treatments	Germination (%)	Number of Plants	Height (cm)	Number of branches	Number of Flowers	Number of pods
	CH average	CH average	CH average	CH average	CH average	CH average
FO1: Humus	84.47	68.25	100.39	12.18	21.32	19.13
FO2: Bovine manure	81.06	64.25	99.49	11.12	21.02	20.63
FO3: Poultry manure	77.65	61.50	99.73	11.32	21.25	18.38
S1: Control treatment	74.81	59.58	99.49	11.95	21.18	17.00
Total	317.99	253.58	399.10	46.57	84.77	75.15
Average	79.50	63.40	99.78	11.64	21.19	18.79

In terms of average yield, Pisum sativum showed a net yield of 1,080.67 kg/ha over the two crop cycles, equivalent to 90.06% of the probable yield (1,200 kg/ha). In contrast, Lupinus mutabilis presented a net yield of 746.32 kg/ha, reaching only 54.73% of the probable yield. These results reflect significant differences in the response of both crops to soil conditions and fertilization sources (see Table 11).

Table 11: Average yield of two crop cycles of Pisum sativum and Lupinus mutabilis. Source: Own elaboration.

Indicators	Total yield Pisum sativum	Total yield Lupinus mutabilis
Total, in kg.	11.67	16.12
Average in kg/Terrace	0.97	0.67
Total, land use	in 108 m2	in 216 m2
kg/ha Probable Yield	1 200	1 363.64
Net yield obtained from the crop per hectare	1 080.67	746.32
Production percentage	90.06	54.73

Neutrosophic membership degree: The neutrosophic membership degree of each intersection of the sets of $P(T_i)$ is determined as shown in Table 12 (intersections within the scope of the study). For this, the t_{norm} and t_{conorm} were used to determine the intersection through the aggregation operation, which is the value $\left(min_j\{\mathrm{T}_{ij}\}, max_j\{\mathrm{I}_{ij}\}, max_j\{\mathrm{F}_{ij}\}\right)$. Each resulting value is defined by a single-valued neutrosophic number (SVNN) that reflects the level of membership of this element or set within $P(T_i)$.

Table 12: Neutrosophic membership degree of each set of $P(T_i)$. Source: Own elaboration.

$P(T_i)$	$x(d^0)$	$P(T_i)$	$x(d^0)$
$\{S_1, A_i\}$	(0.38,0.54,0.63)	$\{\{\{S1, A_i\}, C_i\}, \{B_i, Z_1\}\}$	(0.38,0.54,0.63)
$\{FO1, A_i\}$	(0.98,0,0.03)	$\{\{\{FO1, A_i\}, C_i\}, \{B_i, Z_1\}\}$	(0.88,0.04,0.13)
$\{FO2, A_i\}$	(0.78,0.14,0.23)	$\{\{\{FO2, A_i\}, C_i\}, \{B_i, Z_1\}\}$	(0.78,0.14,0.23)
$\{FO3, A_i\}$	(0.68,0.24,0.33)	$\{\{\{FO3, A_i\}, C_i\}, \{B_i, Z_1\}\}$	(0.68,0.24,0.33)
$\{C_1\}$	(0.68,0.24,0.33)	$\{\{\{S1, A_i\}, C_i\}, \{B_i, Z_2\}\}$	(0.38,0.54,0.63)
$\{C_2\}$	(0.88,0.04,0.13)	$\{\{\{FO1, A_i\}, C_i\}, \{B_i, Z_2\}\}$	(0.78,0.14,0.23)
$\{C_1, C_2\}$	(0.88,0.04,0.13)	$\{\{\{FO2, A_i\}, C_i\}, \{B_i, Z_2\}\}$	(0.78,0.14,0.23)
$\{B_i\}$	(0.88,0.04,0.13)	$\{\{\{FO3, A_i\}, C_i\}, \{B_i, Z_2\}\}$	(0.68,0.24,0.33)
$\{Z_1\}$	(0.88,0.04,0.13)	$\{\{\{S1, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$	(0.38,0.54,0.63)
$\{Z_2\}$	(0.78,0.14,0.23)	$\{\{\{FO1, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$	(0.88,0.04,0.13)
$\{Z_1, Z_2\}$	(0.88,0.04,0.13)	$\{\{\{FO2, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$	(0.78,0.14,0.23)
		$\{\{\{FO3, A_i\}, C_i\}, \{B_i, \{Z_1, Z_2\}\}\}$	(0.68,0.24,0.33)

The results of the neutrosophic membership degrees have highlighted two points of interest within $P(T_i)$. For example, the intersections between the set groups $\{\{\{FO1, A_1\}, C_2\}, \{B_1, Z_1\}\}$, as well as $\{\{\{FO1, A_1\}, C_2\}, \{B_1, \{Z_1, Z_2\}\}\}$, are defined by a membership degree of (0.88, 0.04, 0.13). Therefore, vermicompost emerges as the most efficient organic source, due to its consistent effects on the soil, regardless of the management used, confirming its positive impact compared to other treatments.

On the other hand, the correlation analysis revealed that the weight and number of grains of Lupinus mutabilis correlate 1, with weight being a key indicator of yield. Parameters such as electrical conductivity (0.96), organic matter (0.69), and nitrogen (0.71) showed significant positive correlations with yield. Meanwhile, phosphorus (1) and zinc (0.83) influenced germination, which correlated 0.60 with production. Consequently, the use of organic fertilizers consolidates as a key strategy to optimize agricultural productivity and preserve the quality of the degraded soils of Salache.

4 Conclusion

The study confirmed that the use of vermicompost as an organic fertilizer source significantly improves soil properties such as pH, electrical conductivity, organic matter, and essential nutrients, which positively impacts the yield of Pisum sativum and Lupinus mutabilis. The applied dose (30 tons per hectare) achieved outstanding results, reaching 90% and 55% of the average for each crop, respectively, demonstrating its effectiveness in degraded soils. Furthermore, the analysis using Neutrosophic SuperHyperSoft Sets allowed for the identification of set groups corresponding to chemical and physical properties (a_i), as well as plant indicators (b_i) and crop cycles (c_i). This enabled the design of a structure for evaluating multiple attributes, with neutrosophic membership degrees inherent in sustainable agricultural practices. Lastly, it is suggested to explore new organic sources and promote the use of decomposed fertilizers in eroded areas, to encourage sustainable soil management for resilient agriculture.

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Sentiment Analysis and NeutroAlgebra to Evaluate Organizational Strategies and Performance Levels of Basic Education Teachers

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Abstract. The study addresses the problem of evaluating the organizational strategies and performance levels of basic education teachers through an innovative approach in sentiment analysis. In the current context, there is a growing need for accurate and reliable methods to measure the impact of organizational strategies on educational performance, especially given the importance of educational quality for social development. The existing literature has shown limitations in integrating methods that can handle the complexity and subjectivity of teachers' emotional and attitudinal perceptions, creating a gap that this study aims to fill. Using the neutrosophic sentiment analysis method based on neutroalgebra, the study offers a new perspective to evaluate how organizational strategies affect teacher performance, thus addressing the limitations of traditional approaches. The results indicate that the neutrosophic method provides a more complete and nuanced assessment of teachers' feelings and the impact of organizational strategies on them. This approach reveals patterns of performance that were not evident with conventional methods. The main contribution of the study lies in its ability to capture the emotional and organizational complexity in the evaluation of teaching performance, thus offering theoretical and practical tools to improve educational management. Additionally, the study provides practical recommendations for adjusting organizational strategies based on teachers' perceptions and feelings, which can lead to significant improvement in the educational environment and performance outcomes.

Keywords: Organizational Strategies, Educational Performance, Neutroalgebra, Prospector , Sentiment Analysis.

1. Introduction

Sentiment analysis in the context of organizational strategies and teaching performance in basic education is a topic of increasing relevance. In a constantly changing educational environment, understanding how organizational strategies impact teacher performance and how teachers, in turn, perceive and react to these strategies is crucial to improving educational quality. Research in this area is significant because it offers tools to adapt and optimize educational policies, based on the perception and sentiment of key actors [1]. This research, focused on the neutrosophic sentiment analysis method based on neutroalgebra, seeks to fill a important gap in the literature by providing an advanced methodology to evaluate these dynamics.

Historically, the evaluation of teaching performance has moved from purely qualitative and descriptive methods to more quantitative and structured approaches. Since early attempts to measure performance through direct observations and subjective evaluations, the field has evolved toward techniques that include statistical analyzes and standardized surveys. However, despite these advances, current methods still do not fully capture teachers' emotional complexity and its impact on performance. Developments in sentiment analysis and organizational evaluation techniques have been important, but

the integration of these into a neutrosophic framework is still incipient [2]. The main problem that this study addresses is the lack of a comprehensive methodology that combines sentiment analysis. feelings with the evaluation of organizational strategies in the educational context. The central research question is: how can the neutrosophic sentiment analysis method based on neutroalgebra provide a more complete and accurate assessment of organizational strategies and teaching performance? This issue is crucial given that most existing methods fail to adequately capture the intersection between teachers' emotions and the effectiveness of organizational strategies [3]. The purpose of this study is to develop and validate a neutrosophic sentiment analysis method that uses neutroalgebra to accurately assess how organizational strategies affect teachers in basic education. The first objective is to design a tool that effectively integrates sentiment analysis with strategy evaluation, allowing a deeper understanding of the relationship between these variables. The second objective is to apply this method in a real context to obtain detailed insights about the impact of organizational strategies on teaching performance [4]. In addition to these objectives, the study seeks to provide practical recommendations for the improvement of organizational strategies based on the results obtained. By providing an advanced analytical tool, it is intended to offer education policy makers a more precise and nuanced approach to decision making. The findings of this research could have significant implications for the way organizational strategies are designed and implemented in the education sector [5]. In summary, the proposed approach seeks to fill an important gap in the evaluation of organizational strategies through a method that captures both emotions and the impact of policies on teacher performance. This innovative approach has the potential to transform how the dynamics between organizational strategies and educational performance are understood and addressed. The article will be structured around the detailed explanation of the proposed neutrosophic methodology, its practical application and an analysis of the results obtained [6]. The research hopes to contribute to the field with a new analytical model that can be adapted and used in various educational contexts to improve the effectiveness of organizational strategies and, ultimately, the quality of education.

2. Preliminaries

2.1. Sentiment analysis

Sentiment analysis is the application of natural language processing, text analysis, and computational linguistics to discern and extract subjective information from sources [7]. In the realm of text mining, Sentiment Analysis is focused on the extensive classification of information polarity.

Sentiment analysis encompasses three primary categories: lexical affinity, statistical methods, and concept-level techniques. Nonetheless, we cannot evade the intricacies associated with quantifying the emotions of an individual or a collective, owing to the inherent subjectivity of the issue. This is due to the transient nature of emotive states, which may be represented in one manner in a survey, yet can subsequently shift to a different state moments later.

Experts concur that a neutral opinion is an indispensable component of the measurement scale, as individuals may be unable to distinguish their sentiments as either negative or positive, or their feelings may reside in a neutral state that is neither positive nor negative. The relevance of Neutrosophy lies in its incorporation of neutrality with positive and negative elements. This can also be beneficial for analyzing the connotation of terms inside the text.

2.2 Neutral algebra generated by the join function in Prospector

For a natural number $n > 0$, NeutroGroup is specified from the Prospector combinator function . Prospector is the well-known expert system utilized to model mining problems [7,8]. The set NeutroGroup consists of all integers between $-n$and $n$plus the symbolic element I to symbolize indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$and $\oplus_5$is used. This is expressed according to the following Cayley table:

Table 1. Cayley table corresponding to ⊕_5. Source: [8].

$\oplus_5$	-5	-4	-3	-2	-1	0	Yo	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	I
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
I	-5	-4	-3	-2	-1	I	I	I	I	I	I	I
1	-5	-4	-2	-1	0	1	I	2	3	4	4	5
2	-5	-3	-1	0	1	2	I	3	3	4	5	5
3	-5	-2	0	1	2	3	I	4	4	4	5	5
4	-5	0	2	3	4	4	I	4	5	5	5	5
5	Yo	5	5	5	5	5	Yo	5	5	5	5	5

$\oplus_5$fulfills the properties of commutativity and associativity and has 0 as a null element. Furthermore , it satisfies each one of the following properties :

- If x, y < 0then $x \oplus_5 y \leq min(x, y)$,
- If x, y > 0then $x \oplus_5 y \geq max(x, y)$,
- If x $< 0$and y > 0or if x $> 0$and y < 0, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

The sentiment analysis technique theory analyzes opinions and perceptions based on levels of positivity, negativity, and indeterminacy. This facilitates the capture of positive, negative, neutral, and ambiguous emotions, yielding a precise evaluation and enhancing comprehension of how these elements are viewed within businesses.

The proposed method is applicable to brief informal texts, similar to the aforementioned strategy. A collection of words must be identified and categorized as positive, negative, or neutral, with a strength value assessed on a scale from -5 to 5 or deemed undetermined.

The evaluation of the indeterminate form occurs due to our inability to ascertain the individual's perspective on the subject. This may occur due to the language being incomprehensible or the intended meaning being ambiguous.

Another alternative is that within the same sentence, each of the aforementioned variables is expressed with a maximum positive evaluation (+5) and a maximum negative evaluation (-5), resulting in a contradiction and therefore deemed indeterminate. I.

It is important to note that the function underlying the PROSPECT expert system signifies the extent of evidence the expert possesses for a certain issue. When this evidence is consolidated across two features exhibiting the highest degree of opposition, a contradiction arises. This is why we discuss the benefits of this strategy utilizing NeutroOperation, where the indeterminacy may arise from two

sources. Some aspects we have used are taken from the sentiment strength detection algorithm SentiStrength [9].

- The terminology associated with each preceding variable is categorized as Positive, Negative, or Neutral within a list composed of linguistic values. Each of these phrases is connected with a value ranging from -5 to 5, reflecting its degree of strength and its positive or negative connotation. The term "like" is enhanced with a stronger positive connotation in the phrase "I like it a lot," yet it is also increased with a more negative connotation in the expression "I don't like it very much.".

What concerns is that for the word "a lot" or "a lot" that modifies one of the positive or negative classifier words, is used $x \oplus_5 x$, and for "too much" $x \oplus_5 x \oplus_5 x$, where $x$is the value that is connected with the word. For example, $x > 0$ results in "very" with an even more positive value. On the other hand, when $x < 0$, the result is more negative.

Additionally, the modification of "quite" is converted to $\left[sig(x)\sqrt{|x|}\right]$ [10].

- They consider words that invert the meaning of what was said. In this case the sign is changed. For example, "I like" with a value of $x = 3$, when it comes to "I don't like" it is calculated as $x = -3$, both have the same strength, but with opposite meanings.
- In this algorithm, complex scenarios using exclamation or question marks are disregarded, as the objective is to assess the coherence of the organization members' or clients' comments about each of the twelve ethical aspects previously mentioned. Another aspect that is considered in the proposed algorithm taken from the precedent is the evaluation of emoticons.
- Spell checking is also applied.

The next step is the evaluation of a short informal text written by one person. To do this, natural language processing is carried out where words are searched that express feelings or opinions about each of the twelve aforementioned aspects. Let us denote these aspects as: $V = \{v_1, v_2, \cdots, v_{12}\}$.

Then, within the text processing, the words indicating to each of these variables are detected. These words are labeled with a value from -5 to 5 or I. Let us denote this as follows, for the ith variable, the set $X_i$of evaluations of words that appear in the text [11,12]:

$v_i \rightarrow X_i = \{x_{i1}, x_{i2}, \ldots, x_{im_i}\}$, where$x_{ij}$ is the set of elements between -5 and 5 o I, used to qualify words that refer to the ith variable.

It is important to note that assessing each word individually might be intricate. For instance, the presence of modifiers like "very" alters the value of the modified term. Additionally, when spelling problems render certain evaluations illegible, it is imperative to utilize the value *I*.. The final value associated with eachv_i is:

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \ldots \oplus_5 x_{im_i} \quad (1)$$

We should remember that deriving a collective ethical value for all variables is impractical, as individual values provide a clearer understanding of personal opinions or sentiments..

If we have a group of people whose opinion is analyzed. Let's call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i,j}$it is the total value of the ith ethics variable in the organization, according to the j-th person.
It is calculated [13]:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} \quad (2)$$

That is, the arithmetic mean of each of the variables.

3. Case Study.

This study applies the **method of neutrosophic sentiment analysis** to evaluate the organizational strategies and performance levels of basic education teachers. A series of variables are used that reflect different organizational and teaching performance aspects. These variables are evaluated based on the opinions and perceptions of teachers, classified in terms of positivity, negativity and indeterminacy. Evaluations are assigned values ranging from -5 to 5, or are classified as indeterminate (I) when the opinion is unclear.

Organizational Aspects Evaluated:

1. **Internal Communication**
2. **Support for Innovation**
3. **Knowledge Management**
4. **Resources and Equipment**

Evaluated Performance Levels:

1. **Class Preparation**
2. **Teaching Methodology**
3. **Relationship with Students**
4. **Student Evaluation**

2. Data Collection

A survey was conducted with a sample of 20 teachers, who evaluated each of the aforementioned variables. The responses were processed using the neutrosophia methodology to determine the sentiment values in each variable.

Evaluation Data Collected

The data is presented in the following table, where each teacher has evaluated the variables with a numerical value corresponding to the feeling associated with each aspect.

Internal Communication:

Table 2. Evaluation of Internal Communication.

Teaching	Evaluation 1	Value 1	Evaluation 2	Value 2	Total
1	Clear	+4	Efficient	+3	+7
2	Confused	-4	Deficient	-3	-7

Teaching	Evaluation 1	Value 1	Evaluation 2	Value 2	Total
3	Clear	+4	Neutral	0	+4
4	Efficient	+3	Clear	+4	+7
5	Deficient	-3	Confused	-4	-7
6	Moderate	+3	Efficient	+3	+6
7	Clear	+4	Low	-4	0
8	Efficient	+3	Confused	-4	-1
9	Clear	+4	Clear	+4	+8
10	Confused	-4	Deficient	-3	-7
11	Clear	+4	Moderate	+3	+7
12	Deficient	-3	Efficient	+3	0
13	Efficient	+3	Clear	+4	+7
14	Confused	-4	Deficient	-3	-7
15	Clear	+4	Efficient	+3	+7
16	Moderate	+3	Deficient	-3	0
17	Clear	+4	Clear	+4	+8
18	Deficient	-3	Confused	-4	-7
19	Efficient	+3	Efficient	+3	+6
20	Clear	+4	Clear	+4	+8

Support for Innovation:

Table 3. Evaluation of Support for Innovation

Teaching	Evaluation 1	Value 1	Evaluation 2	Value 2	Total
1	Strong	+5	Moderate	+3	+8
2	Low	-4	Null	-5	-9
3	Moderate	+3	Strong	+5	+8
4	Strong	+5	Low	-4	+1
5	Null	-5	Moderate	+3	-2
6	Strong	+5	Moderate	+3	+8
7	Low	-4	Null	-5	-9
8	Moderate	+3	Strong	+5	+8
9	Strong	+5	Low	-4	+1
10	Null	-5	Moderate	+3	-2
11	Moderate	+3	Strong	+5	+8
12	Strong	+5	Low	-4	+1
13	Low	-4	Null	-5	-9
14	Moderate	+3	Strong	+5	+8
15	Strong	+5	Moderate	+3	+8
16	Low	-4	Null	-5	-9
17	Moderate	+3	Strong	+5	+8
18	Strong	+5	Low	-4	+1
19	Null	-5	Moderate	+3	-2
20	Strong	+5	Moderate	+3	+8

Class Preparation:

Table 4. Evaluation of Class Preparation:

Teaching	Evaluation 1	Value 1	Evaluation 2	Value 2	Total
1	Excellent	+5	Good	+4	+9
2	Regular	0	Insufficient	-3	-3
3	Good	+4	Excellent	+5	+9
4	Insufficient	-3	Regular	0	-3
5	Good	+4	Excellent	+5	+9
6	Regular	0	Insufficient	-3	-3
7	Excellent	+5	Good	+4	+9
8	Insufficient	-3	Regular	0	-3
9	Good	+4	Excellent	+5	+9
10	Regular	0	Insufficient	-3	-3
11	Excellent	+5	Good	+4	+9
12	Insufficient	-3	Regular	0	-3
13	Good	+4	Excellent	+5	+9
14	Regular	0	Insufficient	-3	-3
15	Excellent	+5	Good	+4	+9
16	Insufficient	-3	Regular	0	-3
17	Good	+4	Excellent	+5	+9
18	Regular	0	Insufficient	-3	-3
19	Excellent	+5	Good	+4	+9
20	Insufficient	-3	Regular	0	-3

Teaching Methodology:

Table 5. Evaluation of Teaching Methodology .

Teaching	Evaluation 1	Value 1	Evaluation 2	Value 2	Total
1	Innovative	+4	Efficient	+3	+7
2	Traditional	-3	Inefficient	-4	-7
3	Efficient	+3	Innovative	+4	+7
4	Inefficient	-4	Traditional	-3	-7
5	Efficient	+3	Innovative	+4	+7
6	Traditional	-3	Inefficient	-4	-7
7	Innovative	+4	Efficient	+3	+7
8	Inefficient	-4	Traditional	-3	-7
9	Efficient	+3	Innovative	+4	+7
10	Traditional	-3	Inefficient	-4	-7
11	Innovative	+4	Efficient	+3	+7
12	Inefficient	-4	Traditional	-3	-7
13	Efficient	+3	Innovative	+4	+7
14	Traditional	-3	Inefficient	-4	-7
15	Innovative	+4	Efficient	+3	+7
16	Inefficient	-4	Traditional	-3	-7
17	Efficient	+3	Innovative	+4	+7
18	Traditional	-3	Inefficient	-4	-7
19	Innovative	+4	Efficient	+3	+7
20	Inefficient	-4	Traditional	-3	-7

3. Analysis of the Results

For each variable evaluated, the total value is calculated using the neutrosophic aggregation operation. This operation is performed by adding the values of each evaluation.

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \dots \oplus_5 x_{im_i} \quad (1)$$

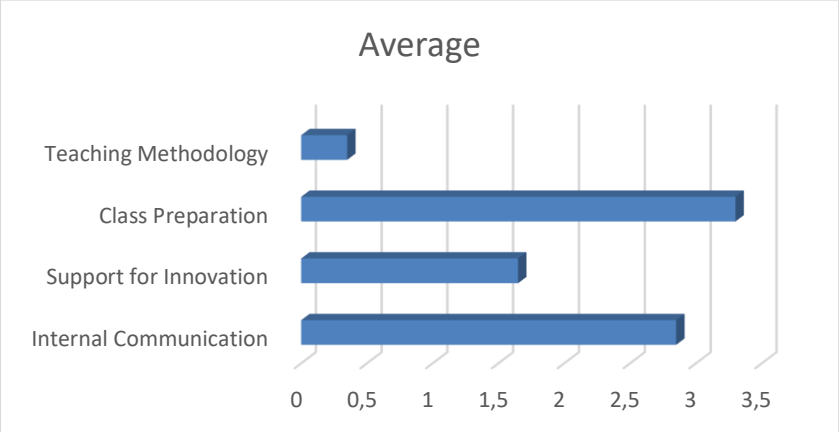
It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} (2)$$

This value represents the average evaluation of internal communication between teachers.

Table 6: Averages of Evaluations by Variable.

Variable	Average
Internal Communication	2.85
Support for Innovation	1.65
Class Preparation	3.3
Teaching Methodology	0.35



Graph 1: Averages of Evaluations by Variable.

Internal Communication and **Class Preparation** positively , with averages higher than 2.5. However, challenges are observed in **Support for Innovation** and **Teaching Methodology** , which present values close to 1 or lower, which indicates a more neutral or even negative perception in these aspects. The neutrosophic analysis has made it possible to identify key areas for improvement in organizational strategies and levels of teaching performance, providing a clear and nuanced view of teacher perceptions in these areas. The results of the study reveal that teachers positively value **Internal Communication** and **Class Preparation** , with averages of 2.85 and 3.3 respectively. However, areas of improvement were identified in **Support for Innovation** and **Teaching Methodology** , which obtained averages of 1.65 and 0.35, reflecting a more neutral or negative perception . These findings suggest that, although teachers consider **Internal Communication** and **Class Preparation** as satisfactory aspects within the educational environment, significant challenges persist in the implementation of pedagogical innovations and teaching methodologies. The high valuation of **Internal Communication** can be explained by an organizational environment that facilitates the exchange of information and coordination between staff members. On the other hand, the low score in **Support for Innovation** could indicate a resistance

to change or the lack of resources necessary for the implementation of new ideas. **Teaching Methodology** was the variable with the lowest score, which could reflect a reliance on traditional practices and limited adoption of modern pedagogical approaches, possibly due to a lack of ongoing training or incentives to innovate in teaching.

When comparing these results with previous studies, a similar trend is observed in the assessment of **Internal Communication** as a key component in teacher satisfaction, in line with research that highlights the importance of effective communication for the functioning of educational organizations. However, the low valuation of **Innovation Support** contrasts with studies that indicate a growing interest in the adoption of educational technologies, which could indicate a gap between intention and implementation in specific contexts. The **Teaching Methodology** obtained particularly low results, coinciding with research that highlights the need to reform traditional pedagogical practices in basic education. This study has several limitations that should be considered when interpreting the results. First, the sample used is relatively small and specific, which could limit the generalizability of the findings to other educational contexts. Furthermore, the subjectivity inherent in teachers' responses could have influenced the evaluations, especially in aspects related to innovation and teaching methodology. Finally, the neutrosophic approach, although effective in managing uncertainty, may not fully capture the complexity of teacher perceptions in all cases.

The results of this study highlight the need to continue exploring mechanisms to promote educational innovation and methodological updating among teachers. Future research could focus on identifying specific barriers that prevent the adoption of new teaching methodologies and developing more effective training programs. Furthermore, the implementation of larger scale studies is suggested to validate these findings in different contexts and improve the understanding of how to improve internal communication and support for innovation within educational institutions. A result that deserves special attention is the low rating of the **Teaching Methodology** , which was not only the lowest score, but also showed a marked divergence with respect to other variables. This anomaly could be related to factors not evaluated in this study, such as teachers' perception of the relevance of continuous training or the pressure to comply with established curricula. However, the lack of a definitive explanation for this result suggests the need for additional research that addresses this phenomenon in greater depth. This discussion provides a comprehensive framework for understanding the study findings, while highlighting key areas for future research and improvements in educational practice.

5. Conclusion

The present study introduces a neutrosophic model of sentiment analysis applied to the evaluation of twelve ethical variables in an educational environment. Using the NeutroGroup generated by an offuninorm , it was possible to accurately analyze the perception of teachers on key aspects such as Internal Communication, Class Preparation, Support for Innovation and Teaching Methodology. The results reflect a positive perception towards Internal Communication and Class Preparation, with averages higher than 2.5, while significant areas of improvement were identified in Support for Innovation and Teaching Methodology, with values close to 1 or lower. These findings highlight the usefulness of the model in identifying and addressing indeterminacy in teachers' perceptions, allowing for a more nuanced understanding of their opinions and attitudes. From a practical perspective, the developed model not only facilitates the identification of strengths and weaknesses in organizational strategies, but also highlights the need to promote pedagogical innovation and methodological updating in the educational field. The low assessment of Support for Innovation and Teaching Methodology suggests barriers in the adoption of new practices, possibly derived from the lack of training or resources. These results invite future research to further explore these challenges, proposing more effective training programs and additional studies to validate and generalize the findings in different educational contexts. This model represents a significant contribution to ethical and organizational analysis, offering new

ways to improve performance and satisfaction in educational environments.

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Applying Neutrosophic Natural Language Processing to Analyze Complex Phenomena in Interdisciplinary Contexts

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Abstract. The paper addresses the challenge of integrating natural language processing (NLP) with neutrosophic logic to solve problems where uncertainty, ambiguity, and indeterminacy play a critical role. This approach becomes particularly relevant in an increasingly digitalized world, where human-machine interaction requires a deeper understanding of linguistic and contextual nuances. Despite advances in conventional NLP, many existing methodologies lack tools to effectively manage undefined or contradictory elements present in real data. In this context, neutrosophic logic offers an innovative framework to model and analyze complex and multifaceted human perceptions. The study applies a methodology that combines advanced NLP techniques with neutrosophic analytical tools, allowing to processing of texts with simultaneous degrees of truth, falsity, and indeterminacy. The results reveal a significant capacity to identify linguistic patterns in high-uncertainty scenarios, with practical applications in areas such as artificial intelligence, decision-making, and semantic analysis. This approach not only extends the boundaries of traditional NLP, but also provides an adaptable framework for studying complex phenomena in interdisciplinary contexts. Ultimately, this work contributes to the development of more intelligent systems, capable of accurately handling the ambiguity inherent in human language.

Keywords: natural language processing, neutrosophic logic, uncertainty, ambiguity, indeterminacy, human-computer interaction, semantic analysis, linguistic patterns, artificial intelligence, decision making.

1. Introduction

The semi-open regime is the process of social rehabilitation of the convicted person who meets the requirements and norms of the progressive system to carry out his activities outside the center of execution of the sentence, under the supervision of the Technical Body. In this way, it allows him to carry out family, labor, social, and community reintegration activities.

Penitentiary law seeks to establish a balance between punishment and the resocialization of convicted prisoners, to promote their reintegration into society. In this context, the semi-open regime has emerged as an alternative that allows prisoners to serve their sentence in a less restrictive environment and with greater opportunities for social reintegration. However, in the case of those convicted of homicide, there is a worrying violation of their right to a semi-open regime. These individuals are frequently excluded from this type of prison, which raises questions about the justification and proportionality of this practice.

The exclusion of those convicted of homicide from the semi-open regime raises ethical and legal questions. On the one hand, it is argued that the seriousness of the crime and the perceived dangerousness of the prisoners justify the denial of this penitentiary benefit. However, this approach poses a challenge to the fundamental principles of criminal law and human rights [1], among which are social reintegration and proportionality of sentences [2].

Mandatory requirements for the semi-open regime:

1. Having served sixty percent (60%) of the sentence imposed by a final conviction; except in cases where the person deprived of liberty is the only appellant in an extraordinary appeal for cassation.
2. Evaluation and qualification report that has a minimum average of five (5) points during the execution of the individualized sentence fulfillment plan.
3. Certificate of not having been sanctioned for committing serious or very serious disciplinary offences during the sentence, issued by the highest authority of the centre.
4. Certificate of being classified at a minimum-security level, issued by the highest authority of the detention center.
5. Document proving the fixed address where the person deprived of liberty will reside, which may consist of a lease agreement, a commitment contract signed by the person deprived of liberty or a third party, or any other supporting document.
6. A legal report from the center indicates that the person deprived of liberty does not have another pending criminal process with preventive detention or a final conviction. If the person deprived of liberty has a process with conditional suspension of the sentence or that will require conditional suspension of the procedure other than that requesting the change of regime, the respective resolution declares the sentence extinguished by compliance with the conditions and terms established by the competent authority.
7. Psychological report from the center, which concludes the conditions for the reintegration of the person deprived of liberty. In addition, if there are certificates of participation in group support, individual psychotherapy, or therapeutic communities during the time of deprivation of liberty, these will be attached to the report.

Persons deprived of liberty who have been convicted of homicide [3], femicide [4] [5], contract killing, crimes against personal integrity and freedom resulting in death, robbery with fatal consequences, crimes against sexual and reproductive integrity, human trafficking and migrant smuggling, crimes of violence against women or members of the family unit cannot access this regime. Bribery, extortion, embezzlement, illicit enrichment, obstruction of justice, overpricing in public contracts, acts of corruption in the private sector [6], money laundering, unjustified private enrichment, crimes of illicit trafficking of controlled substances on a high and large scale, terrorism, organized crime, cattle rustling resulting in death, and serious violations of human rights and crimes against international humanitarian law.

A. Ecuadorian Penitentiary System. Regulations and Characteristics.

In Ecuador, the penitentiary system is structured around a legal framework consisting of the Comprehensive Organic Penal Code (COIP) and various laws and regulations, such as the Organic Law of the Penitentiary Public Service and the Law on the Execution of Sentences, which together regulate from general principles to specific aspects of penitentiary management. This set of regulations promotes respect for human rights [7] [8] [9], individualized treatment of inmates, classification by categories according to risk factors, and the implementation of rehabilitation and social reintegration programs. Thus, it includes paid prison work and active participation in civil society. Despite these provisions, the system faces significant challenges such as overcrowding and

poor living conditions, in response to which the government has initiated improvement efforts, including the construction of new facilities and decongestion policies [10].

B. Violation of constitutional rights of prisoners in semi-open or closed regimes

In Ecuador, the penitentiary system has the responsibility of guaranteeing the respect and protection of the constitutional rights of inmates [11], even in semi-open and closed penitentiary regimes. However, in practice, situations of violation of these rights may arise. Some of the possible violations of constitutional rights in these penitentiary regimes are:

1. Overcrowding: Overcrowding is a recurring problem in Ecuadorian prisons and can affect respect for the constitutional rights of prisoners. The lack of adequate spaces and decent living conditions can lead to problems with health, safety, and access to basic services.
2. Violence and security: Semi-open and closed prison regimes may face challenges in terms of violence and security within prisons. The lack of effective control and adequate security measures may expose prisoners to risky situations and violate their right to personal integrity.
3. Access to basic services: Prisoners in semi-open and closed regimes must have access to basic services, such as medical care, adequate nutrition, drinking water, hygiene, and sanitary conditions. Inadequate provision of these services may constitute a violation of prisoners' constitutional rights.
4. Restriction of freedoms: In semi-open and closed regimes, inmates have certain restrictions on their freedom of movement and personal autonomy. However, these restrictions must be justified and proportionate, always respecting the fundamental rights of inmates.
5. Right to rehabilitation and social reintegration: Prisoners have the right to participate in rehabilitation and social reintegration programs during their period of incarceration. If these programs are not implemented effectively, they may limit the ability of prisoners to properly reintegrate into society once they have served their sentence.

The purpose of this article is to analyze in depth the issue of the violation of the right to a semi-open prison regime for persons convicted of homicide. It also examines the possible justifications put forward for denying them this type of prison regime, as well as the implications and consequences of this exclusion for both individuals and society.

Proposals and recommendations are presented to seek alternatives that ensure the right to a semi-open regime for persons convicted of homicide, following the principles of criminal law and human rights. The ultimate goal is to promote a more just and effective penitentiary system that encourages the resocialization of prisoners without neglecting the seriousness of the crimes committed.

This study focuses on the violation of the right to semi-open regime in persons convicted of homicide, by analyzing legislation, judicial practices, and individual cases through a descriptive and analytical approach. Through the review of files and interviews with experts and convicted persons, a wide range of data was collected, allowing for a sentiment analysis. This allowed the identification of patterns and themes relevant to understanding the attitudes and emotions associated with the semi-open regime.

Despite some limitations related to the sample and data accessibility, the results obtained provide valuable insight into the challenges faced by people convicted under this regime, thus offering significant contributions to the debate on penal reforms focused on rehabilitation and social reintegration.

2. Related work.

2.1 Neutrosophic analysis.

Neutrosophics is a new branch of philosophy that studies the origin, nature and scope of neutralities, created by Professor Florentin Smarandache. Neutrosophic logic and sets, on the other hand, constitute a generalization of fuzzy logic and Zadeh's sets, and especially of Atanassov's intuitionistic logic, with multiple applications in the field of decision-making [12,13].

A neutrosophic set is defined by the following elements: μ true, γ indeterminate and τ false of x in S, respectively, and their images constitute standard or nonstandard subsets within the range (0;1). For N in the universe of discourse, the single-valued neutrosophic set S over N is defined as an object in the representation $S = \{\langle n, \mu_s(n), \gamma_s(n), \tau_s(n)\rangle : n \epsilon N\}$.

Where $\mu_S(n), \gamma_S(n), \tau_S(n)$the following condition 0 holds $0 \leq \mu_S(n), \gamma_S(n), \tau_S(n) \leq 3$for all $n \in N$. Therefore, to define each neutrosophic number to be expressed in the form v, i, f for neutrosophic sentiment analysis modeling. Therefore, the following functions are defined[14,15] :

$v = \mu_S(n)$ for true membership functions, where ∈ {0,1}.

$i = \gamma_S(n)$ for indeterminate membership functions, where ∈ {0,1}.

$j = \delta_A(n)$ for false membership functions, where ∈ {0,1}.

Therefore, the neutrosophic number defined for the study is determined as $S = (v, i, f)$, $v, i, f \in \{0,1\}$and satisfies the following condition $0 \leq v + i + f \leq 3$. Therefore, it is defined as the scoring function Y of a neutrosophic number as defined by the methodology analyzed [13,16,17].

Sentiment analysis, VADER.

VADER (Valence Aware Dictionary and Sentiment Reasoned) is a sentiment analysis tool specifically designed for social media texts, but can also be applied to a wide range of contexts. It is particularly popular due to its focus on Internet-specific language, including handling emoticons, acronyms, and specific web jargon.

The integration of Neutrosophic with the VADER method for sentiment analysis in NLP opens new avenues for exploring public opinion on sensitive and multifaceted issues [14] [15]This approach is a powerful tool for researchers and analysts interested in unraveling the complex dynamics of perception and opinion. It therefore offers valuable insights for policy formulation, communication strategies, and the promotion of a more inclusive social dialogue.

The study of public perception regarding the violation of the right to a semi-open regime of persons convicted of homicide crimes is significantly enriched by the use of advanced natural language processing (NLP) techniques [16] [17], in particular by incorporating Neutrosophy into sentiment analysis[19,20,21]. This approach allows for a more nuanced and in-depth understanding of opinions and emotions expressed in various texts, such as articles, social media comments, and public speeches. Neutrosophy offers a new dimension in sentiment analysis by considering states of indeterminacy alongside positive and negative sentiments. An explanation of how to integrate this approach with the VADER method for sentiment analysis is presented below[22, 23, 24].

Step 1: Data collection.

The first step is to collect a diverse set of texts addressing the topic of interest, from online forum posts to social media comments and op-eds, court transcripts, news articles, and discussion forums. These texts provide a rich database for analysis.

Step 2: Data preprocessing.

Before analyzing the data, it is essential to perform preprocessing to clean and normalize the text. This includes removing irrelevant elements (such as links and special characters), term normalization, and tokenization to break down the text into manageable units.

Step 3: Integrating Neutrosophic with VADER

Traditionally, VADER assigns scores to texts reflecting positive, negative, and neutral sentiments. The integration of Neutrosophic involves extending this analysis to also consider the degrees of indeterminacy present in sentimental expressions. This is achieved by adapting the algorithm to assess the uncertainties and indeterminacies of the language used, which is especially relevant in speeches addressing complex or sensitive topics.

Step 4: Analysis and interpretation

Analysis using this enriched method allows not only to identify general sentiment trends (positive, negative, or neutral/indeterminate), but also to capture the presence of ambivalent or uncertain opinions, reflecting a more complete picture of public discourse. This detailed analysis helps to identify areas of consensus and controversy, as well as to better understand the complexity of social perceptions.

3. Results and discussion

To represent the results, a table is used that reflects the neutrosophic scores of each text analyzed, categorizing the opinions as positive, negative, neutral, and indeterminate. Using the VADER method adapted to include Neutrosophy, the texts are evaluated to determine the emotional tone and the prevalence of positive, negative, neutral or indeterminate opinions. Each text receives scores in these dimensions (see Table 1).

Table 1: Linguistic terms to represent emotional tone.

Linguistic Scale (LS)	**SVNN (v, i, f)**	**Category**
Extremely Positive (EP)	(0.95, 0.15, 0.10)	It reflects expressions of support or approval towards the semi-open regime.
Positive (P)	(0.75,0.30,0.25)	
Indeterminate (I)	(0.50,0.45,0.50)	Texts that present facts or arguments without expressing a clear emotional inclination.
Negative (N)	(0.25,0.85,0.75)	Indicates criticism or rejection of exclusion from this regime.
Extremely negative (ES)	(0.15,0.90,0.95)	

To develop a natural language processing (NLP) and sentiment analysis tool using VADER, adapted to the semi-open regime topic and enriched with a neutrosophic perspective, a set of keywords or phrases is first defined (see Table 2).

Table 2: Classification of keywords or phrases obtained from the data source.

No.	Keywords or phrases	Classification close to LS	Relationship
1	Social reintegration	P	Related to the objective of the semi-open regime.
2	Overpopulation	N	It reflects problems within the prison system.
3	Opportunity for rehabilitation	P	Focus on rehabilitation.
4	Violation of rights	IS	Legal or ethical issues.
5	Second chance	EP	Concept of redemption.
5	Insecurity	ES	Concerns inside the prison.
7	Penitentiary reform	I	Depending on the context, it can be seen as a solution or a challenge.
8	Stigmatization	north	Problems of social reintegration.
9	Restorative Justice	N	Focus on repairing the damage.
10	Recidivism	IS	Recidivism of crimes.
11	"perhaps"/"possibly"	I	-

Interpretation and discussion:

- Activity: Analyze the results to understand how the captured narratives and sentiments influence legal decisions and public opinion.
- Expected outcome: Knowledge of the public and legal perception of exclusion from the semi-open regime, together with the identification of areas for action and reform.

Proposals and suggestions:

- Activity: Formulate recommendations based on the findings to promote more just and effective prison policies.
- Expected outcome: Proposals to improve access to the semi-open regime and support social reintegration.

The results indicate a clear predominance of negative opinions and a high degree of indeterminacy in the narratives analyzed. The main concerns revolve around social reintegration and criminal justice, with recurrent criticism of the rigidity of policies that exclude those convicted of homicide from the semi-open regime. Linguistic patterns suggest a strong correlation between the language used in legal discourses and public perception, especially in terms of the dehumanization and stigmatization of convicts. [23]

Neutrosophic sentiment analysis highlights the complexity of views on exclusion from the semi-open regime for people convicted of murder. The indeterminacy captured in this study reflects uncertainty in society and the legal system about how best to approach the reintegration of people

who have committed serious crimes. It is argued that indeterminacy and negativity in narratives could hamper penal reform efforts, emphasizing the need for a more balanced and evidence-based approach. Some strategies are therefore proposed:

Together, these strategies aim to create a more humane and effective criminal justice system that recognizes the dignity and potential for the reintegration of each individual, thus contributing to a safer and more just society. Among the alternatives to be developed are:

- Implementation of tailored reintegration programs: Develop and implement social and work reintegration programs tailored to the individual needs of inmates. This includes vocational training, continuing education, and psychological and mental health support. It requires a detailed analysis of each inmate's profile to determine the most appropriate interventions, facilitating effective reintegration into society and reducing recidivism rates.
- Legislative reform to promote semi-open prisons: Amend criminal legislation to make access criteria to semi-open prisons more flexible, taking into account not only the nature of the crime committed but also the conduct of the inmate, his commitment to rehabilitation, and the risk of reoffending. This includes removing unnecessary legal barriers and promoting a culture of restorative justice focused on repairing harm and reconciliation.
- Strengthening prison infrastructure and resources: Increase investment in prison infrastructure and human resources to ensure that prisons have the capacity and programs necessary to facilitate the rehabilitation and social reintegration of inmates. This includes expanding facilities suitable for semi-open regimes, increasing the number of personnel trained in rehabilitation, and implementing technologies that allow for the ongoing monitoring and support of inmates in the reintegration process.

The results obtained through sentiment analysis, using the combination of Neutrosophic and the VADER method, reveal deficiencies in the Ecuadorian penitentiary system, especially in the perception and access to the semi-open regime by inmates convicted of homicide. This analysis highlights not only the lack of knowledge of inmates about their rights and the semi-open regime but also the influence of the rigidity of the system and social stigmatization on their negative perceptions. The combination of legal restrictions, strict criteria, and the scarcity of rehabilitation programs, together with stigmatization, highlights how these perceptions and realities hinder effective penal reforms. It therefore underlines the need to promote strategies to achieve a more balanced, inclusive and evidence-based approach to improving the criminal justice system.

4. Conclusion

This study highlights the important issue of the violation of the right to a semi-open penal regime for persons convicted of certain crimes, pointing out that the unjust limitation of access to this penal regime seriously compromises the possibility of effective social reintegration. It is evident that such a violation negatively affects both the fundamental rights of convicted persons and social well-being in general. In addition, it prevents the implementation of appropriate rehabilitation programs, increasing the risk of recidivism and hindering the social reintegration of individuals.

Sentiment analysis integrating Neutrosophy with the VADER method highlights the negative and indeterminate perception around criminal justice and reintegration, underlining the urgency of informed and balanced penal reforms. At the same time, it emphasizes the critical importance of adhering to the principle of individualization of punishment, which requires a detailed examination of the circumstances of each case to personalize sanctions according to the needs and characteristics

of convicts. The rigidity in the application of sentences, especially the exclusion of the semi-open regime, contradicts the principles of proportionality and humanity. Thus, it reveals a significant gap between current penitentiary policies and the effective need for rehabilitation and social reintegration of convicts.

The urgent need to develop policies and mechanisms to ensure the right of convicted prisoners to a semi-open regime is stressed. To this end, effective rehabilitation programs must be implemented, monitoring measures strengthened, and social reintegration promoted as a central objective of the penitentiary system. Guaranteeing human rights and promoting a penal system that prioritizes effective reintegration is essential to progress toward a more equitable and humane administration of justice, capable of reducing recidivism and improving security and social cohesion.

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Application of the Plithogenic IADOV Approach to Evaluate Satisfaction with the Implementation of Indigenous Rights

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Abstract. The present study addresses a crucial challenge in the evaluation of complex systems: the need to incorporate a holistic view that contemplates not only defined elements but also those that are intrinsically ambiguous or uncertain. The research focuses on the application of the Plithogenic IADOV approach, a model based on neutrosophic logic, to overcome the limitations of traditional methodologies that often fall short when addressing phenomena containing contradictory or unknown dimensions. In the existing literature, despite the wealth of approaches used for the evaluation of systems, there is a perceived lack of methodologies that effectively manage the indeterminacy inherent in many evaluation processes. This study seeks to fill that gap, proposing a method that expands the boundaries of analysis by integrating aspects of uncertainty that have commonly been ignored. The methodology employed is based on the implementation of the Plithogenic IADOV model, which merges neutrosophic analysis techniques with dynamic evaluation tools, allowing for more complete and accurate results in highly complex contexts. Through this application, the results obtained not only provide a clearer view of the elements evaluated but also allow identifying areas for improvement that other approaches would not have detected. The contribution of this study is twofold: first, it offers a new tool for the evaluation of complex systems, capable of effectively handling ambiguity; and second, it provides practical applications that can be implemented in various fields, from business management to public policy. Thus, this approach not only offers a richer theoretical view but also provides tangible solutions for making informed decisions in uncertain scenarios.

Keywords: IADOV Plithogenic, neutrosophic logic, evaluation, complex systems, indeterminacy, ambiguity, uncertainty, traditional models, neutrosophic analysis, methodology, dynamic evaluation.

1. Introduction

Ecuador is distinguished by its rich cultural and ethnic diversity, where Indigenous peoples play an essential role in its history, society, traditions and ancestral knowledge [1] . The indigenous legal worldview, which delves into its connection with the land and nature, contrasts with the Western perspective. Thus, it offers a unique vision of justice and law based on its understanding of the cosmos and life.

Historically, Ecuadorian indigenous peoples have maintained their legal systems in the face of injustices caused by the usurpation of their lands and resources [3] . Despite this, they have fought for the recognition of their customary law and legal practices [4,16] , which vary among different ethnic groups, reflecting the cultural plurality of the country. This struggle has led to important constitutional reforms and international recognition of their rights [5] , evidencing progress towards justice and equity in multicultural societies.

The 2008 Constitution marked a milestone by promoting plurinationality and Buen Vivir, principles that seek to harmonize the relationship between human beings and nature [6,17] . This approach has made it possible to strengthen indigenous collective rights, ensuring their participation in the

management of natural resources and the exercise of their legal traditions within a framework of respect and collaboration with the state judicial system.

However, there are challenges in the implementation of these rights [7] , especially concerning the exercise of the executive, legislative, and jurisdictional powers of indigenous communities [8] . These tensions between constitutional theory and everyday practice motivate the present study, which seeks to evaluate the satisfaction of the Indigenous communities of Ambato regarding the exercise of their executive, legislative and jurisdictional powers through a plithogenic analysis . Among the specific objectives to be developed:

- Analyze the challenges in the effective implementation of legally recognized indigenous rights based on the IADOV Plithogenic analysis.
- To examine the perceptions of Indigenous communities regarding the exercise of their executive, legislative and jurisdictional powers.
- Evaluate the participation, consultation and representation of Indigenous communities in decision-making processes at the legislative and executive levels, through the plithogenic integration of the coherences and inconsistencies of said processes.
- Propose solutions based on plithogenic analysis to improve the implementation of Indigenous rights in Ecuador.

2. Related work.

2.1. Plithogenic environment

The study uses the IADOV Plithogenic method to analyze the diversity and complexity of perceptions of indigenous communities in Ambato, Ecuador, regarding their rights and legal practices. This innovative approach highlights how the interaction between ancestral worldviews and the current legal system influences community satisfaction and underlines the need for inclusive approaches that harmonize cultural plurality with indigenous justice.

Mathematical modeling through neutrosophic logic to plithogenic logic is a methodology that focuses on including indeterminacy and contradiction in the evaluation of sets and systems. Plithogenic logic has the following characteristics:

1. Neutrosophic sets: These sets allow quantifying indeterminacy (I) through a third parameter, in addition to true membership (T) and false membership (F) [9] . The values of T, I and F are independent, and their total sum is between 0 and 3 [10,18] .
2. Membership functions: Within a universe of discourse U, a Neutrosophic Set (NS) is defined by three functions : $u_A(x), r_A(x), v_A(x) : X \rightarrow]0-, 1+[$; which satisfy the condition $0 \leq -\inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$for all $x \in X$. $u_A(x), r_A(x), v_A(x)$are the truth, indeterminacy and falsity membership functions of x in A, respectively, and their images are standard or nonstandard subsets of $]0-, 1+[$.
3. Plithogeny : Represents the creation and evolution of entities from dynamics and fusions of previous entities that may be contradictory, neutral or non-contradictory [11, 12] . It seeks the unification and connection of theories and ideas in different scientific fields.
4. plithogenic : an extension of the classical, fuzzy, intuitionistic fuzzy and neutrosophic sets. A plithogenic set (P, a, V, d, c) :
    a) Where "P" is a set, "a" is an attribute (usually multidimensional), "V" is the range of attribute values, "d" is the membership degree of the attribute value of each element x to the set P for some given criteria ($x \in P$), and "d" represents " d_F", or " d_{IF}", or " d_N", when it is a

fuzzy membership degree, an intuitionistic fuzzy membership, or a neutrosophic membership degree, respectively, of an element x to the plithogenic set P;

b) "c" means " c_F", or " c_{IF}", or " c_N", when it is a fuzzy attribute value contradiction degree function, an intuitionistic fuzzy attribute value contradiction function, or a neutrosophic attribute value contradiction function, respectively.

c) The functions are defined according to the applications that the experts need to solve. $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ Then, the following notation is used: $x(d(x,V))$ where $d(x,V) = \{d(x,v)$, for all $v \in V\}, \forall x \in P$. The attribute value contradiction function is computed between each attribute value for the dominant attribute value (denoted by) in particular, and also for other attribute values v_D.

1. Plithogenic - These include union (OR), intersection (AND), and other aggregation operators that combine attribute values based on t_{norm} and . t_{conorm} Both linear and nonlinear aggregation operations can be created.
2. Contradiction and aggregation calculation: The contradiction function c evaluates the contradiction between attribute values. Therefore, they influence how t_{norm} and t_{conorm} are applied to create aggregation operators.
3. If t_{norm} is applied on the value of the dominant attribute denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then it is applied on the value of the attribute v_2 as follows:

$$[1 - c(v_D, v_2)] \cdot t_{norm}(v_D, v_2) + c(v_D, v_2) \cdot t_{conorm}(v_D, v_2), \quad (1)$$

4. Or according to the following symbology:

$$[1 - c(v_D, v_2)] \cdot (v_D \wedge_F v_2) + c(v_D, v_2) \cdot (v_D \vee_F v_2), \quad (2)$$

5. Similarly, if t_{conorm} applied to the value of the dominant attribute denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then it is applied to the value of the attribute v_2:

$$[1 - c(v_D, v_2)] \cdot t_{conorm}(v_D, v_2) + c(v_D, v_2) \cdot t_{norm}(v_D, v_2), \quad (3)$$

6. Or, according to the following symbology:

$$[1 - c(v_D, v_2)] \cdot (v_D \vee_F v_2) + c(v_D, v_2) \cdot (v_D \wedge_F v_2), \quad (4)$$

7. neutrosophic intersection and union : They are defined in such a way that one criterion is applied for membership and its opposite for non-membership, while for indeterminacy the average is taken.
8. plithogenic is defined as:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right) \quad (5)$$

9. plithogenic is defined as:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right), \quad (6)$$

10. Resolution and decision matrix: Formulas are used to calculate the median of the plithogenic numbers, allowing a single decision matrix to be constructed for all specialists.

$$\text{median}_{i=1}^{m}\{PN_i\} = (\text{median}_{i=1}^{m}\{T(PN_i)\}, \text{median}_{i=1}^{m}\{I(PN_i)\}, \text{median}_{i=1}^{m}\{F(PN_i)\}), \quad (7)$$

Where the elements analyzed consist of plithogenic numbers , showing the components of truth, indeterminacy and falsity. In other words, it means that the median of a set of plithogenic numbers is defined as the plithogenic number of the medians of its components $PN_i, T(PN_i), I(PN_i)$, and$F(PN_i)$

To compare relationships between quadrants, the following formula is used to blur a neutrosophic number:

$$\mathcal{S}([T, I, F]) = \frac{2 + T - I - F}{3} \quad (8)$$

• For each row of the pairwise comparison matrix, calculate a weighted sum based on the sum of the product of each cell by the priority of each corresponding alternative or criterion (see Table 1).

Table 1: Linguistic expression to determine the level of importance of the factor on the variable

Linguistic expression	Scale	plithogenic (T, I, F)	S
Poor significance (PS)	0	(0,0,9,1)	0.03
Least significant (LS)	1	(0,2,0,8,0,8)	0.20
Low significance (LS)	2	(0.4,0.7,0.6)	0.37
Moderately significant (MS)	3	(0.5,0.5,0.5)	0.50
Significant (S)	4	(0,6,0,3,0,4)	0.63
Most significant (MS)	5	(0.8,0.2,0.2)	0.80
Very significant (VS)	6	(0.9,0,0.5)	0.95

3. Plithogenic IADOV method

The IADOV plithogenic technique is an assessment method that uses five questions, three multiple choice and two open, to measure respondents' satisfaction [13,19] . The particularity of this method lies in its "IADOV Logical Table", which connects three of the questions in a way that is hidden from the participant to infer satisfaction through their interrelations. By extending this technique to the plithogenic context and using a neutrosophic scale [14] , the ability to measure aspects that are indeterminate or inaccessible with conventional methods is introduced. This makes it possible to address the complexity of respondents' perceptions. It requires an assessment system adapted to the neutrosophic model to accurately capture experts' opinions (see Table 2). This system and its neutrosophic equivalents are defined as the scoring function A of a neutrosophic number according to the proposal of Basset.

Table 2: Expert evaluation system.

Linguistic term	SVNN	Scale
Clearly satisfied	(1,0,0)	0.50
More satisfied than dissatisfied	(0.75,0.20,0.25)	0.40
Indefinite	Yo	0.25

Linguistic term	SVNN	Scale
More dissatisfied than satisfied	(0.25,0.70,0.75)	0.15
Clearly dissatisfied	(0,0,1)	0.00
Contradictory	(1,0,1)	1.00

The term *I* in Neutrosófica is interpreted as a unit of indeterminacy. Another component of the method is the IADOV Logic Table, which assigns numerical values to three closed questions applied to experts (according to the references consulted [15,20]). If necessary, open questions can also be applied to surveys. Among the questions used in this study are:

1. Do you think that indigenous rights in Ecuador are adequately implemented in the community?
2. What areas do you think require urgent attention to strengthen the rights of indigenous communities?
3. What are the most significant advances you have observed in the implementation of indigenous rights in your community?
4. Can you describe a specific experience where you felt that indigenous rights were well defended or promoted?
5. Are indigenous rights implemented in the community?

To calculate the Neutrosophic Plithogenic Global Satisfaction Index (NPGSI) of the respondents, $H_N^P$the aggregation operator was used, considering the evaluations of each element X to the plithogenic set P ; $x \in Pd_F\, d_{IF}\, d_N$. Thus, the NPGSI is obtained as the sum of the elements analyzed within the plithogenic subset (S_i^P) evaluated[21, 22].

$$H_N^P\left(S_{1,}^P S_2^P, \ldots, S_n^P\right) = \sum_{i=1}^{n} \left[w_j\, , S_i^P\right] \tag{11}$$

4. Results and discussion.

The research reveals the composition and distribution of Indigenous nationalities in Ecuador, their relationship with the land and nature, and their perceptions of community justice contrasted with Western legal visions. It highlights the holistic connection with the environment, the value of community justice focused on restoration and reconciliation, and the importance of ancestral wisdom in conflict resolution, differing notably from the individualism and punitive approach of the Western legal system. For the modeling of the Plithogenic IADOV method, a sample of 35 representatives of the Pilahuines , Chibuleos , Tomabelas and Salasacas ethnic groups was taken . Among them, they act as authorities to administer justice in their community for family, civil or administrative conflicts.

The effective implementation of indigenous rights in Ecuador, despite significant legal and constitutional advances, faces multiple challenges. By using a plithogenic approach with neutrosophic numbers, the disparity between legislation and practice can be assessed from a perspective that recognizes complexity and indeterminacy. Key challenges are identified below and quantified using the IADOV plithogenic method to assess satisfaction with the effective implementation of Indigenous rights (see Figure 1).

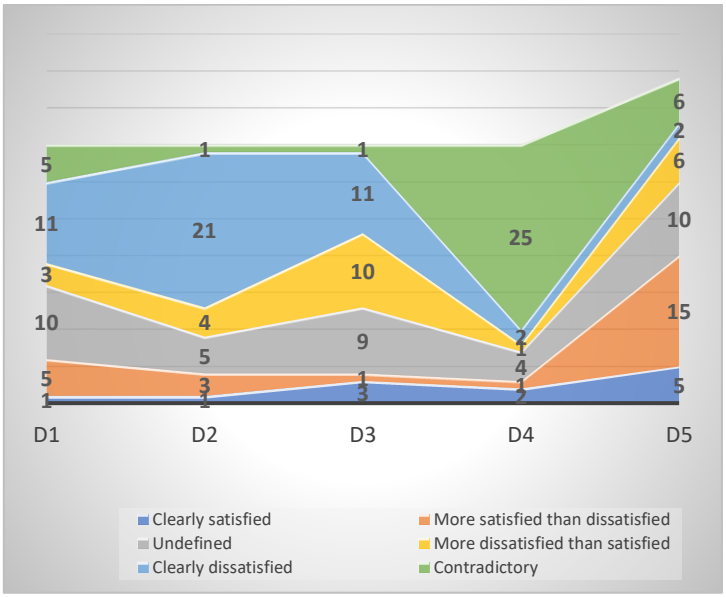

Figure 1: Plithogenic areas of satisfaction with the implementation of indigenous rights.

The satisfaction levels (GS) are observed according to the plithogenic areas related to the implementation of indigenous rights. Based on the challenges and criteria presented by the respondents, the following information was obtained:

1. Access to land and natural resources (D1) $H_{D1}^{P} = 0.30$is at a sub-level between I and MSI and CS. Therefore, respondents tend to feel *more satisfied than dissatisfied* with the demarcation of their lands and the concession of these to third parties.
    - GS-(0.75, 0.20, 0.25) : Although legislation recognizes indigenous rights over their ancestral lands and resources, the practice is often hampered by economic and extractive interests.
    - S-(I): There is a high level of uncertainty regarding the demarcation of indigenous territories and the granting of rights over natural resources, affected by government policies and pressure from extractive companies.
    - GS-(1, 0, 1) : The contradiction between recognized rights and their implementation reflects important challenges in the real protection of these rights.
2. Participation in decision-making (D2): $H_{D2}^{P} = 0.13$Located between the MSI and the CI, the respondents tend to feel *even more dissatisfied than satisfied* with the exercise of executive, legislative and jurisdictional powers in Ecuador.
    - GS-(1, 0, 0) : Mechanisms for indigenous participation in political and legislative matters have been established.
    - GS-(I): The effectiveness of this participation is variable, with cases of symbolic participation or limited real influence on important decisions.
    - GS-(1, 0, 1) : The discrepancy between inclusion intention and operational practice shows limitations in allowing genuine and meaningful participation.

3. Intercultural Bilingual Education (D3): $H_{D3}^{P} = 0.19$Located between the MSI and I areas, respondents show a tendency towards dissatisfaction and indetermination regarding the implementation of intercultural bilingual education. This is due to the variation in its implementation, resulting in a classification of *less dissatisfied and indefinitely satisfied* .
    - GS-(1, 0, 0) : Bilingual intercultural education is legally recognized and promoted as a right.
    - GS-(I): Implementation varies significantly across regions and is affected by resource availability, teacher training, and appropriate content.
    - GS-(0.25, 0.70, 0.75) : There are gaps between educational policies and their practical implementation, which reflects challenges in the universalization of quality education that respects cultural diversity.
4. Indigenous justice and legal pluralism (D4): $H_{D4}^{P} = 0.79$This index is located in the areas of CS C. The trend is towards contradictions in legal systems, so, although it is recognized, it is not applied correctly.
    - GS-(1, 0, 0) : The constitutional recognition of indigenous justice represents a step towards legal pluralism.
    - GS-(I): Coordination between ordinary jurisdiction and indigenous justice presents uncertainties, especially in the definition of competencies and mutual respect for decisions.
    - GS-(1, 0, 1) : Conflicts and misunderstandings between legal systems highlight the difficulties in the harmonious implementation of legal pluralism.
5. Conservation of heritage and traditional knowledge (D5): $H_{D5}^{P} = 0.20$It can be classified as a classification of *least dissatisfied and not defined satisfied* .
    - GS-(1, 0, 0) : There are legislative efforts to protect indigenous cultural heritage and traditional knowledge.
    - GS-(I): The effectiveness of these measures is uncertain, facing challenges of unauthorized appropriation and commercialization.
    - GS-(1, 0, 1) : The discrepancy between normative protection and violations in practice underscores the need to strengthen conservation mechanisms.

This Plithogenic IADOV analysis highlights that while there is progress in the recognition of indigenous rights, effective implementation faces a complex interplay. The Plithogenic areas used illustrate the disparity between legislation and practice, emphasizing the need to address indeterminacy to improve the protection of indigenous rights in Ecuador. Furthermore, the method introduces new scales to understand the results and respondents' feelings regarding the effectiveness of indigenous rights in Ecuador.

Another critical point to highlight is the perception of indigenous communities regarding the exercise of their executive, legislative and jurisdictional powers in Ecuador, revealing a complex panorama characterized by variations in satisfaction and internal contradictions. Through the use of plithogenic areas, these perceptions can be quantified and analyzed, reflecting the diverse realities and challenges that these communities face within the framework of their recognized rights. The results visualized in the plithogenic areas allow a more nuanced and in-depth evaluation of the degrees of satisfaction (see Table 3).

Table 3: Perception of indigenous communities regarding the exercise of their executive, legislative and jurisdictional powers.

Observation of respondents	Perceptions on the exercise of executive powers	Perceptions on the exercise of legislative powers	Perceptions on the exercise of jurisdictional powers
(1,0,0)	There is moderate recognition of progress in autonomy and the exercise of executive powers at the local level, especially in the management of natural resources and community projects.	-	There is a high degree of satisfaction and appreciation of the ability to exercise community justice based on traditional systems and norms, considering it crucial for the preservation of culture and social cohesion.
(0.75,0.20,0.25)	-	Participation in legislative spaces is valued but is limited by structural barriers and the lack of effective representation in key decisions.	-
I	There is uncertainty about the consistency and continuity of these executive powers, especially in relation to political changes and the variability of institutional support.	There is great uncertainty about the real influence of indigenous participation in the creation of laws and policies that directly affect their communities.	Uncertainties persist regarding the interaction and recognition of indigenous justice within the national legal system, especially in cases of conflict between jurisdictions.
(1,0,1)	There is a perceived contradiction between the formal recognition of these powers and the practical limitations on their effective exercise, due to resource and capacity restrictions.	Expectations about the ability to influence the legislative process often clash with the reality of a political system that does not always adequately reflect or integrate indigenous perspectives and needs.	Tensions and contradictions arise when the application of community justice faces legal challenges or external perceptions of incompatibility with universal human rights.

This analysis of the Plithogenic IADOV highlights the complexity of indigenous perceptions regarding the exercise of their recognized powers. While there are positive aspects, especially at the jurisdictional level, significant challenges persist in the executive and legislative spheres, reflected in

degrees of uncertainty and contradiction. The neutrosophic evaluations used illustrate not only levels of satisfaction but also areas of conflict, emphasizing the need for a more inclusive and equitable approach to the implementation of indigenous rights and powers.

Furthermore, the complexity of indigenous participation, consultation and representation in Ecuador is evident, showing both significant progress and challenges. While efforts to include Indigenous communities in decision-making processes are acknowledged, uncertainties and contradictions in their implementation indicate the need to strengthen these mechanisms to ensure effective and respectful participation that truly reflects the needs and aspirations of indigenous peoples.

Plithogenic integration for the implementation of indigenous rights in Ecuador.

Based on plithogenic analysis in a neutrosophic setting, solutions can be proposed to improve the implementation of indigenous rights in Ecuador. These solutions should focus on addressing areas of high uncertainty and contradiction, while reinforcing what has proven to be effective. The following sequence of plithogenic integration is presented:

1. Design the structure of the plithogenic set and its characteristics (see Table 4),
2. Propose strategies to mitigate challenges within each plithogenic subset.
3. Evaluate interactions to determine those that are close or of greater importance, which will be integrated into the proposed solutions.

Table 4: Structure of the plithogenic set for the implementation of indigenous rights in Ecuador.

Plithogenic set	Subset	Attributes	Variables	Factors
Implementation of indigenous rights	V_1:Prevention of violations.	a. Prevention policies. b. Awareness programs.	1. Number of programs executed. 2. Level of community participation.	A. Existing legal framework. B. Assigned resources.
	V_2:Rights education.	c. Curriculum contents. d. Teacher training.	3. Coverage of intercultural education. 4. Comprehension assessments.	C. Educational policy. D. Investment in education.
	V_3:Political participation.	e. Access to political spaces f. Effective representation.	5. Number of indigenous representatives. 6. Impact of legislative initiatives.	E. Electoral legislation. F. Political culture.
	V_4:Territorial management.	g. Community management systems. h. Protection of the earth.	7. Land conflicts resolved. 8. Formally recognized territories.	G. Land claims. H. Development policies.
	V_5:Justice and legality.	i. Recognition of indigenous justice. j. Access to legal services	9. Cases handled by indigenous justice. 10. Rulings in favor of indigenous rights.	I. Indigenous legal framework. J. Judicial infrastructure.

The structure of the neutrosophic plithogenic set allows for a neutral analysis of the situation of indigenous rights in Ecuador. This helps to identify where and how policies and programs can be improved to achieve a more effective implementation of indigenous rights. To establish the structure $d_N$and values of the attributes within the set, neutrosophic values are assigned to each attribute based on the responses of the representative sample of respondents (see Table 5). The structure of the plithogenic set for the analysis is as follows:

Plithogenic set: Implementation of indigenous rights in Ecuador

1. Subset (V_1): Violation prevention.
   - Attributes: Prevention policies and awareness programs.
   - Attribute Value: Neutrosophic values are assigned based on the perceived effectiveness of these programs.
2. Subset (V_2): Rights education.
   - Attributes: Curricular content, teacher training.
   - Attribute Value: Evaluated in terms of the coverage and quality of intercultural education.
3. Subset (V_3): Political participation.
   - Attributes: Access to political spaces, effective representation.
   - Attribute value: Measured by the number and impact of indigenous representatives.
4. Subset (V_4): Territorial management.
   - Attributes: Community management systems, land protection.
   - Attribute value: It is reflected in conflict resolution and recognition of the territory.
5. Subset (V_5): Justice and legality.
   - Attributes: Recognition of indigenous justice and access to legal services.
   - Attribute value: Based on the frequency and outcome of cases handled by indigenous justice.

Table 5: Values of the attributes $d_N$in the plithogenic set.

No.	Subset	Attribute	Attribute valued_N
1	Prevention of violations	Prevention policies	(0.8,0.2,0.2)
		Awareness programs	(0,6,0,3,0,4)
2	Education on rights	Curricular contents	(0.9,0,0.5)
		Teacher training	(0.8,0.2,0.2)
3	Political participation	Access to political spaces	(0.5,0.5,0.5)
		Effective representation	(0.4,0.7,0.6)
4	Territorial management	Community management systems	(0,6,0,3,0,4)
		Protection of the earth	(0.5,0.5,0.5)
5	Justice and legality	Recognition of indigenous justice	(0,6,0,3,0,4)
		Access to legal services	(0.5,0.5,0.5)

It can be observed that the multi-attribute neutrosophic plithogenic set with dimension 5 and cardinality 2x2x2x2x2 = 32 presents dominant values in the attributes v_a, v_c, v_e, v_g, and v_i for each subset. Additionally, the evaluation of the measurement scale of the plithogenic set establishes the neutrosophic values assigned to each attribute in relation to the effectiveness of policies related to indigenous rights in Ecuador. The purpose of this structure is to provide a tool for decision-making based on the analysis of the complexity and contradictions inherent to the subject. Based on each subset, the following priority strategies and actions are proposed:

A. Strengthen prior consultation: Implement effective and binding mechanisms for prior consultation with indigenous communities on projects that affect them.
   Action: Develop and implement a clearer and more rigorous regulatory framework for prior consultations to ensure that they are binding, systematic and culturally appropriate. Training of indigenous officials and leaders in negotiation and intercultural dialogue.
   Time: Short term.
B. Promote intercultural education: Ensure the inclusion of intercultural and bilingual education programs throughout the education system.
   Action: Increase investment in the training of bilingual educators and in the development of teaching materials that reflect Ecuador's cultural and linguistic diversity. Ensure the effective implementation of intercultural bilingual education in all regions.
   Time: Medium term.
C. Enhance political and legislative participation: Facilitate platforms for effective political representation of indigenous peoples in the legislative sphere.
   Action: Create permanent spaces for dialogue between indigenous representatives and legislators to ensure that indigenous proposals and concerns are effectively considered in the creation of laws. Implement a platform to monitor legislative initiatives proposed or supported by indigenous representatives.
   Time: Short to medium term.
D. Recognize and implement indigenous justice: Strengthen recognition and coordination between indigenous justice and the national judicial system.
   Action: Promote the development of coordination protocols between indigenous justice and the national legal system, recognizing and respecting the powers and decisions of each. To this end, training in human rights and intercultural justice is promoted for operators of both systems.
   Time: Medium term.
E. Legally protect lands and resources: Implement and enforce laws that protect the lands and natural resources of indigenous communities.
   Action: Establish a robust legal framework for the demarcation and titling of ancestral lands that prioritizes indigenous rights over natural resources, complemented by surveillance and monitoring programs to prevent unauthorized exploitation.
   Time: Long term.

This reformulated framework directly addresses the context of indigenous rights in Ecuador by addressing key areas such as consultation and participation, education and representation, and justice and legality. Prioritised strategies and actions aim to address existing challenges and promote the effective and equitable implementation of indigenous rights.

These solutions, prioritized through a plithogenic approach and quantified with neutrosophic numbers, seek to comprehensively and effectively address the challenges identified in the implementation of indigenous rights in Ecuador. This approach allows efforts to be concentrated in the areas of greatest need, promoting a more coherent and fair implementation of indigenous rights.

Implementation of the plithogenic intersection.

Neutrosophic plithogenic intersection involves the combination of two or more subsets and their attributes, where each aspect is common between them. The study identifies intersections where there is an overlap of efforts, objectives or results. Among them, three are classified for development:

a. Curricular content and access to political spaces.
b. Protection of lands and recognition of indigenous justice.
c. Prevention policies and access to legal services.

The degrees of contradiction between the values of each attribute within the intersection presented in the subsets are defined (see Table 6). The results show that:

- Subset $V_2; V_3: c_N(v_c, v_e) = 0.45$
- Subset $V_5; V_4: c_N(v_i, v_h) = 0.20$
- Subset $V_1; V_5: c_N(v_a, v_j) = 0.30$

Table 6: Plithogenic neutrosophic intersection between subsets according to equation (5). Source: own elaboration.

Intersection attributes	(a1, a2, a3) ∧ p(b1, b2, b3)	S_N	Assessment
Curricular content and access to political spaces	(0.68,0.25,0.5)	0.64	It is located in a sublevel closer to I than to MSI.
Protection of lands and recognition of indigenous justice	(0.52,0.27,0.48)	0.59	It is located in a sublevel closer to I than to MDI.
Policies on prevention and access to legal services	(0.59,0.35,0.41)	0.61	It is located in a sublevel closer to I than to MSI.

The results obtained define the three intersections close to an important classification for plithogenic integration in terms of the implementation of indigenous rights in Ecuador. Therefore, the characteristics of the three intersections within the subsets are determined (see Table 7).

Table 7: Plithogenic intersections in the implementation of indigenous rights in Ecuador.

Subsets	Intersection attributes	Plithogenic intersection	Cause of intersection	Advantages	Cons	Benefits
Education in Rights and Political Participation	Curricular content and access to political spaces	(0.68,0.25,0.5)	Education is essential for informed political	Strengthens the foundations for active civic	It can be challenging if education is not practically applied in	Improve the quality of democracy and

Subsets	Intersection attributes	Plithogenic intersection	Cause of intersection	Advantages	Cons	Benefits
			participation.	participation.	the political context.	empower citizens.
Territorial Management and Justice and Legality	Protection of lands and recognition of indigenous justice	(0.52,0.27,0.48)	Effective territorial management and justice are essential for sovereignty and autonomy.	It ensures coherent management of the territory and its resources.	Lack of integration can lead to duplication or legal conflicts.	It contributes to stability and respect for territorial rights.
Prevention of Violations and Justice and Legality	Policies on prevention and access to legal services	(0.59,0.35,0.41)	Prevention and a strong legal response are essential to protecting rights.	Create a comprehensive framework for prevention and justice.	If not well coordinated, preventive measures may not have legal backing.	Reduces the incidence of rights violations and improves legal response.

The intersection between education and political participation indicates that a solid foundation in rights education can significantly improve the quality of political participation. However, if education is not translated into effective political practices, the full potential cannot be achieved .

Regarding land management and justice, the intersection suggests that effective land management that respects indigenous justice can lead to greater sovereignty and autonomy. However, it is critical that laws and justice systems support land management rather than contradict it.

Finally, the intersection between crime prevention and legal justice suggests that for preventive policies to be effective, they must be supported by robust access to legal services. Lack of legal support can undermine preventive policies.

Each intersection seeks to harness the benefits of cooperation between different subsets to improve the implementation of indigenous rights in Ecuador. It is vital to ensure that these intersections not only exist in theory but also translate into coordinated and coherent actions in practice.

6. Conclusion

The analysis carried out using the Plithogenic IADOV method allows us to understand the perceptions of the indigenous peoples of Ambato regarding their rights. The results suggest legislative progress that has not yet been fully translated into practice. Although a positive trend in satisfaction with the jurisdiction is observed, the executive and legislative spheres show a lower degree of

satisfaction. This situation reflects a scenario of uncertainty regarding the effective exercise of rights, which shows the difficulty of homogenizing experiences within the framework of indigenous cultural diversity.

The study also reveals significant concerns about the participation and representation of indigenous peoples in legislative and executive decisions. It highlights the need to improve indigenous participation and strengthen their ancestral justice systems. It proposes intensifying the participation of these communities in political, legal and economic life by promoting an environment of mutual respect. This will contribute to an Ecuador that adequately values cultural diversity and promotes a more just social and legal balance.

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Use of Neutrosophic Cognitive Maps for a more complex representation of human perceptions

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Abstract. The research addresses a central problem in the modeling and analysis of human perceptions: the need for tools that capture more richly and accurately the complexity inherent in opinions, especially in contexts of high uncertainty and contradiction. In a scenario where traditional methodologies such as conventional cognitive maps lack the flexibility to represent nuances such as ambiguity or non-linear relationships, a key opportunity arises to explore innovative approaches. This methodological gap raises the question of how to integrate neutrosophic logic into analytical tools to improve the representation and analysis of complex perceptions in diverse environments. The study proposes and validates the use of Neutrosophic Cognitive Maps (NCM) as an advanced methodology that incorporates degrees of truth, falsity, and indeterminacy simultaneously. The results show that NCMs allow for modeling more dynamic and realistic scenarios, overcoming the limitations of traditional techniques. Moreover, its implementation not only contributes to the theory of neutrosophic logic but also presents significant practical applications in solving complex problems, from strategic decision-making to organizational planning. This contribution highlights how NCMs, by fusing mathematical rigor with interpretive flexibility, constitute an essential tool to address contemporary challenges in disciplines such as management, social sciences, and engineering.

Keywords: Neutrosophic Cognitive Maps, perceptual modeling, uncertainty analysis, neutrosophic logic, complex representation, indeterminacy, analytical tools, strategic decision-making.

1. Introduction

Neutrosophic cognitive maps emerge as fundamental tools in the judicial field, especially in the litigation process, due to their unique ability to capture, represent, and analyze the complexity of the mental models of the various actors involved (judges, plaintiffs, defenders, etc.). These maps allow for a richer and more nuanced representation of each actor's perceptions, beliefs and expectations, facilitating a deeper understanding of their positions and motivations. This ability to represent not only binary aspects (true/false) but also uncertainty and degrees of truth and falsity is crucial to addressing the inherently complex and multifaceted nature of legal cases [1] .

The use of neutrosophic cognitive maps contributes significantly to legal certainty. By offering a clear view of the perspectives of all actors, these maps help judges and parties to identify areas of consensus and disagreement, thereby reducing the chances of misunderstandings or misinterpretations of the parties'

intentions or arguments [2] . This clarity and transparency increase the predictability of judicial processes, a cornerstone of legal certainty, by ensuring that decisions are made based on a comprehensive understanding of all the aspects involved.

In terms of procedural fairness, neutrosophic cognitive maps play a crucial role in ensuring that all voices are heard and considered equally [3] . By explicitly visualizing the perceptions and arguments of each actor, these maps promote a more balanced and fair approach to decision-making, avoiding bias towards any one party and ensuring that the process is inclusive and representative of all viewpoints. This fairness is essential to maintaining trust in the justice system and to ensuring that all actors feel that their perspectives have been adequately valued.

The impact of the abandonment order on access to justice can be significantly modulated with neutrosophic cognitive maps. By identifying and understanding the perceived barriers and challenges faced by actors, especially plaintiffs, these maps can help identify proactive solutions to prevent abandonment of the process. Furthermore, by offering a clear representation of actors' mental models, they facilitate the identification of areas in which the justice system may be perceived as inaccessible or unfavorable, thus allowing the implementation of corrective measures that promote greater access to justice for all.

The use of neutrosophic cognitive maps to identify and represent the mental models of different actors in a judicial process is an innovative approach that seeks to capture the complexity of perceptions and attitudes toward abstract concepts such as legal certainty, procedural fairness, and access to justice, especially in the context of the abandonment order. This methodology is based on the neutrosophic theory, which extends classical logic and fuzzy logic by introducing the degree of indeterminacy along with the degrees of truth and falsity.

The steps to implement Neutrosophic Cognitive Maps are described (Figure 1):

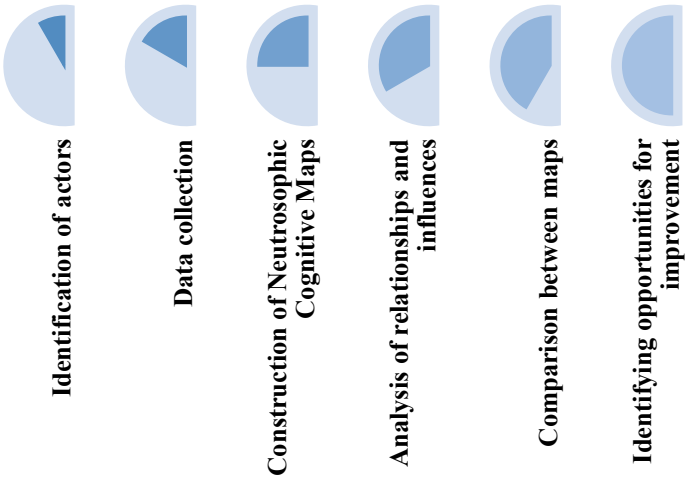

Figure 1: Steps for the implementation of Neutrosophic Cognitive Maps. Source: Elaboration own

1. Start by identifying the key players in the legal process of reporting: judges, complainants (victims), defenders (lawyers for the accused) and prosecutors.

2. Through interviews, questionnaires or workshops, collect data on the perceptions of these actors regarding legal certainty, procedural fairness and the impact of the abandonment order on access to justice.
3. Use the information you have gathered to build a cognitive map for each stakeholder group. Each map should represent key concepts as nodes and perceptions or beliefs about these concepts as links, with labels reflecting the degree of truth, falsity, and indeterminacy of these perceptions.
4. Analyze how these concepts interrelate and influence each other within each map.
5. Compare the neutrosophic cognitive maps of the different actors to identify areas of consensus, conflict or significant indeterminacy. This may reveal, for example, whether judges and complainants have very different mental models regarding the impact of the abandonment order on access to justice.
6. Based on comparative analysis, identify opportunities to improve the perception of legal certainty, procedural fairness and access to justice.

By applying the Neutrosophic Cognitive Maps methodology, it is possible to improve the perception and actions of the different actors involved in the judicial process of the complaint. In turn, this can inform more effective strategies to improve legal certainty and access to justice. The implications of using cognitive maps are described below in (Figure 2).

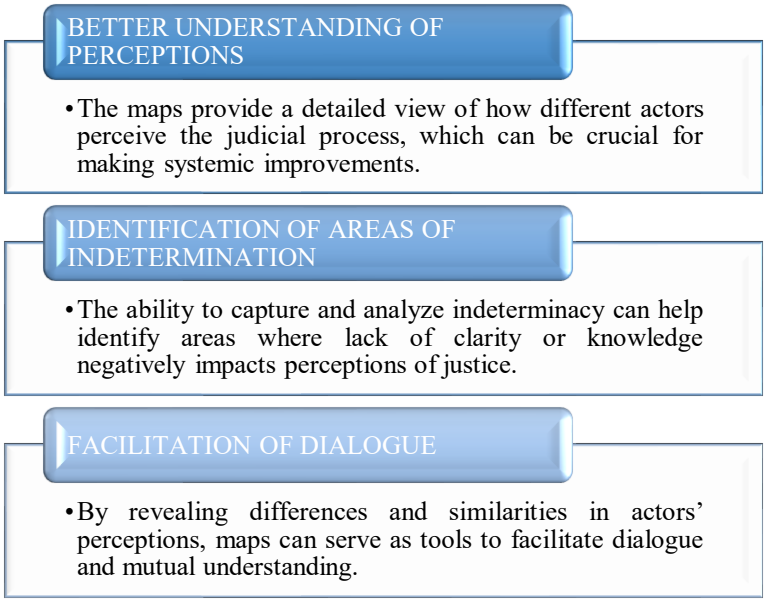

Figure 2: Implications of neutrosophic cognitive maps. Source: Own elaboration.

1.2 Legal framework on abandonment in the judicial process

The Ecuadorian criminal legal framework is aimed at promoting equal access to justice, as stipulated by the National Constituent Assembly [4,12] . It seeks to guarantee due process and safeguard the essential rights of citizens. Within this spectrum, the right of appeal takes on special importance, which empowers the parties involved in judicial disputes to challenge judicial decisions that they consider adverse.

Regarding the relevant Ecuadorian criminal legal provisions on the dismissal order within the framework of the complaint and its respective enforcement procedure, the Comprehensive Organic Criminal Code (COIP) and its subsequent reforms, enacted by the National Assembly in 2014, stand out. These legislations outline the principles and procedures that govern the criminal judicial framework in Ecuador, incorporating specific regulations on the complaint and the dismissal order. Fundamentally, article 647 of the COIP constitutes a cornerstone for understanding the complaint within the Ecuadorian criminal system. It defines and regulates the complaint as a special mechanism for the private exercise of criminal action, through which an individual can file a complaint with the judicial authority after being the victim of a private crime [5] .

The aforementioned article 647 defines the requirements and conditions for filing a claim, including the deadline for filing it and the formalities required. On the other hand, article 651 of the COIP refers directly to the order of withdrawal of the claim, establishing that, at the request of the accused party, the judge may issue the order of withdrawal when the complainant has withdrawn from pursuing the claim for a period of thirty days from the last procedural request. This implies that, in the event of inaction by the complainant to continue with the process within said period, the judge is empowered to declare the withdrawal of the claim, thereby concluding the procedure.

It is imperative to point out that the criminal law does not expressly provide for the possibility of appealing the decision of abandonment in the COIP. This omission is crucial in the context of the right to appeal, since, as it is a conclusive decision that puts an end to the process, it deprives the complainant of the possibility of challenging said determination before higher instances.

The lack of an explicit provision on the appeal of the order of abandonment creates uncertainty about the right of the complainant to appeal, which could violate the guarantee of due process and effective judicial protection. Such a situation could generate legal uncertainty and restrict access to justice for persons involved in a criminal process that culminates in the order of abandonment.

The figure of withdrawal from the claim has significant implications for the right of the complainant to appeal in the Ecuadorian judicial system. It is essential to examine both the advantages and the limitations that this legal provision may entail in terms of effective judicial protection and due process. Withdrawal is interpreted as a mechanism aimed at ensuring the agility and effectiveness of the criminal process, avoiding the indefinite prolongation of cases without substantial progress, which could contribute to the decongestion of the courts and the prompt resolution of other disputes.

However, from the perspective of the right to appeal, the order to abandon raises significant concerns, as it is considered a final decision that puts an end to the process and deprives the complainant of the right to challenge the decision before a higher instance, restricting the exercise of the right of defense and the possibility of a more detailed review of the judicial decision. The absence of a specific avenue of appeal for the order to be abandoned may lead to varied interpretations by the judicial body and generate disparities in the application of the law, affecting the uniformity and predictability of judicial decisions, which, in turn, could negatively affect legal certainty.

The research aims to analyze and improve the understanding of the complexities in the judicial litigation process with neutrosophic cognitive maps. Through this, it is intended to identify and mitigate the barriers faced by the different actors involved in access to justice, legal certainty and procedural fairness. This will be achieved by capturing, representing and analyzing the mental models of these actors regarding key concepts.

2. Related Work .

2.1. Neutrosophic cognitive maps (NCM).

Neutrosophic cognitive maps (NCMs) represent an evolution of conventional cognitive schemes by incorporating neutrosophic principles for the effective management of elements such as uncertainty, ambiguity, and contradiction. Originated by Florentin Smarandache in the late 1990s, neutrosophic cognitive maps extend the foundations of earlier systems such as fuzzy logic, intuitionistic sets, and fuzzy sets, providing a more robust framework for dealing with uncertainty and indeterminacy.

In a fuzzy cognitive map, both causal connections and concepts are characterized not only by binary values (true or false) or degrees of membership (as in fuzzy cognitive maps), but also by degrees of truth, falsity, and indeterminacy. This feature facilitates the modeling of scenarios in which connections between concepts are not fully defined or are inherently uncertain.

Effective application of NCMs requires an interdisciplinary approach spanning disciplines such as psychology, engineering, computer science, and mathematics to adequately simulate the complexity of real systems in uncertain contexts.

By fusing the elements of cognitive maps with the guidelines of neutrosophic theory, NCMs employ a set of mathematical formulations intended to facilitate the modelling and analysis of causal interactions between concepts in situations of uncertainty, ambiguity and contradiction. These formulations, based on neutrosophic logic and operations with neutrosophic sets, seek to quantify how variations of one concept affect others within the map structure.

A neutrosophic set is characterized by the presence of three fundamental parameters assigned to each element: the degree of truth (T), the degree of falsity (F) and the degree of indeterminacy (I), where each of these parameters varies from 0 to 1. The fundamental premises related to neutrosophic set are described in [8], [9] .

A neutrosophic cognitive map is a directed neutrosophic graph, where vertices represent concepts and edges represent causal relationships between vertices [10,13] .

Vertex Representation: If there are k vertices $C_1, C_2, \ldots, C_k$, each can be represented by a vector $(x_1, x_2, \ldots, x_k)$ where $x_i \in \{0, 1, I\}$ according to the state of vertex C_i at a specific time or situation:

- $x_i = 0$: Vertex C_i is in an activated state.
- $x_i = 1$: Vertex C_i is disabled.
- $x_i = I$: The state of vertex C_i is indeterminate.

Connections between vertices: A directed edge from C_m to C_n is called a connection and represents causality from C_m to C_n.

Assigning weights to each vertex: Each vertex in the NCM is associated with a weight within the set { 0 , 1, -1, I}. The edge weight $C_m C_n$, denoted as α_{mn}, indicates the influence of C_m on C_n and can be:

- $\alpha_{mn} = 0$: C_m *has no* effect on C_n.
- $\alpha_{mn} = 1$: An increase (decrease) in C_m produces an increase (decrease) in C_n.
- $\alpha_{mn} = -1$: An increase (decrease) in C_m produces a decrease (increase) in C_n.
- α_{mn} = I: The effect of Cm on Cn *is* indeterminate .

If $C_1, C_2, \ldots, C_k$ are the vertices of an NCM. The neutrosophic matrix N(E) is defined as $N(E) = \alpha_{mn}$), where α_{mn} denotes the weight of the directed edge $C_m C_n$, where $\alpha_{mn} \in [0.1]$ [11,14] N(E), is called *the neutrosophic adjacency matrix* of the NCM.

The coding used to identify the nodes resulting from the exchange with the study participants is described below:

N1 - Abandonment Order: Represents the judicial decision that ends the trial process without admitting any appeal, based on the inactivity of the plaintiff.

N2 - Right to Appeal: The legal capacity of plaintiffs to challenge judicial decisions, crucial for effective judicial protection and due process.

N3 – Legal uncertainty: Reflects the uncertainty and limitations in accessing justice generated by the impossibility of appealing the abandonment order.

N4 - International Human Rights Standards: International standards and principles that underline the importance of the right to appeal and to effective judicial protection.

N5 - Implications for plaintiffs: The direct consequences for people who file lawsuits, including their ability to defend their rights and obtain justice.

N6 - Judicial Training and Education: The need to educate judicial operators about regulatory changes and the importance of the right to appeal.

3. Results and discussion

For the development of the research, surveys were conducted separately to professional staff (judges, prosecutors, and lawyers) and non-professional parties (plaintiffs and defendants). Based on the results, the criteria displayed in the following tables and graphs were obtained, according to their degree of importance to them.

Table 1: Adjacency matrix (Group of legal professionals). Source: Prepared by own

	N1	N2	N3	N4	N5	N6
N1	0	0	1	0	0	0
N2	0.8	0.7	0	0	0	0
N3	0.7	1	Yo	0	0	Yo
N4	0.1	1	Yo	0	0	0
N5	0.5	0	0.9	Yo	0	0
N6	0.5	0	0.9	Yo	0	0

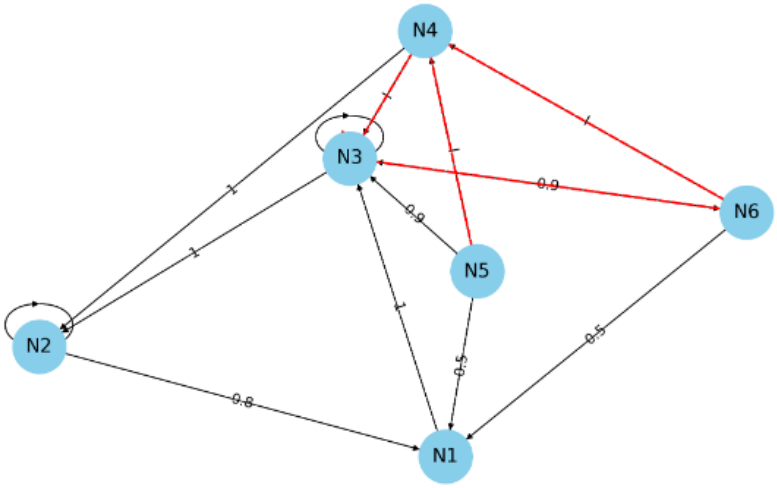

Figure 3: Neutrosophic cognitive map. Source: Own elaboration

Table 2: Centrality analysis (Legal Professionals Group). Source: Prepared by own

Node	$of(v_I)$	$id(v_i)$	$td(_{vi})$
N1	1.1	2.6	3.7
N2	1.5	2.7	4.2
N3	1.7+2I	2.8+2I	4.5+4I
N4	1.1+I	0+2I	1.1+3I
N5	1.4+I	0.4	1.8+I
N6	1.4+I	0+me	1.4+2I

This analysis reveals the complex interdependence between the right of appeal, judicial training and international human rights standards in the context of ensuring justice and fairness. It highlights how legal uncertainty emerges as a central problem, influenced by the ability to appeal and the need for judicial training. Neutrosophic values and centrality measures reflect the uncertain and multifaceted nature of these concepts and their impact on the judicial system. [15, 16]

Table 3: Adjacency matrix (Group of non-legal professionals). Source: Own elaboration

	N1	**N2**	**N3**	**N4**	**N5**	**N6**
N1	0.5	0	0	0.4	0.3	0
N2	Yo	0.2	0	0	0	0

	N1	**N2**	**N3**	**N4**	**N5**	**N6**
N3	1	Yo	0	0	0	0.8
N4	0	0.9	Yo	0	0.1	0
N5	0	0.9	Yo	0	0.1	0
N6	0	1	0	0	0.5	Yo

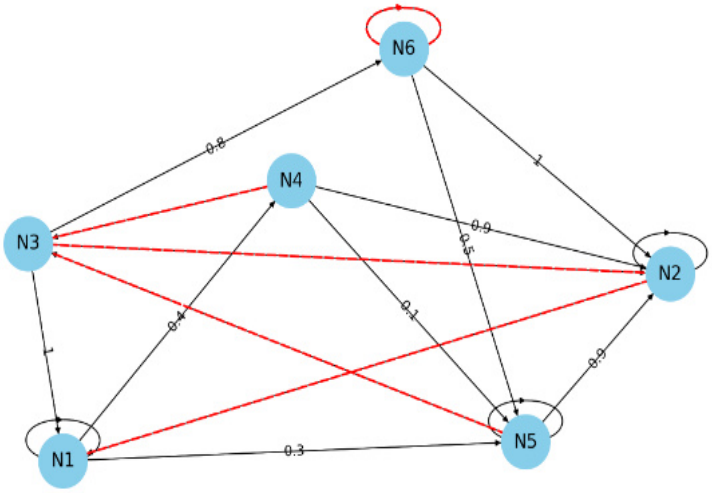

Figure 4: Neutrosophic cognitive map. Source: Own elaboration

Table 4: Centrality analysis (Non-legal professionals group). Source: Prepared by own

Node	***of*(*v* $_I$)**	***id*(*v* $_i$)**	***td*($_{vi}$)**
N1	1.2	1.5+yo	2.7+yo
N2	0.2+I	3+me	3.2+2I
N3	1.8+I	0+2I	1.8+3I
N4	1+me	0.4	1.4+I
N5	1+me	1,2	2.2+I
N6	1.5+yo	0.8+yo	2.3+2I

The analysis of the neutrosophic cognitive map, in the context of a study involving non-legal professionals, reflects the complex interactions and perceptions about the judicial system and the appeal process. The adjacency matrix and centrality analysis provide both quantitative and qualitative insight into

these relationships, using specific coding for each node, which relates to different aspects of the judicial process and the rights of complainants.

Together, the cognitive map and centrality analysis illustrate the perception of a judicial system that could significantly benefit from greater clarity, training and alignment with international human rights standards to improve access to justice and legal certainty.

The combination of neutrosophic analysis (Figure 5) with centrality data provides a multidimensional view of the impact of certain nodes within both groups: legal professionals and non-legal professionals. This integration allows us to understand how certain topics and concepts are interconnected and how they influence the judicial system from different perspectives.

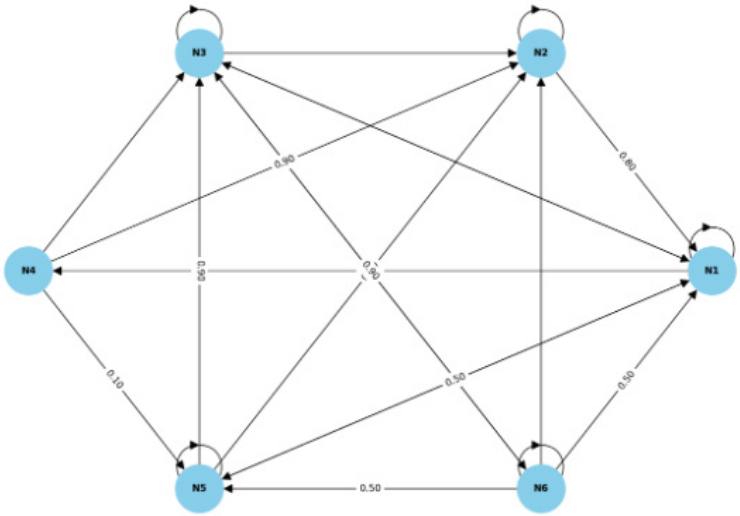

Figure 5: Combined neutrosophic cognitive map. Note: Own elaboration.

The combined neutrosophic cognitive map visualizes the relationships between different nodes, which represent the perspectives of both legal professionals and non-legal subjects. In this map, each node reflects a particular aspect of the judicial system, and the connections between them indicate the interactions and perceptions about their importance and functioning. The weights on the edges show the perceived strength of these connections, and the absence of a specific weight indicates areas of uncertainty or indecision (previously denoted by 'I') [15].

This map integrates the complexity and differences in perception between groups, offering a visual representation that highlights both areas of consensus and areas of uncertainty. It reflects how certain nodes, such as N3, can play a central role in the network, suggesting their importance in the justice system from both perspectives. The visualization facilitates the identification of key points to focus on for improvement, clarity, training and alignment with international human rights standards, in pursuit of fairer and safer access to justice.

Focusing on these areas could therefore significantly help improve people's ability to defend their rights and access justice. It is important to highlight the need to take measures to address these issues both in the training of legal professionals and in raising public awareness.

As a culmination of this research, the strategy for Improving Access to Legal Resources in Ecuador is proposed, which would address the points of convergence detected as a result of the study. It is essential to address the various barriers that citizens face in accessing justice. The implementation of this strategy can be structured in several key phases :

Needs assessment and mapping of existing resources:

- Conduct a national diagnosis to identify the main barriers to access to justice, including geographical, economic, linguistic and cultural barriers.
- Map existing legal resources across the country to identify areas of deficiency and oversaturation, ensuring equitable distribution of new resources.

Development of Online Platforms with Information and Legal Advice:

- Implement a comprehensive digital platform that provides clear and accessible legal information, tutorials and interactive tools to help people understand their rights and legal processes. This platform could also include an online consultation system to provide free basic legal advice.
- Ensure that the platform is accessible on multiple devices, including mobile phones, and is available in multiple languages, including the predominant indigenous languages in Ecuador.

Availability of Specialized Public Defenders:

- Expand the public defense program to include specializations in high-demand and complex areas.
- Implement a system of ongoing training for these lawyers, ensuring that they are up to date with the latest laws and legal practices.

4. Conclusions

This article explored the influence of perceptions of legal certainty and access to justice in the context of complaint processes through the use of Neurosophic Cognitive Maps (NCM). This innovative methodology allowed for a detailed and nuanced representation of the complexity inherent in judicial systems, capturing the various facets of truth, falsehood and indeterminacy that characterise the perceptions and experiences of those involved, both legal professionals and non-professionals.

The research showed that legal uncertainty (N3) constitutes a central axis in the perception of both groups. These findings underline the critical importance of addressing this aspect in order to improve access to justice and strengthen trust in the judicial system. The centrality of these nodes in the neutrosophic cognitive maps reflects not only the perceived relevance of these issues, but also their potential as intervention points for systemic reforms and improvements.

The application of NCMs revealed that the inclusion of degrees of truth, falsity and indeterminacy offers a deeper understanding of the dynamics and perceptions at play, facilitating the identification of priority areas for intervention. This approach allows for a richer and more detailed analysis than traditional methods, opening up new avenues for designing specific, evidence-based strategies to improve the justice system.

In response to these findings, a multifaceted strategy focused on improving access to legal resources in Ecuador was proposed. This strategy, designed to address the barriers identified in the study, includes the implementation of online information and advice platforms, the expansion of the availability of specialized public defenders, and the creation of free legal aid centers. Each of these components is geared toward addressing the specific needs of citizens and strengthening the judicial system from an inclusive and equitable perspective.

Adopting a proactive approach, as suggested in this study, has the potential not only to improve legal certainty and access to justice but also to increase the predictability of judicial proceedings. This, in turn, can contribute to preventing whistleblowers from abandoning proceedings, promoting a more fair, balanced and trustworthy justice system.

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Using the neutrosophic N-AHP method to prioritize variables and expert analysis

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Abstract. The study focuses on addressing a growing challenge in complex decision-making: how to prioritize key variables in an environment of inherent uncertainty and contradiction. In this context, the research explores the implementation of the neutrosophic N-AHP method, a variant of Hierarchical Process Analysis (HPA) that incorporates neutrosophic logic to handle ambiguity and indeterminacy in the evaluation of critical variables. Although traditional HPA has been widely used to prioritize options, it has been limited in contexts where expert perceptions are not clear-cut or are marked by high levels of uncertainty. This work seeks to overcome those limitations, proposing a more flexible and robust approach that allows integration of the inherent complexity in strategic and operational decisions. Through the use of the neutrosophic N-AHP, the study not only introduces an innovative methodology for expert analysis but also provides a powerful tool for prioritizing variables in a wide range of applications, from business management to public policy formulation. The results indicate that this approach significantly improves the accuracy and consistency of decision-making by allowing experts to express their judgments more completely, adapting to the reality of ambiguous decisions. The contributions of the study are vast: first, they open up new possibilities in research on decision-making under uncertainty; second, they offer practical applications that can be adopted in different sectors, improving the quality and effectiveness of decisions. In conclusion, this research not only advances the methodological field but also offers useful tools for those seeking to navigate highly complex scenarios.

Keywords: Neutrosophic N-AHP, prioritization, variables, expert analysis, uncertainty, contradiction, decision making, traditional AHP, neutrosophic logic, ambiguity, indeterminacy, hierarchization.

1 Introduction

Labor laws play a fundamental role in protecting and ensuring the job stability of workers in any society. These laws establish a legal framework that seeks to balance the relationships between employers and employees, granting rights and responsibilities to both parties. The influence of labor laws on job stability lies in creating a work environment where workers can be confident of the continuity of their employment and protection against unfair practices.

First, labor laws provide a protection mechanism against arbitrary and unjustified dismissal. They establish valid legal grounds for termination of employment, ensure that workers are not unfairly dismissed, and provide a sense of security in their jobs [1]. This protection is essential to enable workers to plan their lives and careers with confidence, without the constant fear of suddenly losing their jobs [2].

Furthermore, labor laws promote equal opportunities and non-discrimination in the workplace. These laws prohibit differential treatment based on personal characteristics such as gender, age, race, religion, or disability. This contributes to a more inclusive and fair work environment, where workers feel valued for their skills and contributions, without worrying about being discriminated against.

Labor laws also set minimum working conditions, such as the length of the working day, mandatory breaks, and regulation of overtime. By ensuring that workers are not exploited and have an adequate work-life balance, these laws contribute to greater job satisfaction and overall well-being. Workers can count on reasonable hours and adequate working conditions, which in turn promote emotional stability and productivity [3, 4].

In the area of collective bargaining, labor laws allow workers to join unions and negotiate with employers to improve their working conditions. This strengthens workers' voice in decision-making related to their work and provides the opportunity to reach agreements that support long-term stability. The ability to negotiate and reach collective agreements fosters a more harmonious and stable work environment [5, 6].

Labor laws play an essential role in protecting and promoting the job security of workers. These laws not only set minimum standards for labor relations but also reinforce workers' confidence in their jobs and fairness in the workplace. By regulating key aspects such as dismissal, non-discrimination, working conditions, and collective bargaining, labor laws contribute to the creation of a more stable, fair, and equitable working environment for all involved.

In Ecuador, labor legislation is designed to establish a set of guarantees and rights that seek to protect the interests and well-being of the country's workers. These guarantees ensure that employees have fair and safe working conditions. These guarantees not only benefit employees, but also contribute to a more fair, equitable, and productive work environment in Ecuador [7].

Job stability is a crucial aspect of the workplace, both for employees and employers. Ensuring a stable work environment involves compliance with a series of legal aspects that protect the rights and responsibilities of both parties [8]. Below are some of the key legal aspects related to job stability:

1. Employment contract: The basis of any employment relationship is the employment contract. It must specify the terms and conditions of employment, such as salary, work schedule, duration of the contract, job responsibilities, and other relevant aspects.
2. Protection from wrongful termination: In many jurisdictions, employees have the right to job security and protection from wrongful termination. Employment laws often provide valid legal grounds for termination, such as poor performance, misconduct, or financial reasons. In the event of termination, proper procedures must be followed.
3. Severance pay: In some countries, there is an obligation to pay severance pay for unfair dismissal or in specific circumstances. These compensations may vary depending on the employee's seniority and local legislation.
4. Stability of fixed-term employment contracts: In some places, fixed-term or temporary employment contracts may be subject to restrictions regarding their duration and renewal. Termination of a fixed-term contract may require valid justification.
5. Trade union rights: Workers have the right to join trade unions and participate in trade union activities to protect their labor interests and negotiate more favorable working conditions.
6. Leave and work permits: Labor laws often provide for sick leave, maternity or paternity leave, paid vacations, and other types of leave. These permits contribute to job stability by allowing employees to address personal situations without fear of retaliation.
7. Equal Opportunity and Non-Discrimination: Employers must comply with laws prohibiting workplace discrimination based on gender, age, race, religion, disability, or other protected characteristics.
8. Occupational health and safety: Employers are responsible for providing a safe and healthy work environment. This includes complying with safety regulations, providing personal protective equipment when necessary, and ensuring that employees are informed about occupational hazards.
9. Work-family balance rights: Some jurisdictions recognize employees' rights to request flexible schedules, teleworking, or other forms of work-family balance.

10. Personal data protection: Employees have the right to the protection of their data in the workplace and employers must comply with data privacy and security laws.

These are just some of the key legal aspects to ensure job stability. Laws and regulations may vary by country and jurisdiction, so it is important to check local legislation and seek legal advice to ensure compliance with all applicable requirements. In Ecuador, job stability is an important principle in the workplace and is supported by specific laws and regulations, including:

- Labor Code: This law is the main labor legislation in Ecuador and establishes the rights and obligations of employers and workers. It contains provisions related to job stability, employment contracts, justified and unjustified dismissals, severance pay, and other important labor aspects [8, 9].
- Constitution of the Republic of Ecuador: The Ecuadorian Constitution recognizes and guarantees the labor and social rights of workers, including the right to job stability. Additionally, it establishes the right of workers to collective bargaining and protection against arbitrary dismissal [10].
- Organic Law for Labor Justice and Recognition of Work in the Home: This law complements the Labor Code and establishes specific procedures for the resolution of labor disputes, including unjustified dismissal [11].
- General Regulations of the Organic Labor Law: This regulation provides additional details on specific aspects of the labor context and its application, including provisions on job stability and dismissal.
- Regulation on Unjustified Dismissal of Workers: This regulation establishes the procedures to follow in the event of unjustified dismissal and details the steps to file claims with the labor authorities.
- Organic Law of Public Service: This law regulates labor relations in the Ecuadorian public sector and establishes provisions related to job stability, hiring processes, and the rights of public sector workers [12].

The purpose of this research is to determine the influence of job stability on the guarantees that labor laws must provide and their effects in Ecuador. To carry out the study, the unstructured interview technique is used to collect data, as well as the multicriteria decision-making method Hierarchical Analytical Process for the selection of the most important variables to be analyzed. This method allows the measurement of both subjective and objective factors from numerical and verbal estimates, providing great flexibility in the analysis to be performed.

2. Related work.

2.1 Neutrosophic Concepts

Definition 1 : ([13,14]) The Neutrosophic set N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership function I_A, and the falsity membership function F_A, where U is the Universe of Discourse and $\forall x [\in U, T_A(x), I_A(x)$, and $F_A(x) \subseteq]^-0,1^+[$, $y^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$. Note that by the definition, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are standard or nonstandard real subsets of $]^-0, 1^+[$ and hence $T_A(x)$, $I_A(x)$, and $F_A(x)$ can be subintervals of [0, 1].

Definition 2 : The single-valued neutrosophic set (SVNS) N over U is A = {<x; TA(x), IA(x), FA(x)>: x ∈U}, where TA: U →[0, 1], IA: U →[0, 1] and FA: U →[0, 1], 0 ≤TA(x) + IA(x) +FA(x) ≤3. The single-valued neutrosophic number (SVNN) is represented by N = (t, I, f), such that 0 ≤t, I, f ≤1 and 0 ≤t + I + f ≤3.

Definition 3 : The single-valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set in $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined in [13, 15].

Definition 4 : Given $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$two $\tilde{b} = \langle(b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$single-valued trapezoidal neutrosophic numbers and λany nonzero number on the real line, the operations are defined in [13, 15].

Definitions 3 and 4 refer to the *single-valued triangular neutrosophic number* when the condition a2 = a3 . For simplicity, we use the linguistic scale of triangular neutrosophic numbers, see Table 1 and also compare with the scale defined in.

Table 1: Saaty scale translated into a neutrosophic triangular scale. Source [15]

Definition	Neutrosophic triangular scale
Equally influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
Very strongly influential	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
Absolutely influential	$\tilde{9} = \langle(9,9,9); 1.00, 1.00, 1.00\rangle$
Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

For verification of the Analytic Hierarchy Process model methodology in a neutrosophic environment (N-AHP), see [13, 15-17] .

2.2 Neutrosophic social choice theory

This subsection summarizes the main concepts of the neutrosophic social choice theory developed in [18].

Definition 5: ([18]) Let a = (Ta, Ia, Fa) be a single-valued neutrosophic number with truth value Ta, indeterminacy value Ia and falsity value Fa. *The distributed indeterminacy form* (DIF) of *a* is defined as DIF = (Ta − TaIa, 0, Fa − FaIa).

The DIF aims to distribute the result of indeterminacy regarding truth and falsehood, that is, it measures the degree of affectation of truth and falsehood, when indeterminacy varies.

Definition 6: ([18,19]) Let *a be* a univalent neutrosophic number. A *precision function* H of *a* is:

$H(a) = \frac{(1+T_a-I_a\,(1-T_a)-F_a(1-I_a)}{2}$(1)

Where for all *a* , H (a) Є [0, 1]. H is an order relation representing an information accuracy score of *a* . If H(a_1) = H(a_2), then $a_1 = a_2$, i.e., they have the same information, whereas, if H(a_1)< H(a_2), then $a_2$is greater than a_1.

Let S = {s1, s2, . . . , sn} be a set of alternatives and m a set of individuals. Each individual declares his preferences over S, which are represented by an individual neutrosophic preference relation Rk, where N R_k:

S × S → [0,1] × [0,1] × [0,1] and matrix Rk =[r_{ij}^k], j= 1,2,3,…, n; k = 1,2,3, . . . , m, where r_{ij}^k= N R_k(r_i^k, r_j^k).

$$R_k = \begin{matrix} (0.5, 0.5, 0.5) \\ r_{21}^k \\ r_{n1}^k \end{matrix}$$

The function H (called *the neutrosophic index* or *neutrosophic hesitation function*) assigns a_{ij} to each neutrosophic value a number in [0, 1]. Therefore, the *neutrosophic index* or *neutrosophic hesitation function* is defined as follows:

$H(a) = \frac{(1+T(a_{ij})-I\,(a_{ij})\,(1-T(a_{ij}))-F(a_{ij})(1-I(a_{ij}))}{2}$(2)

The matrix R_k^H= [H(r_{ij}^k)], i, j = 1,2,3, . . , n; k = 1, 2, 3, ..., m. R_k^H is *quasi-reciprocal* if and only if $H(r_{ij}^k) \leq 1-H(r_{ij}^k)$. If R_k^Hit is not quasi-reciprocal, we call ak an *irrational individual* . Other definitions are stated in [19].

Definition 7: ([19]) *:* $S_i \in W$ It is called *a consensus winner* if and only if $\forall S_j \neq S_i$: $r_{ij} > 0.5$,where$r_{ij} \in H_\pi$.

Definition 8: ([20]) The *average social aggregation function C* is defined to calculate the order of $S_i$in the group to the extent that individuals are not against the option yes, using the following equation:

$C(S_i) = \frac{1}{m-1}\sum_{i\neq j} r_{ij}$(3)

Where i, j = 1, 2, ⋯ , m.

3. Material and Methods

The population of individuals selected for the study was made up of workers from different backgrounds. A total of 63 workers, aged between 25 and 50 years, participated in the study, of which 56% were women and 44% men. Additionally, the sample included 8 professionals in Labor Law and 2 university professors in the same area, as part of the sampling group to provide their considerations on the subject analyzed and provide information from a professional perspective. After being informed about the objective of the study, 100% of them gave their consent to participate and collaborate.

The study was conducted based on the actors from whom the information was collected. The fundamental legal aspects to guarantee job stability, determined from the survey carried out on the workers, are analyzed, examining the effect of a set of variables. The Analytic Hierarchy Process (AHP) method is applied with the help of experts to determine the influential variables for the analysis.

In this section, the proposed NAHP+NSC method for the analysis of the object of study of the article is presented. The first element is defined for any neutrosophic triangular number ã as the triangular precision function of ã = ‹(a1, a2, a3); αa, βa, γa›, which is the TA function defined as follows[20]:

$TA(\tilde{a}) = A(\langle(a1, a2, a3);\ DIF((\alpha a, \beta a, \gamma a))\rangle)$ (4)

This is the degree of accuracy of equation 6 calculated for the DIF of the neutrosophic number contained in ã. The inclusion of the DIF follows the idea of, where the accuracy function H calculates the effect of indeterminacy on truth and falsity.

It can be seen that the reciprocal or quasi-reciprocal properties in NSC theory are similar to the reciprocal property in NAHP, from the perspective of the rationality of the decision maker.

The method analyzed consists of the following steps [20]:

1. The objective of the problem is established and, accordingly, the group of experts is selected. Attributes, sub-attributes and alternatives are then specified.
2. The expert group is divided into M interest subgroups, denoted by IG = {IG 1, IG 2, ⋯ , IG M}. In the analysis, it is assumed that the members of each subgroup form a homogeneous decision group.
3. Each expert evaluates his/her own NAHP. However, for each IG i, the equivalent matrices of the subgroup members are aggregated using formula 5.

Given {Ãi1, Vi2, …, Ã in_i} as a set of n iSVTNNs representing the evaluation of each member of the i-th subgroup, where Ã ij = ‹(a ij , a ij , a ij); α ã$_{ij}$, β ã$_{ij}$, γ ã$_{ij}$› (i = 1, 2, …, M)(j = 1, 2, …, n i), the weighted average of the SVTNNs is calculated using the following equation:

$$\tilde{A}_i = \sum_{j=1}^{n_i} \lambda_{ij}\, \tilde{A}_{ij} \quad (5)$$

where λ ijis the weight of Ã ij, $\lambda_{ij} \in [0,1]$and$\sum_{j=1}^{n_i} \lambda_{ij} = 1$.

Note that λ ijmeasures the relative importance of the j-th expert in the i-th subgroup.

Each Ã i represents the pairwise comparison matrix of the NAHP method in IGi, to aggregate the pairwise comparison matrices of criteria, subcriteria and alternatives.

$\tilde{A}_i$ is converted to $\tilde{A}_i$ using equation 13. This process can be repeated until the results are consistent according to the consistency index of the NAHP method. According to this method, a preference vector of the alternatives is obtained.

Individual judgment aggregation (IJA) is used here because there is interest in measuring the subgroup judgments as a synergistic unit.

Let be $O_i = \{O_{i1}, O_{i2}, \dots, O_{iN}\}$the position of each alternative when evaluated by the members of the i-th subgroup. For example, O1 ={1, 1, 3, 5, 4} means that according to the first subgroup, alternatives 1 and 2 are equally preferred, while subsequent alternatives are the third, fifth, and fourth alternatives, in that order.

4. For each S_{il}(l = 1, 2, ..., N), $V_{il}$the following triplet is formed $P_{il} = card\ (k \neq l: S_{il}$= (, where P_{il}, I_{il}, N_{il}) is strictly preferable to S_{ik}}), I_{il}= c ard ({$k \neq l: S_{il}$is equally preferred to S_{ik}}), and N_{il}= card ({k ≠ l: $s_{ik}$is strictly preferable over s_{il}}).

It is observed that, V_{il} ∈[0, N − 1] × [0, N − 1] × [0, N − 1] and P_{il}+ I_{il}+ N_{il} = N − 1.

Finally, V l ∈ [0, 1] × [0, 1] × [0, 1], V l = (P $_l$, I $_l$, N $_l$), sums the lth alternative preference for all subgroups, where P_l= $\frac{\sum_{i=1}^{M} P_{il}}{M(N-1)}$, $I_l = \frac{\sum_{i=1}^{M} I_{il}}{M(N-1)}$,and$N_l = \frac{\sum_{i=1}^{M} N_{il}}{M(N-1)}$.

Please note that this is a neutrosophic voting method.

5. H(V l) (l = 1,2, ⋯ , N) and the alternatives are ranked by preference, such that V $l_1$is preferable to V $l_2$if and only if H(V l_1) > H(V l_2). When H(V l_1) = H(V l_2), then " V is considered $l_1$equally preferable to V l_2". (See [20])

4. Results and discussion

The bibliographic analysis carried out on the subject of study, as well as the experience of the selected experts, allowed us to identify a set of factors that can influence job stability and the effectiveness of labor law in Ecuador. Among the most notable are:

1. Employment contract
2. Protection against unfair dismissal
3. Protection of personal data
4. Stability in fixed-term contracts
5. Trade union rights
6. License and work permits
7. Equal opportunities and non-discrimination
8. Safety and health at work
9. Rights to work-family conciliation
10. Severance pays

After several rounds of questions and processing of information through Delphi as shown in Table 2, with responses and adjustments to the data initially obtained, a consensus was reached to carry out the analysis with the variables shown, considering that they are the ones with the greatest influence and provide results of greater importance in the analysis of the research object in development.

Table 2: Delphi method. Source: own elaboration

Delphi	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Average	Standard deviation
Equal opportunities and non-discrimination	8	7	9	8	7	7.8	0.74833147
Employment contract	7	8	8	9	7	7.8	0.74833147
Protection against unfair dismissal	9	9	10	8	9	9	0.63245553
Health and safety at work	8	9	7	8	8	8	0.63245553
License and work permits	8	8	9	7	8	8	0.63245553
Severance pay	5	5	6	5	4	5	0.63245553
Stability in fixed-term contracts	10	9	10	9	10	9.6	0.48989794
Trade union rights	7	8	8	7	7	7.4	0.48989794
Protection of personal data	6	7	7	6	6	6.4	0.48989794
Rights to work-family conciliation	6	5	5	6	6	5.6	0.48989794

For the specific situations that occur most frequently and contravene labor legislation, the following measures or alternatives are proposed to analyze the level of influence and decision-making that favor the protection of labor rights:

1. Clear contractual conditions. (Review the terms and stipulations of the contract to protect against unfair dismissal)
2. Stability period. (Provide protection to the worker ensuring compliance during the time stipulated in the contract)
3. Administrative advice. (Through proper guidance from management, the employee can perform his/her work without difficulty)
4. Flexible negotiation. (Facilitate flexible adaptation of terms and conditions as necessary through mutual agreement between the contracted individual and the employer)
5. Supervision. (Contracted personnel must be periodically supervised by management to provide information and guidance aimed at improving the quality of service.)
6. Working conditions. (Minimum conditions necessary for carrying out the work must be provided, guaranteed by management)

Once the variables to be analyzed have been obtained, the experts are asked to make pairwise comparisons between them, so that it can be determined which ones have the greatest influence on the others concerning labor law. Once the individual assessments of all the experts have been obtained, an average of all the assessments is calculated. The result obtained from the analysis is shown in Table 3.

Table 3: Pairwise comparison matrix. Source: Own elaboration.

N-AHP	Clear contractual conditions	Stability time	Administrative advice	Flexible negotiation	Supervision	Working conditions
Clear contractual conditions	$\tilde{1}$	$\tilde{7}$	$\tilde{7}$	$\tilde{7}$	$\tilde{8}$	$\tilde{6}$
Period of stability	$\frac{1}{\tilde{7}}$	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{4}$	$\tilde{7}$
Administrative advice	$\frac{1}{\tilde{7}}$	$\frac{1}{\tilde{3}}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$	$\tilde{3}$
Flexible negotiation	$\frac{1}{\tilde{7}}$	$\frac{1}{\tilde{5}}$	$\frac{1}{\tilde{3}}$	$\tilde{1}$	$\tilde{1}$	$\tilde{3}$
Supervision	$\frac{1}{\tilde{8}}$	$\frac{1}{\tilde{4}}$	$\frac{1}{\tilde{3}}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$
Working conditions	$\frac{1}{\tilde{6}}$	$\frac{1}{\tilde{7}}$	$\frac{1}{\tilde{3}}$	$\frac{1}{\tilde{3}}$	$\tilde{1}$	$\tilde{1}$

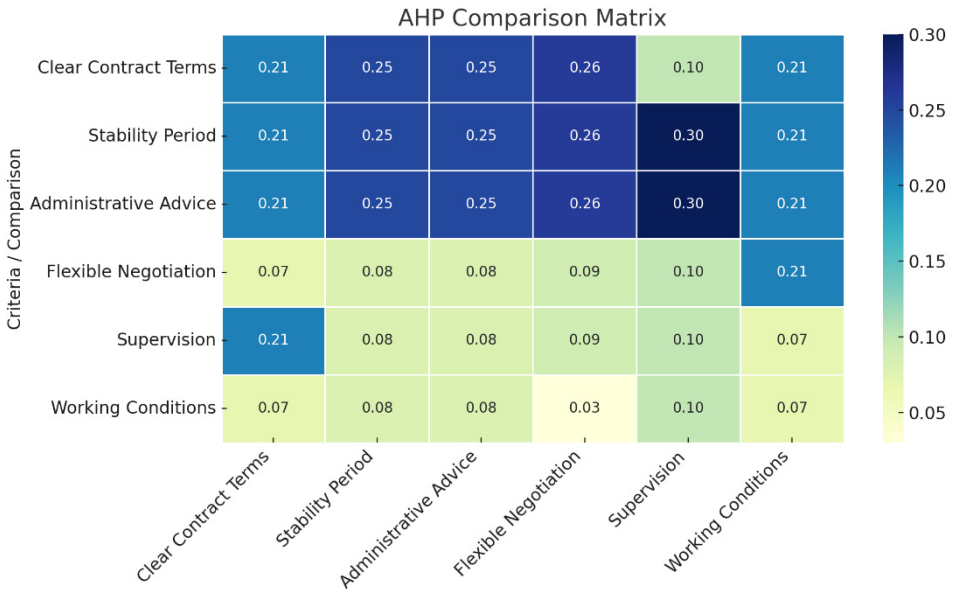

Figure 1. Heat map of the weighted matrix. Source: Own elaboration using scoring equation.

In Figure 1, each cell shows the relative importance of one criterion compared to another, with colors indicating the degree of importance: darker shades of blue represent higher values, while lighter shades indicate lower values. As can be seen, the weighted combination of 0.3 is the highest value and 0.03 is the lowest combination. The weight of each was then determined, as shown in Figure 2. After developing the

pairwise comparison matrix and continuing with the logic of the method, the weight vector of each of the factors analyzed in the consistency study is obtained, with an eigenvalue of 0.2815358, an IC coefficient of 0.06, and a CR of 0.04, which is considered consistent and therefore valid for the study.

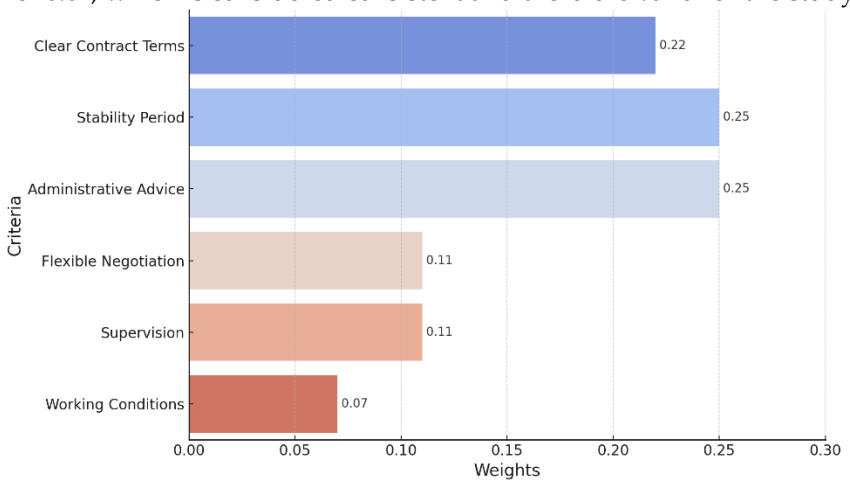

Figure 2: Weighting of criteria according to the N-AHP method. Source: Own elaboration

The image shows a horizontal bar chart representing the weights of various criteria in an AHP-N analysis. The criteria evaluated include "Clear contractual conditions", "Stability period", "Management advice", "Flexible negotiation", "Supervision" and "Working conditions". Each bar represents the relative weight of these criteria, indicating their importance in the overall decision or evaluation.

The weights range from 0.07 to 0.25, showing a clear differentiation between the most and least important criteria. “Stability period” and “Administrative advice” have the highest weight (0.25), indicating that they are considered very important in this analysis. On the other hand, “Working conditions” has the lowest weight (0.07), suggesting that although it is still important, it has less influence on the outcome of the analysis compared to the other criteria.

By applying these results to decision theory data, it can be verified that in a second iteration of data processing, the following weight vector P = [0.533, 0.209, 0.110, 0.058, 0.049, 0.039] is obtained, with the following order:

1. Clear contractual conditions
2. Period of stability
3. Administrative advice
4. Flexible negotiation
5. Supervision
6. Working conditions

As can be seen, there is a slight variation in the order, but the first three parameters remain the same. Therefore, it can be said that evaluating the effectiveness of labor legislation in terms of job stability involves analyzing a series of factors that can influence the way in which legal provisions are implemented and complied with in practice. The study revealed that for workers, aspects such as protection against dismissal, contractual relations and equal opportunities are essential to maintaining a safe work environment that allows them to feel supported by labor laws. [16,18,19]

Other key elements that influence the evaluation of the effectiveness of labor legislation on job stability in Ecuador are its compliance and application, since its effectiveness depends largely on its proper and consistent compliance and application. It should be assessed whether labor authorities have the resources and capacity to monitor and ensure that both employers and workers comply with the provisions related to job stability.

The clarity and accessibility of procedures and processes related to job stability are crucial. If workers and employers do not understand how to file complaints, claims for unfair dismissal, or how to conduct

collective bargaining processes, the effectiveness of the law will be compromised. Workers must have effective access to dispute resolution mechanisms and impartial judicial systems in the event of labor disputes. [20] The ease and effectiveness of these mechanisms will influence how issues related to job stability are addressed and resolved.

Labor legislation should provide clear and proportionate sanctions for non-compliance. The consequences of violating provisions relating to job stability should be significant enough to deter employers from taking unjustified measures. Both employers and employees should be informed about their rights and obligations regarding job stability. Education and awareness about labour legislation can help prevent misunderstandings and conflicts.

Overall economic conditions and job prospects can influence job stability. When economic conditions are precarious, employers may face financial difficulties that could affect employees' job stability. A country's broader economic and labor policies can affect job stability. An emphasis on sustainable economic development and employment promotion can support stability in the labor market.

A country's employment culture can influence how employment relations are handled. A corporate culture that values employees and an active union culture can contribute to job stability. Reforms and changes in labor laws can have a significant impact on job stability. It is essential to assess how these reforms are implemented and how they fit with the changing needs of workers and employers. Active involvement of employers, workers, and their representatives in formulating and revising labor policies can improve the effectiveness of laws related to job stability. Ultimately, assessing the effectiveness of labor legislation in terms of job stability is a multifaceted process that requires a detailed analysis of how legal provisions are implemented and adapted to the labor reality of a specific country [24,25].

5. Conclusion

This article has explored the impact of job stability on the effectiveness of labor legislation in Ecuador. Through the analysis and synthesis of information, it has been shown that job stability is an important aspect in the workplace, both for employees and employers. Labor legislation plays a fundamental role in protecting and promoting job stability by establishing standards that ensure fair and safe working conditions for workers. Through observation, analytical analysis, synthesis, inductive and deductive methods, and historical-logical analysis, various legal aspects that contribute to job stability have been examined in depth.

Mathematical methods of information processing, combined in a neutrosophic environment, effectively prioritize variables in the decision-making process, and experts in the field have participated in the identification of the most influential variables in job stability. The results of this analysis have revealed that the employment contract and protection against unjustified dismissal are the most influential variables in job stability, followed by equal opportunities and non-discrimination, as well as safety and health at work.

The study also highlighted the importance of compliance and effective enforcement of labor laws, the existence of clear and accessible procedures, access to justice, adequate sanctions for non-compliance, and education and awareness of labor rights and obligations. It emphasized how the effectiveness of labor legislation is interconnected with socio-economic, political and cultural factors, and how legislative reforms and social dialogue are essential to maintaining effective labor stability.

Labor laws in Ecuador guarantee fair and safe working conditions, and key aspects such as the employment contract, protection against unjustified dismissal, equal opportunities and job security are essential for job stability.

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Analysis of Genetic Components of Late Blight Resistance (Phytophthora infestans) in Potatoes (Solanum phureja) from Cutuglahua, Pichincha, Using Neutrosophic SuperHyperSoft Sets

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Abstract. Late blight (Phytophthora infestans) resistance in Solanum phureja has influenced potato genetic improvement due to the high incidence of this disease. This study analyzed key resistance attributes, such as lesion growth rate, lesion size, latency period, and sporulation intensity, through field and laboratory tests. To integrate these attributes and optimize the selection of resistant genotypes, Neutrosophic SuperHyperSoft Sets modeling was used. Among the results, genotypes such as Chaucha Amarilla (with high general combining ability) and progenies such as 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo) were identified, exhibiting high specific combining ability values. In conclusion, lesion size was identified as a crucial indicator, emphasizing the priority of integrating key attributes in genetic improvement programs for the development of more resistant and sustainable varieties.

Keywords: potato, infectious agent, potato rot, genetic resistance, Neutrosophic SuperHyperSoft Sets.

1 Introduction

Potato is a key crop globally, known for its high protein and energy yield per unit of land area [1]. In Ecuador, its economic and nutritional importance is reflected in the cultivated hectares and the yields achieved [2]. However, the crop faces critical challenges due to late blight [3], caused by Phytophthora infestans, a devastating disease that destroys plantations in just a few days, worsened by the constant emergence of new strains of the pathogen that overcome existing genetic resistances [4].

Therefore, the success in developing resistant varieties lies in the selection of parents with high general combining ability (GCA) and specific combining ability (SCA) [5][6][7]. This process is complex due to the tetra-allelic inheritance of potatoes, the polygenic nature of resistance, and the environmental influence on phenotypic expression [8][9]. To address these limitations, diallel methods, such as the one proposed by Griffing, are employed, which allow the evaluation of genetic interactions and the selection of promising hybrid combinations [10].

Therefore, this study analyzes key resistance parameters, such as lesion growth rate, lesion size, latency period, and sporulation intensity, through field and laboratory tests [11]. Additionally, Neutrosophic SuperHyperSoft Sets modeling is incorporated to facilitate the integration of multiple attributes, optimizing decision-making in the selection of genotypes. In fact, the goal is to identify progenitors and crosses with the potential to develop potato varieties resistant to late blight, contributing to the sustainability of the crop [12][13][14].

2 Preliminaries

2.1 Neutrosophic SuperHyperSoft Sets.

This section serves the purpose of remembering the basic notions of Fuzzy Extension SuperHyper-Soft Sets and neutrosophic theory [10].

Definition 1 ([1, 9, 10]**).** Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set [11] [12].

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A using the following formula:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, if the family F is known, the function μ_A can be reconstructed using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 2 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *HyperSoft Set* over U.

Definition 3 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([4, 5, 13-15])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}\left(U\big(x(d^0)\big)\right)$ is called a *Fuzzy_Extension SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc [13].

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set N* is characterized by three membership functions [14], which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [15].

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets

of $]^{-}0, 1^{+}[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^{-}0$ and 1^{+} belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$ and $F_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Single-Valued Neutrosophic Sets (SVNS) were introduced with the idea of applying neutrosophic sets for practical purposes. Some operations between SVNSs are expressed below [18, 9,20]:

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the sum of A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \quad (1)$$

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, two SVNNs, the multiplication of A_1 and A_2 is defined as:

$$A_1 \ A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \quad (2)$$

The product of a positive scalar SVNN, A = (a, b, c) is defined as:

$$A = (1 - (1 - a), b, c) \quad (3)$$

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

3 Results

3.1 Case study

This research was carried out in the greenhouse of the Santa Catalina Experimental Station, located at 3050 meters above sea level, with controlled conditions of temperature (21 °C) and relative humidity (70%). Three native potato progenitors (Chaucha Negra, Chaucha Amarilla, and Yema de Huevo) were selected for their resistance to late blight and good agronomic quality, attributes that make them suitable for the genetic improvement of potatoes.

For the experimental design, a diallel design according to Model I, Method 2 of Griffing, combined with a completely randomized block design with four repetitions, was used. The crossings between the selected progenitors generated progenies that were evaluated in relation to the key genetic components (identified as attributes a_i in section 3.2). These parameters were analyzed using the Pearson correlation coefficient at 5%. This allowed for the establishment of the relationship between the late blight resistance components.

The crossings were carried out under controlled conditions, both in the field and using the bottle method. In the case of field crossings, five plants per progenitor were planted, while in the bottle method, branches with inflorescences were kept in water with water-soluble fertilizers. Subsequently, the flowers were emasculated and pollinated under optimal temperature and humidity conditions.

The seeds obtained from the crosses were germinated after being treated with gibberellic acid (1500 ppm) to standardize germination. The seedlings were grown in disinfected substrate and transplanted into plastic pots. The isolation and inoculation of Phytophthora infestans were carried out using a standard inoculum (40,000 sporangia/ml), which was applied to leaflets under controlled temperature conditions (18 °C) and a 14-hour light photoperiod.

3.2 Modeling with Neutrosophic SuperHyperSoft Sets

To improve the accuracy in the evaluation of the genetic resistance of the genotypes, the modeling of Neutrosophic SuperHyperSoft Sets was incorporated. This tool allows for the integration of the multiple relationships associated with the study attributes, as well as the neutrosophic mebership degrees in the evaluation of both progenitors and progenies (see Tables 1 and 2), in the resistance to the pathogen Phytophthora infestans.

Table 1: Progenitors for the analysis of the genetic components of late blight resistance. Source: Own elaboration.

Code	Progenitors
P1	Chaucha Negra
P2	Chaucha Amarilla
P3	Yema de Huevo

Table 2: Crosses derived from the combination of progenies. Source: Own elaboration.

Treatment	Cross	Progeny	Genealogy
T1	1x1	1	Ch. Negra x Ch. Negra
T2	1x2	11	Ch. Negra x Ch. Amarilla
T3	1x3	9	Ch. Negra x Yema de Huevo
T4	2x2	2	Ch. Amarilla x Ch. Amarilla
T5	2x3	15	Ch. Amarilla x Yema de Huevo
T6	3x3	5	Yema de Huevo x Yema de Huevo

On the other hand, the measurement attributes, as well as the combinatory aptitude attributes directed towards genetic improvement programs, are presented (see Tables 3 and 4). In fact, they help in selecting both the best progenitors individually and the best crosses between them to obtain the best possible varieties in terms of desired characteristics, such as disease resistance.

Table 3: Genetic attributes of progenitors and progenies. Source: Own elaboration.

Attribute	Name
a_1	General Combining Ability (GCA)
a_2	Specific Combining Ability (SCA)

Table 4: Measurement attributes for each progenitor and progeny. Source: Own elaboration.

Attribute	Genetic components of late blight resistance
b_1	Lesion growth rate (LGR)
b_2	Lesion size (LS)
b_3	Latency period (LP)
b_4	Sporulation intensity (SI)

Each group of determined attributes forms a Neutrosophic SuperHyperSoft, which is formed by $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2))$, as well as $(G, \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_4))$. Thus, both functions return neutrosophic $\mathcal{P}\left(\mathrm{U}(x(d^0))\right)$ for each progenitor and progeny. Therefore, the following Neutrosophic SuperHyperSoft $(H, (\mathcal{P}(A_1) \times \mathcal{P}(A_2)) \times (\mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_4)))$ is proposed, as well as a new neutrosophic degree of membership $x(d^0)$, obtained from the intersection of the attributes $a_i \cap b_i$. The following functions are proposed to determine which treatment shows greater resistance to *late blight*, as well as their values in the estimation of progenitors and progenies of potatoes (see Tables 5 and 6).

- $F(a_1 \cap b_i) = \{P_i\}, \left\{\left(x_{a_1}(d^0)\right) \cap (x_{b_i}(d^0))\right\}$
- $F(a_2 \cap b_i) = \{T_i\}, \left\{\left(x_{a_2}(d^0)\right) \cap (x_{b_i}(d^0))\right\}$

Table 5: Values in the estimation of the attributes ($a_1 \cap b_i$) for potato progenitors (Solanum phureja). Source: Own elaboration.

Progenitor	$a_1 \cap b_1$		$a_1 \cap b_2$		$a_1 \cap b_3$		$a_1 \cap b_4$
	LGR averages (mm2)	Effects of GCA	LS averages (mm2)	Effects of GCA	LP averages (days)	Effects of GCA	Average.
P1	14.54	-4.49	69.30	-2.58	3.75	-2.67	1.00
P2	11.28	14.68	55.49	20.99	4.25	21.22	1.00
P3	18.60	-10.19	77.48	-18.41	3.25	-18.55	2.00

Table 6: Values in the estimation of the attributes ($a_2 \cap b_i$) for potato progenies (Solanum phureja). Source: Own elaboration.

Progeny	$a_2 \cap b_1$		$a_2 \cap b_2$		$a_2 \cap b_3$		$a_2 \cap b_4$
	LGR averages (mm2)	Effects of SCA	LS averages (mm2)	Effects of SCA	LP averages (days)	Effects of SCA	Average.
T1	23.85	42.40	83.62	50.91	3.13	44.49	1.89
T2	9.27	-54.09	75.88	-65.58	3.85	-60.28	1.27
T3	13.53	-30.71	82.65	-36.25	3.60	-28.69	1.54
T4	21.08	15.37	77.51	20.74	3.44	21.51	1.34
T5	29.27	23.35	88.82	24.09	3.16	17.26	1.80
T6	29.05	3.68	99.92	6.08	3.00	5.71	2.00

Comparative analysis:

- $a_1 \cap b_1$: In Table 5, it is observed that Chaucha Amarilla (P2) is the progenitor with the lowest lesion growth rate, followed by Chaucha Negra, and finally, Yema de Huevo shows the highest values in lesion growth rate. On the other hand, the knowledge of general combining ability (GCA) and the estimation of the effect in all crosses show that the progenitor Chaucha Amarilla achieved a positive value of 14.68 in its GCA effects, indicating its superiority over the other progenitors. This parent originated more resistant progenies to late blight, presenting a high probability of using this progenitor in the INIAP Potato Program's genetic improvement programs, as it possesses the ability to transmit resistance to late blight to its descendants.
- $a_2 \cap b_1$: The progenies from crosses 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo) showed high absolute values for Specific Combining Ability (SCA). This indicates that the progenies performed better than the average GCA of their progenitors.
- $a_1 \cap b_2$: The high GCA value of 20.99 for Chaucha Amarilla recognizes this progenitor, indicating a high probability of being employed in genetic improvement programs. According to the reviewed literature, it is possible to select four times more potato clones from a progenitor with high GCA than from one with low GCA.
- $a_2 \cap b_2$: High absolute values in the effects of SCA were observed in progenies 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo), indicating that the offspring showed a better or superior performance compared to the average GCA of their parents.
- $a_1 \cap b_3$: When considering the number of days in the development of symptoms (LP) of Phytophthora infestans (P. infestans), progenitors with high resistance levels were identified, with Chaucha Amarilla standing out, followed by Yema de Huevo and Chaucha Negra, respectively. Therefore, Chaucha Amarilla was chosen as the progenitor with a high GCA effect of 21.22, indicating that it is the best parent capable of transmitting resistance to late blight to its offspring.
- $a_2 \cap b_3$: By accounting for a higher number of days in the appearance of symptoms, progenies 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo) showed high absolute values for SCA. This indicates that the progenies performed better or

superior to the average GCA of their parents, showing a degree of heterosis for resistance to P. infestans.

- $a_1 \cap b_4$: The averages obtained for the progenitors, based on the visual scale of sporulation intensity, determined that Chaucha Amarilla and Chaucha Negra exhibited the lowest sporulation intensity with 1.00 (low sporulation). Meanwhile, the progenitor Yema de Huevo showed the highest sporulation intensity with 2.00 (medium sporulation), and the overall average for the progenitors was 1.33, considered as low sporulation.
- $a_2 \cap b_4$: The averages obtained for the offspring in terms of sporulation intensity established that the progenies from cross 11 (Ch. Negra x Ch. Amarilla) showed the lowest sporulation intensity with 1.27 (low sporulation). Meanwhile, the progenies from family 5 (Yema de Huevo x Yema de Huevo) showed the highest sporulation intensity with 2.00 (medium sporulation).

Correlation analysis: The genetic components of resistance to late blight in Solanum phureja showed significant relationships between the variables LGR, LS, and SI, with positive correlations at 1% between LGR - LS and LGR - SI, and a moderate relationship between LS - SI at 5%. These correlations indicate that an increase in one variable is associated with an increase in the others (see Table 7). In contrast, the negative correlations between LGR - LP, LP - SI, and LS - LP suggest that a higher value in one variable reduces the other, such as a shorter latency period (LP) with an increase in LGR and LS.

Table 7: Correlation analysis of components a_i. Source: Own elaboration.

	LGR	LS	LP	SI
LGR	-----			
LS	0.58**	-----		
LP	-0.54**	-0.45*	-----	
SI	0.54**	0.51*	-0.64**	-----
** Significant at 1% probability				
* Significant at 5% probability				

LGR was identified as the most relevant component in the resistance to P. infestans, and its relationship with other resistance variables (LS, LP, SI) suggests that these components are useful for evaluating and selecting resistant germplasm. The results reinforce the need to consider both additive and non-additive effects in the inheritance of resistance. It suggests that both the progenitors and the progenies show significant differences in the expression of this resistance. Therefore, the study highlights the value of phenotypic evaluations for genetic improvement in late blight resistance programs.

Neutrosophic membership degree: The neutrosophic degree of membership for each intersection of the sets of $P(T_i)$ is determined, as shown in Tables 8 and 9. Each neutrosophic degree of membership belongs to the set P(A_i) and P(B_i).

Table 8: Values of $x(d^0)$ in the estimation of the attributes (a_1) and (b_i). Source: Own elaboration.

	a_1	b_1	a_1	b_2	a_1	b_3	$a_1 \cap b_4$
P1	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.2,0.75,0.8)	(0,0.95,1)	(0.7,0.25,0.3)
P2	(0.7,0.25,0.3)	(0.4,0.55,0.6)	(0.6,0.35,0.4)	(0.4,0.55,0.6)	(0.3,0.65,0.7)	(0.4,0.55,0.6)	(0.7,0.25,0.3)
P3	(0.5,0.45,0.5)	(0.6,0.35,0.4)	(0.5,0.45,0.5)	(0.5,0.45,0.5)	(0.2,0.75,0.8)	(0.3,0.65,0.7)	(0.4,0.55,0.6)

Table 9: Values of $x(d^0)$ in the estimation of the attributes (a_2) and (b_i). Source: Own elaboration.

	a_2	b_1	a_2	b_2	a_2	b_3	$a_1 \cap b_4$
T1	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.1,0.85,0.9)	(0.2,0.75,0.8)	(0.7,0.25,0.3)
T2	(0.8,0.15,0.2)	(0.6,0.35,0.4)	(0.8,0.15,0.2)	(0.6,0.35,0.4)	(0.2,0.75,0.8)	(0.3,0.65,0.7)	(0.8,0.15,0.2)
T3	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.2,0.75,0.8)	(0.1,0.85,0.9)	(0.7,0.25,0.3)

	a_2	b_1	a_2	b_2	a_2	b_3	$a_1 \cap b_4$
T4	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.8,0.15,0.2)	(0.8,0.15,0.2)	(0.1,0.85,0.9)	(0.1,0.85,0.9)	(0.8,0.15,0.2)
T5	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.1,0.85,0.9)	(0.1,0.85,0.9)	(0.7,0.25,0.3)
T6	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.7,0.25,0.3)	(0.9,0.05,0.1)	(0.1,0.85,0.9)	(0,0.95,1)	(0.7,0.25,0.3)

To determine the new neutrosophic membership degree of $(H, (\mathcal{P}(A_i)) \times (\mathcal{P}(B_i)))$ P(A,B), the t_{norm} and t_{conorm} were used to determine the intersection between the functions defined in the study (see Tables 10 and 11). For this, the aggregation operation $(min_j\{T_{ij}\}, max_j\{I_{ij}\}, max_j\{F_{ij}\})$ Was used.

Table 10: Values of $x(d^0)$ in the estimation of the attributes $(a_1 \cap b_i)$ in $\mathcal{P}\left(U(x(d^0))\right)$. Source: Own elaboration.

	$a_1 \cap b_1$	$a_1 \cap b_2$	$a_1 \cap b_3$	$a_1 \cap b_4$
P1	(0.6,0.35,0.4)	(0.6,0.35,0.4)	(0,0.95,1)	(0.7,0.25,0.3)
P2	(0.4,0.55,0.6)	(0.4,0.55,0.6)	(0.3,0.65,0.7)	(0.7,0.25,0.3)
P3	(0.5,0.45,0.5)	(0.5,0.45,0.5)	(0.2,0.75,0.8)	(0.4,0.55,0.6)

The results in Table 10 define the progenitor Chaucha Amarilla (P2), which demonstrated greater resistance to late blight. It has a degree of membership of (0.3, 0.65, 0.7) in the intersection of the analyzed function for this treatment $F\ (a_1 \cap b_i) = \{P_i\}, \left\{\left(x_{a_1}(d^0)\right) \cap (x_{b_i}(d^0))\right\}$. In contrast, Chaucha Negra showed a lower degree of neutrosophic membership, although it contributed resistance through its progenies.

Table 11: Values of $x(d^0)$ in the estimation of the attributes $(a_2 \cap b_i)$ in $\mathcal{P}\left(U(x(d^0))\right)$. Source: Own elaboration.

	$a_1 \cap b_1$	$a_1 \cap b_2$	$a_1 \cap b_3$	$a_1 \cap b_4$
T1	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.1,0.85,0.9)	(0.7,0.25,0.3)
T2	(0.6,0.35,0.4)	(0.6,0.35,0.4)	(0.2,0.75,0.8)	(0.8,0.15,0.2)
T3	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.1,0.85,0.9)	(0.7,0.25,0.3)
T4	(0.7,0.25,0.3)	(0.8,0.15,0.2)	(0.1,0.85,0.9)	(0.8,0.15,0.2)
T5	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0.1,0.85,0.9)	(0.7,0.25,0.3)
T6	(0.7,0.25,0.3)	(0.7,0.25,0.3)	(0,0.95,1)	(0.7,0.25,0.3)

In contrast, in Table 11, according to the specific combining ability (SCA) effects, the genotype exhibiting the best absolute value in its effects is the progeny Ch. Negra x Ch. Amarilla (T2). It obtained a neutrosophic degree of membership of (0.2, 0.75, 0.8) over the other treatments, according to the following function $F\ (a_2 \cap b_i) = \{T_i\}, \left\{\left(x_{a_2}(d^0)\right) \cap (x_{b_i}(d^0))\right\}$.

Based on the results obtained, it was projected that the integration of genotypes with high general combining ability (GCA) and specific combining ability (SCA) is essential for optimizing genetic improvement programs for Solanum phureja with resistance to late blight. The identification of outstanding genotypes, such as Chaucha Amarilla (high GCA), along with progenies such as 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo), which demonstrated high values in SCA, suggested that the inheritance of resistance is regulated by both additive and non-additive effects. This observation allowed the identification that a more precise selection, considering the genetic variability of progenies and the specific influence of progenitors, would facilitate the development of genotypes with greater resistance.

Furthermore, the relevance of lesion growth rate (LGR) constitutes a critical indicator of resistance to the pathogen and, therefore, one of the main criteria for future selection programs. As the understanding of interactions between additive and non-additive genetic effects deepens, breeding strategies will be developed in potato crops. These strategies would focus on optimizing specific resistance traits against late blight in different agroecological zones, contributing to the creation of more resistant potato

varieties and the sustainability of production against emerging diseases like Phytophthora infestans.

4 Conclusion

The genetic evaluation of late blight resistance (Phytophthora infestans) in Solanum phureja allowed the identification of genotypes with significant variability in their resistance, highlighting both positive and negative correlations between the attributes a_i. On the other hand, the additive and non-additive genetic effects showed a greater influence of specific combining ability (b_2) over general combining ability (b_1). This emphasized the relevance of non-additive effects in late blight resistance. In fact, the variance and neutrosophic degree of membership analysis reflected that for b_i in the intersection with the attributes a_i, the progenitor Chaucha Amarilla was the most resistant. Meanwhile, Chaucha Negra, with a lower neutrosophic degree of membership, contributed resistance through its progenies. Additionally, progenies 11 (Ch. Negra x Ch. Amarilla), 1 (Ch. Negra x Ch. Negra), and 9 (Ch. Negra x Yema de Huevo) showed the highest b_2 values, while the self-fertilization 5 (Yema de Huevo x Yema de Huevo) exhibited greater susceptibility. Consequently, these results open new lines of research in understanding and leveraging non-additive effects in genetic resistance improvement.

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Analyzing Internal and External Factors for Ecological Preservation in San Juan de Pastocalle: A Combined SWOT and Neutrosophic AHP Approach

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Abstract. This study aimed to analyze the role of Community Circles in preserving the ecological link in the rural parish of San Juan de Pastocalle, identifying both the factors that enhance and those that limit this process. The SWOT analysis combined with the neutrosophic AHP method was applied to evaluate the key elements that affect ecological sustainability in the community. The results revealed that the main weaknesses were the lack of training in adequate recycling and the poor infrastructure for waste management. Strategic actions focused on strengthening local capacities and ensuring economic support were proposed. The use of decision-making methods applying neutrosophic logic allowed for a more precise and objective evaluation. The importance of neutrosophy in addressing uncertainty in the management of complex problems is highlighted.

Keywords: ecological sustainability, decision-making, Neutrosophy, SWOT, AHP.

1 Introduction

The relationship between human communities and their ecological environment has garnered increasing interest in recent decades due to the environmental impacts arising from urbanization, climate change, and the degradation of natural resources. In this context, the preservation of local ecological areas emerges as a key challenge for rural communities, where the integration of inhabitants with their natural environment remains essential for ensuring the sustainability of ecosystems [1]. In many of these communities, Community Circles play a fundamental role by promoting ecological awareness and facilitating active citizen participation in environmental protection.

Community Circles are spaces where community members gather to discuss topics of common interest, with the relationship between humans and the ecological environment occupying a central place. In the rural parish of San Juan de Pastocalle, these activities have proven to be an effective vehicle for raising awareness among residents about the importance of respecting nature, managing waste, and exercising environmental rights. However, there remains concern about the lack of practical application of these values in the community's daily life, limiting the effectiveness of local environmental conservation initiatives. This disconnection between theory and practice, between ecological awareness and real action, represents a key challenge facing the parish in its attempt to preserve the ecological bond with its environment. [2]

The ecological bond is understood as the intrinsic relationship between people and the ecosystems in which they live. The conservation of this bond is essential to ensure that communities can manage their natural resources sustainably. Various studies have shown that local communities, particularly those in ecologically valuable areas, are the best managers of their natural resources due to their traditional knowledge and practices passed down over time [3], [4]. In this regard, the role of Community Circles in San Juan de Pastocalle is crucial, as these initiatives promote the transmission of traditional knowledge and practices that strengthen the bond between inhabitants and their natural environment, while also fostering community cooperation around common environmental goals.

However, in a context where decision-making is characterized by uncertainty and complexity, it is

necessary to employ analytical methods that allow for addressing problems in a structured and systematic way [5]. In this regard, the use of decision-making tools presents itself as a convenient option to evaluate the various factors affecting the effectiveness of Community Circles in preserving the ecological bond. In this framework, the SWOT analysis (Strengths, Weaknesses, Opportunities, and Threats) can help identify strengths to be leveraged, weaknesses to be addressed, opportunities to be explored, and threats to be mitigated. [6]

However, SWOT analysis alone cannot provide a quantitative evaluation or effectively resolve the contradictions and imprecision inherent in decisions that need to be made in situations of uncertainty [7]. In this context, the integration of the AHP (Analytic Hierarchy Process) method in its neutrosophic variant emerges as a solution to address these limitations [8]. AHP is a multi-criteria decision-making technique that allows for ranking different alternatives according to established criteria. Its combination with neutrosophic set theory overcomes problems of indeterminacy and lack of precision in evaluations; as this approach considers not only the truth and falsehood of options but also the degrees of indeterminacy that often characterize real-world situations. [9]

Neutrosophic theory is particularly useful in contexts where evaluation criteria are ambiguous or uncertain [10], [11], such as environmental management in rural communities. By incorporating the notion of indeterminacy, this approach allows for a more accurate representation of the complexity of decisions that must be made in ecological and social fields [12], [13].

The objective of this study is to analyze how Community Circles contribute to the preservation of the ecological bond in the rural parish of San Juan de Pastocalle, evaluating both the factors that enhance and those that limit this process. By applying SWOT analysis combined with neutrosophic AHP, the study seeks to identify the most effective strategies for optimizing the impact of Community Circles on the ecological and social environment of the community.

2 Preliminaries

2.1 Definitions

Definition 1: A Neutrosophic set N is defined by three membership functions: the truth-membership function T_A, the indeterminacy-membership function I_A, and the falsehood-membership function F_A. These functions are applied to the universe of discourse U where for any element $x \in U$, $T_A(x)$, $I_A(x)$, and $F_A(x)$ belong to the interval $]-0,1+[$, and the following condition holds: $-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$. [14]

It is important to note that according to this definition, the membership functions, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]-0,1+[$ implying that these functions can be subintervals of $[0,1]$.

Definition 2: The Single-Valued Neutrosophic Set (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1], I_A: U \to [0,1]$, and $F_A: U \to [0,1]$, with the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The Single-Valued Neutrosophic Number (SVNN) is represented by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3: According to [15], the single-valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, where the truth, indeterminacy, and falsehood membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}} \left(\frac{x - a_1}{a_2 - a_1} \right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, \quad a_2 \leq x \leq a_3 \\ \alpha_{\tilde{a}} \left(\frac{a_3 - x}{a_3 - a_2} \right), a_3 \leq x \leq a_4 \\ 0, \quad otherwise \end{cases} \tag{1}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \gamma_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x - a_2 + \gamma_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3, a_4 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3 \le a_4$.

Definition 4: given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued trapezoidal neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$ (4)

Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$ (5)

Inversion: $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3, a_4 \neq 0$. (6)

Multiplication by a scalar number: $\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$ (7)

Division by a scalar number: $\frac{\tilde{a}}{\lambda} = \begin{Bmatrix} \langle (\frac{a_1}{\lambda}, \frac{a_2}{\lambda}, \frac{a_3}{\gamma\lambda}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle \ if \ (\lambda > 0) \\ \langle (\frac{a_3}{\gamma\lambda}, \frac{a_2}{\lambda}, \frac{a_1}{\lambda}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \beta\gamma_{\tilde{a}} \rangle \ if \ (\lambda < 0) \end{Bmatrix}$ (8)

Division: $\frac{\tilde{a}}{\tilde{b}} = \{ \begin{matrix} \langle (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \theta\beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle \ if \ (a_3 > 0, b_3 > 0) \\ \langle (\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle \ if \ (a_3 < 0, b_3 > 0) \\ \langle (\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle \ if \ (a_3 < 0, b_3 < 0) \end{matrix}$ (9)

2.2 Neutrosophic SWOT-AHP basics

The application of the Neutrosophic Analytic Hierarchy Process (N-AHP) allows the use of a linguistic scale instead of a numerical one, offering a more intuitive approach for experts [16]. The original numerical scale proposed by Saaty is adapted into a linguistic framework, as illustrated in Table 1. This adaptation enables experts to evaluate elements, such as ethical considerations in supply chains, using linguistic terms rather than numerical values, which is often more natural for subjective assessments.

Table 1: Saaty's scale translated into a Triangular Neutrosophic Scale. Source: [8]

Definition	Triangular Neutrosophic Scale
Equally influential	$\tilde{1} = \langle (1,1,1); 0.50, 0.50, 0.50 \rangle$
Slightly influential	$\tilde{3} = \langle (2,3,4); 0.30, 0.75, 0.70 \rangle$
Strongly influential	$\tilde{5} = \langle (4,5,6); 0.80, 0.15, 0.20 \rangle$
Very strongly influential	$\tilde{7} = \langle (6,7,8); 0.90, 0.10, 0.10 \rangle$
Absolutely influential	$\tilde{9} = \langle (9,9,9); 1.00, 1.00, 1.00 \rangle$

Definition	Triangular Neutrosophic Scale
Intermediate values	$\tilde{2} = \langle(1,2,3);0.40,0.65,0.60\rangle$ $\tilde{4} = \langle(3,4,5);0.60,0.35,0.40\rangle$ $\tilde{6} = \langle(5,6,7);0.70,0.25,0.30\rangle$ $\tilde{8} = \langle(7,8,9);0.85,0.10,0.15\rangle$

The SWOT-AHP involves the execution of the following procedures:

Step 1: Form a team of experts familiar with SWOT analysis. This team identifies internal and external factors essential to the analysis through methods such as surveys or interviews.

Step 2: Establish a hierarchical framework for the problem. This hierarchy typically has four levels:

1. The organizational goal or primary objective.
2. Strategic criteria identified by the SWOT framework (Strengths, Weaknesses, Opportunities, and Threats).
3. Sub-criteria, which detail the elements within each strategic criterion.
4. Strategies or alternatives to be evaluated.

Step 3: Develop a neutrosophic pairwise comparison matrix to assess the relative importance of criteria, sub-criteria, and alternatives. Linguistic terms (from Table 1) are used to quantify the relationships, capturing expert judgments effectively. The neutrosophic scale is attained according to expert opinion. The neutrosophic pairwise comparison matrix of factors, sub-factors, and strategies are as follows:

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} \cdots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} \cdots & \tilde{1} \end{bmatrix} \quad (10)$$

The matrix satisfies the condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, b based on inversion principles.

Step 4: Ensure consistency in the pairwise comparison matrix. Consistency is evaluated by verifying if the transitive relationship $a_{ik} = a_{ija_{jk}}$ holds for all combinations of i, and k. This analysis focuses on the lower, middle, and upper bounds of the triangular neutrosophic numbers.

Step 5: Calculate weights for factors (S, W, O, T), sub-factors, and alternatives. This is achieved by converting the neutrosophic pairwise comparison matrix into deterministic values using equations for scoring and accuracy:

Let $\tilde{a}_ij = \langle(a_1, b_1, c_1), \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ represent a singular triangular neutrosophic number; then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (11)$$

and

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (12)$$

Which are the score and accuracy degrees of $\tilde{a}_{ij}$ respectively.

To get the score and the accuracy degree of $\tilde{a}_{ij}$, following equations are used:

$$S(\tilde{a}_{ji}) = 1 \Big/ S(\tilde{a}_{ij}) \quad (13)$$

$$A(\tilde{a}_{ji}) = 1 \Big/ A(\tilde{a}_{ij}) \quad (14)$$

Through the evaluation and scoring of each triangular neutrosophic number within the neutrosophic pairwise comparison matrix, the subsequent deterministic matrix is obtained:

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} \quad (15)$$

Determine the order of precedence, denoted as the Eigen Vector X, from the preceding matrix, through the following steps:

1. Standardize the values within each column by dividing each entry by the sum of the respective column.

2. Calculate the cumulative average for each row.

Step 6: Determine the overall priority of strategies by aggregating weights using Equation (16). The overall weight value for alternative j (j=1,…,) can be expressed as follows:

$$Tw_{\mathrm{Alt}_j} = w_S * \sum_{i=1}^{n} w_{S_i} * w_{\mathrm{Alt}_j} + w_W * \sum_{i=1}^{n} w_{W_i} * w_{\mathrm{Alt}_j} + w_O * \sum_{i=1}^{n} w_{O_i} * w_{\mathrm{Alt}_j} + w_T * \sum_{i=1}^{n} w_{T_i} * w_{\mathrm{Alt}_j} \quad (16)$$

where (i=1,…,n) and (wS,wW,wO,wT) are the weights of Strengths, Weaknesses, Opportunities, and Threats; (wSi,wWi,wOi,wTi) are the sub-factor weights; and wAlt$j$ is the weight of the alternative j, corresponding to its sub-factor.

3 Results

The data collection was based on a sample of 100 individuals with direct connections to elements related to environmental impact and the ecological bond in their surroundings. Through interviews and surveys, internal and external factors were identified that explain how Community Circles influence this bond in the rural parish of San Juan de Pastocalle, evaluating both strengths and limitations. Subsequently, the information was refined with the collaboration of five experts. The identified factors are presented below:

Table 2: Identified internal and external factors

Internal Factors		**External Factors**	
Strengths	**Weaknesses**	**Opportunities**	**Threats**
Active participation in ecological activities (51.85% are willing to participate in environmental volunteering)	Lack of knowledge about the concept of ecological bond (19.37% of respondents do not understand it well)	Growth of environmental awareness at national and global levels	Disinterest in the impact of climate change (53.33% consider climate change not a priority for many)
Interest in environmental conservation (72.59% of respondents support conservation)	Low infrastructure for waste management (34.81% disagree with the existence of collection points)	Recent alliances between public and private sectors for sustainability projects	Destruction of habitats due to human activities (55.56% point to deforestation as a threat)
Community commitment to ecological care (45.93% engage in waste collection)	Lack of training in proper recycling (problems with waste classification)	Strengthening teamwork through Community Circles to implement ecological measures	Lack of financial support for high-impact environmental activities
Support from local authorities for ecological projects (52.22% support governmental conservation initiatives)	Limited participation from certain sectors in decision-making (some voices are not heard in community processes)		
Existence of organized environmental groups (38.89% of the population belongs to an ecological group)			

The identified elements formed the basis of the study, which was developed according to the proposed methodology. After determining the internal and external factors influencing the system, the next step was to evaluate them in order to identify those with the greatest impact on its functioning, based on expert judgment. The analysis was carried out sequentially, starting with the evaluation of the components of the SWOT matrix, as detailed in Table 3, followed by the ranking of the factors according to their relative importance.

Table 3: Evaluation Matrix of the SWOT Criteria (S, W, O, T). Source: Own elaboration.

	S	O	W	T
S	(1,1,1); 0.5, 0.5, 0.5	(2, 3, 4); 0.3, 0.75, 0.7	(4, 5, 6); 0.8, 0.15, 0.2	(2, 3, 4); 0.3, 0.75, 0.7
O	(1/4, 1/3, 1/2); 0.3, 0.75, 0.7	(1,1,1); 0.5, 0.5, 0.5	(1,1,1); 0.5, 0.5, 0.5	(4, 5, 6); 0.8, 0.15, 0.2
W	(1/6, 1/5, 1/4); 0.8, 0.15, 0.2	(1,1,1); 0.5, 0.5, 0.5	(1,1,1); 0.5, 0.5, 0.5	(2, 3, 4); 0.3, 0.75, 0.7
T	(1/4, 1/3, 1/2); 0.3, 0.75, 0.7	(1/6, 1/5, 1/4); 0.8, 0.15, 0.2	(1/4, 1/3, 1/2); 0.3, 0.75, 0.7	(1,1,1); 0.5, 0.5, 0.5

After performing the corresponding calculations with the data presented in Table 3, a numerical matrix was generated to undergo the standard AHP Method process. This procedure included verifying the consistency ratio and obtaining the weight matrix associated with the analyzed criteria, W = (0.22; 0.10; 0.49; 0.19). Similarly, this analysis was applied to each criterion of the SWOT diagnosis, allowing for the derivation of the weight vector for the identified internal and external factors. Table 4 summarizes the obtained results.

Table 4: Evaluation matrix of SWOT criteria. Source: Own elaboration

	Factors	**Global Weight Vector**
Strengths	Active participation in ecological activities	0.03
	Interest in environmental conservation	0.02
	Community commitment to ecological care	0.06
	Support from local authorities in ecological projects	0.02
	Existence of organized groups in favor of the environment	0.09
Opportunities	Growth of environmental awareness at national and global levels	0.05
	Recent alliances between public and private sectors for sustainability projects	0.01
	Strengthening teamwork through Community Circles for ecological measures	0.04
Weaknesses	Lack of knowledge about the concept of ecological link	0.08
	Low infrastructure for waste management	0.16
	Lack of training in proper recycling	0.20
	Limited participation of certain sectors in decision-making	0.06
Threats	Disinterest in the impact of climate change	0.04
	Habitat destruction due to human activities	0.04
	Lack of financial support for high-impact environmental activities	0.11

The SWOT analysis applied to the system revealed a predominance of weaknesses over other evaluated elements. The factors with the highest global weight were the *Lack of training in proper recycling* (0.20) and the *Low infrastructure for waste management* (0.16). This highlighted that, according to expert judgment, these weaknesses represented the main barriers to preserving the ecological link in the rural parish of San Juan de Pastocalle.

Additionally, the threat related to the lack of economic support for high-impact environmental activities (0.11) also received a significant weight. This result emphasized the need to implement survival strategies to mitigate the negative effects of these weaknesses and threats. Although the strengths and opportunities had less influence, it is important to highlight that the *Existence of organized groups in favor of the environment* (0.09) was the main strength. Although this element carried less weight, it could be key in promoting collective and sustainable actions.

In this context, the study underscored the urgency of prioritizing actions that strengthen local capacities in recycling and infrastructure, as well as ensuring adequate economic support. These measures would optimize the impact of the Community Circles in preserving the ecological link.

Additionally, a PEST analysis [17] was conducted based on the detected elements. Table 5 summarizes the main elements developed.

Table 5: PEST analysis for the solution of detected elements

Dimension	Context	Proposed Actions
Political	Insufficient support from local and national authorities for sustainability projects.	Promote public policies that fund waste management infrastructure.
	Incorporate the parish into national sustainability and ecological linkage programs.	
Economic	Limited financial support for high-impact environmental initiatives.	Establish public-private partnerships to finance recycling projects and ecological activities.
	Scarcity of investment in infrastructure for waste management.	Create microcredit programs and specific subsidies to improve community facilities.
Social	Lack of training in recycling and low participation from specific sectors.	Implement educational workshops on recycling within the Community Circles.
	Weak awareness of the importance of ecological linkage.	Develop awareness campaigns through local media and school activities.
Technological	Lack of appropriate infrastructure for waste management.	Introduce accessible technologies for waste separation and recycling.
	Train residents in sustainable technological practices through interactive workshops.	

4 Conclusion

The conducted study demonstrated that the use of the SWOT analysis combined with the neutrosophic AHP method effectively identified the weaknesses limiting the preservation of the ecological linkage in the rural parish of San Juan de Pastocalle. The application of Neutrosophy, in particular,

proved essential for managing the inherent uncertainty in the expert evaluation of the factors, providing a clearer and more objective view of the influence of the various elements. This methodological approach enabled a thorough analysis, facilitating strategic decision-making to strengthen the most critical areas of the system.

The results underscore the importance of addressing weaknesses related to waste management infrastructure and recycling training as priorities to optimize the impact of the Community Circles. Moreover, the application of these decision-making methods highlighted the need for comprehensive solutions that include inter-institutional cooperation and economic strengthening, in order to ensure the sustainability of ecological activities within the community. The study opens new research opportunities, highlighting how multicriteria methods can be valuable tools in managing complex problems in community contexts.

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Use of Delphi-AHP neutrosophic methods for the analysis of school indicators

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Abstract. The study addresses the complex task of analyzing school indicators through the combined application of the Delphi-AHP neutrosophic methods, seeking to overcome the limitations of traditional approaches in educational assessment. The research focuses on the urgent need to develop more robust models that can handle the uncertainty, imprecision, and ambiguity inherent in educational data. Although previous research has been conducted on the analysis of school indicators, conventional approaches fail to effectively integrate the multiple dimensions and degree of uncertainty of the data. This study proposes an innovative solution by integrating neutrosophic tools, which allow for a more flexible and detailed analysis of indicators, considering various perspectives and situations. The use of the Delphi-AHP method, complemented by a neutrosophic perspective, allows for capturing the complexity of the educational environment and generating more accurate and realistic assessments. Through a series of consultation and weighting rounds, results were obtained that highlight the importance of factors such as the social context, educational infrastructure and the participation of actors involved in the school process. The findings not only enrich the theoretical field by applying a neutrosophic approach to the analysis of school indicators but also offer practical applications to improve educational policies and decision-making. This study contributes significantly to the advancement of educational data analysis by providing a more adaptable and accurate methodological framework for future research and practice in the school setting.

Keywords: Neutrosophic methods, Delphi-AHP, School indicator analysis, Data uncertainty, Evaluation models, Decision making, Data analysis.

1 Introduction

Education is an inalienable right, and access to it is guaranteed by the Ecuadorian State, that is, the constitutional norm contemplates its application under the principles of equity, equality, solidarity, and justice [1] . The Ecuadorian Constitution guarantees the right to education by the State, which should promote public policies and execute programs and projects financed with public funds, promoting the schooling of all citizens.

Education is a fundamental human right and is essential for progress. It allows all human beings to live a healthy, creative and meaningful existence. It is a catalyst that seeks to consolidate inclusive societies from university spaces, not only by declaring it valuable in institutional documents but also by training professionals capable of making decisions and facing the complexity of life in society [2] .

The educational transformation is achieving inclusive, renewed and quality education. Although significant changes have been achieved, much remains to be done for the district to have one of the quality educational systems. To this end, the interaction and co-responsibility of the State and the

educational community are essential. The Ministry of Education, based on quality standards, implements the work of educational auditing to make visible the educational development in the territory in coherence with the constitutional principle of Good Living and the sustainable development of the comprehensive training of children, youth and adults.

Education helps human beings to become autonomous, to have a better quality of life, to make decisions and to be supportive. People have the right not only to access education, but also to access quality education oriented towards fundamental values such as humanism, justice, respect for diversity, education for peace and sustainable development. States shall promote international cooperation in education to contribute to the elimination of ignorance and illiteracy throughout the world and to facilitate access to technical knowledge and modern teaching methods. In this regard, special account shall be taken of the needs of developing countries.

In the Ecuadorian context, school dropout has been identified as a problem that demands urgent attention and effective measures from authorities, educators, the community and society in general. It is important to understand the causes and consequences of school dropout to address it comprehensively and develop strategies that promote equitable and sustainable education. School dropout in children and adolescents is a phenomenon that affects the integral development of the individual because it does not allow them to continue with their academic training. It has the characteristic of being multi-causal and causes a mismatch between age and educational level or definitive abandonment.

School dropout in Ecuador is a problem that harms the efficiency of the educational system in low-income sectors and the rural population. One of the factors is the economic part, determined by the lack of permanent employment, which is sometimes the result of a lack of studies. Various investigations show that the causes of school dropout are due to multifactorial reasons that range from personal, family and economic aspects to political, cultural and institutional ones.

Dropping out of school is an act that leads to neglecting studies or enrolling in another institution and is considered school abandonment when the priority is not to study but to meet obligations that are satisfied with various human needs. In other words, school abandonment is determined by statistical figures related to the number of students who drop out of school from one school year to the next.

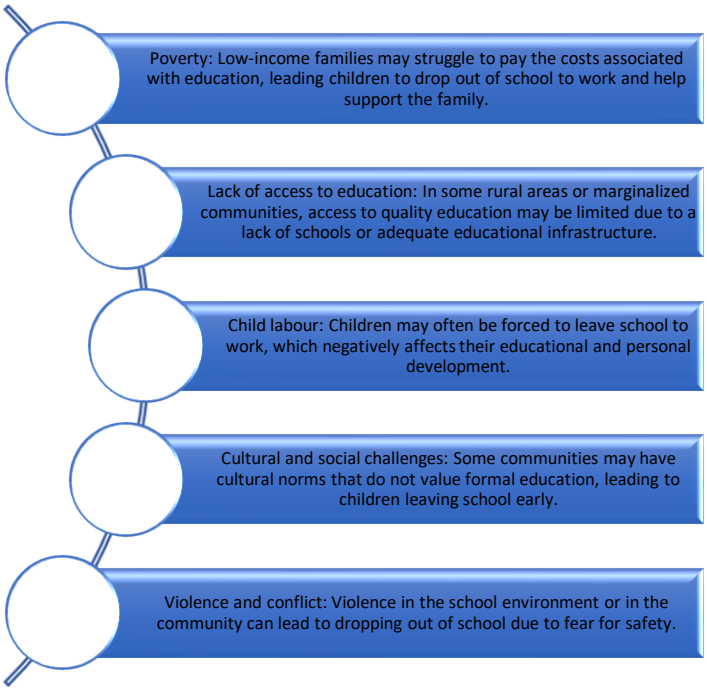

Figure 1. Causes of school dropout. Source: Own elaboration.

Poverty and lack of access to basic resources are determining factors in the phenomenon of school dropout. In many regions of Ecuador, especially in rural areas and marginalized communities, families face economic difficulties that directly affect the educational continuity of their children. The high costs associated with education, such as uniforms, books and school supplies, as well as transportation expenses, can become insurmountable barriers for many low-income families.

Similarly, the need to contribute to family income and child labour are painful realities that force many children to abandon their studies in search of employment to help their families survive. This situation not only negatively impacts their academic development but also limits their future opportunities and perpetuates the cycle of poverty.

School dropout can also be linked to cultural challenges rooted in certain communities, where formal education is not always valued or prioritized. Cultural norms and traditions can encourage children and adolescents to drop out of school early, creating a worrying educational gap. Furthermore, violence in the school environment and in the community can be an important reason for school dropout. Insecurity and fear of physical or psychological aggression can lead students to avoid school and affect their emotional well-being and their ability to learn in a safe and supportive environment.

Young people lose the opportunity to acquire knowledge with which they can get by in life, which directly causes a high unemployment rate, as they cannot apply this knowledge to perform a specific task. This situation endangers the future of the country, because if there are no young people with the basic knowledge that can only be learned in schools and colleges, it will be very difficult for them to develop other technical skills due to all the time wasted in non-educational activities [3] .

The Constitution of the Republic of Ecuador sets out objectives to progressively improve the quality of education, from a rights-based, gender-based, intercultural and inclusive approach, to strengthen unity in diversity and, above all, to guarantee the permanence and completion of studies, as well as to examine other forms of diversity, analyzing the capacity of each person to build relationships of coexistence, equity, dialogue and creativity, but there is still much to be done.

From the perspective of interculturality, it is of utmost importance to understand the diversity of cultures that coexist in the same territory, the worldview and the social needs for full development. In this framework, the right to have educational systems relevant to knowledge, language and reality is recognized, taking into account the diversity that postulates the interculturality of the territories in the process of integration.

School dropout is also an expression of educational inequality and the conditions that compulsory schooling assumes, considering that various intra-family factors have a unique impact on each student when continuing their higher education. It is necessary to demonstrate the relevance of involving the entire community in rethinking education as a need and a right established for Ecuadorians. In this way, it is possible to demand from state and institutional bodies norms and public policies that encourage and support all individuals to continue their path towards education from a culturally appropriate logic that knows how to attend to the needs of peoples and nationalities, rethinking the school format to promote greater educational inclusion.

In many Latin American countries, public education represents the majority of educational services offered. There are approximately 9 times more children in public education than in private education. Thus, differences can be observed between both types of education: in coverage, performance, infrastructure and teachers. The differences become more marked when observed by geographic area.

Education is a relevant intermediary for obtaining means, such as the acquisition and transfer of a good social level and lifestyle, and at the same time, it transforms the community where it is located. Therefore, various educational methods explore ways to prevent school dropouts and fulfill the purpose of students completing their studies satisfactorily and projecting themselves beyond. Based on the above, the general objective of this study is: to analyze the causes of school dropouts [4] .

Specific objectives:

- To investigate the main socioeconomic causes of school dropouts among children and adolescents in Ecuador.

- Identify the factors that may influence the decision to drop out of school.
- Propose educational strategies and policies that effectively address the identified causes of school dropout, with special attention to measures to reduce poverty and improve access to educational resources, as well as awareness-raising and sensitization campaigns on the importance of education.

1. Neutrosophic Delphi Method

The Delphi technique is used in many fields such as program planning, resource utilization, policy analysis, and needs assessment. The Delphi technique has the following advantages:

1. Addressing complex problems effectively.
2. Able to define and modify a wide range of alternatives.
3. Create different judgments about the same issue and use feedback on individual judgments to allow people to revise their views.
4. Achieve a high degree of consensus.
5. Increase coherence by reducing the noise that results from focusing on group and/or individual interests rather than focusing on problem-solving.

This is a structured communication technique designed primarily to gather and consolidate expert opinions on specific topics through a series of iterative questionnaires with controlled feedback. Developed in the 1950s by the RAND Corporation, this method is used to reach consensus on predicting future trends, solving complex problems, strategic planning, and risk assessment, among others.

The process begins with the selection of a panel of experts who possess specialized knowledge in the area of interest. These experts respond to an initial questionnaire, the answers to which are anonymized and summarized by a coordinator or coordinating team. The summarized results are then shared with the group, along with a new questionnaire based on the previous responses. This questionnaire-response-feedback process is repeated in several rounds, to narrow the range of responses and move the group toward consensus.

A key feature of the Delphi method is the anonymity of participants, which helps to avoid the influence or dominance effect of some participants over others, thus facilitating more objective responses and reducing conformity bias. At the end of the process, the convergence of opinions is expected to reveal a consensus or a clearer understanding of the issue at hand, providing valuable information for decision-making [5] .

To establish the knowledge of the analyzed topic and the object of study, a self-assessment process is carried out on a scale (see Table 1). This so-called neutrosophic knowledge coefficient is determined from the information that the expert himself presents on the object of study.

Table 1: Linguistic terms used.

Linguistic term	**SVNN**
Full knowledge of the object of study (FK)	(1,0,0)
Very very good in the subject of study (VVGK)	(0.9, 0.1, 0.1)
Very good in the subject of study (VGK)	(0.8,0.15,0.20)
Good in the subject of study (GK)	(0.70,0.25,0.30)
Moderately good in the subject of study (MGK)	(0.60,0.35,0.40)
Know the topic of study (K)	(0.50,0.50,0.50)
Has moderately poor knowledge of the subject matter (MPK)	(0.40,0.65,0.60)
Little knowledge of the subject of study (PK)	(0.30,0.75,0.70)
Knows very little about the subject of study (VPK)	(0.20,0.85,0.80)

Linguistic term	**SVNN**
Very, very poor knowledge of the subject of study (VVPK)	(0,10,0,90,0,90)
Without knowledge of the subject of study (NK)	(0,1,1)

For the evaluation and validation of the questionnaires through the Delphi method, the scale (see Table 2) was used to achieve greater objectivity in the management of information, which allows the evaluation of the criteria argued by the judges of the panel of experts for each of the items individually. Through linguistic terms with Single-valued Neutrosophic Numbers (SVNN) for consensus based on the expert's opinion, criteria are evaluated using the neutrosophic argumentation coefficient.

Table 2: Linguistic terms used.

Linguistic term I	**SVNN**	**Linguistic term II**
Very suitable (VA)	(0,9,0,1,0,1)	Essential (E)
Fairly adequate (FA)	(0.75,0.25,0.20)	Very useful (VU)
Suitable (A)	(0.50,0.55,0.5)	Useful (U)
Poorly Adequate (PA)	(0.3,0.75,0.80)	Not Very Useful (LU)
Not suitable (NA)	(0,10,0,90,0,90)	Not useful (NU)

To determine the consensus among the participants of the expert panel, the coefficient of agreement was used, determined through the expression:

$$Cc = \left(1 - \frac{V_n}{V_t}\right) 100 \quad (1)$$

Where: V_n is the number of negative votes cast by the judges, and $V_t$is the total number of votes cast by the judges. Therefore, a level of consensus must be reached when the coefficient of agreement Cc obtains a value greater than 75%, and the process must be concluded; if this value is not reached, a new round must be established.

2. Neutrosophic AHP method

The Analytic Hierarchy Process (AHP) is a theory-oriented to the decision-maker and is used to identify the best alternative according to the assigned resources. This method can be applied to situations that involve technical, economic, political, social and cultural factors. That is, it aims to be a scientific tool to address aspects that are difficult to quantify but that sometimes require a unit of measurement. The methodology in its original version can be seen in [11]. However, for this work the need for uncertainty is recognized, and for this purpose its neutrosophic version is adopted, which uses triangular numbers for its execution, whose definition is the following:

Definition 1: Let X be a space of points and x ∈ X. A neutrosophic set A in X is defined by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. U is the Universe of Discourse and $\forall x \in U$, $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq]-0, 1+[$, $y -0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup T_A(x) \leq 3+$. Note that by the definition, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are standard or nonstandard real subsets of]-0, 1+[and hence $T_A(x)$, $I_A(x)$, and $F_A(x)$ can be subintervals of [0, 1] [6] .

Definition 2: Let X be a universe of discourse. A single-valued neutrosophic set A over X is an object taking the form A = {<x; $T_A(x)$, $I_A(x)$, $F_A(x)$>: x ∈U}, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$ and $F_A: U \rightarrow [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all x ∈ X. The intervals $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degree of truth, degree of indeterminacy, and degree of falsehood of x through A, respectively. For convenience, an SVN number is represented by A = (a, b, c), where a, b, c ∈ [0, 1] and a+b+c≤3.

Definition 3: Suppose ã, ã, ã ∈ [0, 1] and a_1, a_2, a_3,a_4 ∈ R where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then, a single-valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a special neutrosophic set in the real line set ℝ, whose truth membership, indeterminacy membership and falsity membership functions are defined as see the revised methodology [7, 8]

Definition 4: Given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$two $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$single-valued trapezoidal neutrosophic numbers and λany nonzero number on the real line, the following operations are defined:

Addition:$\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$

Remaining: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$(2)

Investment: $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3, a_4 \neq 0$.

Multiplication by a scalar number: [9]

This technique models the problem leading to the formation of a representative hierarchy of the associated decision-making scheme. The comparison is made using a scale, according to Table 3. To verify the neutrosophic methodology see [6-10].

Table 3. Saaty scale translated into a neutrosophic triangular scale. Source: [11, 12, 13]

The Saaty scale	Definition	Neutrosophic triangular scale
1	Equally influential	$\langle (1, 1, 1); 0.50, 0.50, 0.50 \rangle$
3	Slightly influential	$\langle (2, 3, 4); 0.30, 0.75, 0.70 \rangle$
5	Strongly influential	$\langle (4, 5, 6); 0.80, 0.15, 0.20 \rangle$
7	Very strongly influential	$\langle (6, 7, 8); 0.90, 0.10, 0.10 \rangle$
9	Absolutely influential	$\langle (9, 9, 9); 1.00, 1.00, 1.00 \rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\langle (1, 2, 3); 0.40, 0.65, 0.60 \rangle$ $\langle (3, 4, 5); 0.60, 0.35, 0.40 \rangle$ $\langle (5, 6, 7); 0.70, 0.25, 0.30 \rangle$ $\langle (7, 8, 9); 0.85, 0.10, 0.15 \rangle$

3. Information processing and results

The Delphi methodology is based on an iterative process of rounds of questionnaires sent to a panel of experts. The idea is that, through several rounds, a closer consensus is reached on the issue in question. In the context of factors involved in school dropout, this process would involve asking experts to evaluate and discuss the relative importance of each factor and how they interact with each other [14, 15, 16].

Round 1: Determination of factors

Below are six factors that influence school dropout:

1. Economic: Lack of permanent employment, often a result of lack of education, negatively impacts the ability to continue education.
2. Personal: Individual aspects of students that may influence their decision to drop out of school.

3. Family: Circumstances of the family environment that may affect the educational continuity of children and adolescents.
4. Political: Public policies and government management of education can have a direct impact on the school dropout rate.
5. Institutional: Factors related to the quality of education, inclusion and the efficiency of the educational system in general.
6. Cultural: Cultural values, beliefs and practices can influence perceptions of education and, therefore, school dropout rates.

Round 2: Identification of key factors and initial assessments

Questions:

1. What do you think is the impact of economic factors on school dropouts?
2. Evaluate the impact of personal factors on school dropout.
3. To determine the influence of family factors on school dropouts.
4. Evaluate the importance of political factors in school dropouts.
5. Indicate how cultural factors affect school dropouts.
6. What is the impact of institutional factors on school dropout?

Answers:

Experts provide their assessments based on their knowledge and experience. These responses are compiled and analyzed to identify areas of consensus and disagreement.

Table 5: Level of validation of criteria.

Expert	Economic	Staff	Family	Political	Cultural	Institutional
E1	(0.35; 0.75; 0.80)	(0.50;0.5;0.50)	(0.75; 0.25; 0.20)	(0.75; 0.25; 0.20)	(0.35; 0.75; 0.80)	(0.50;0.5;0.50)
E2	(0.9;0.1;0.1)	(0.10;0.90;0.90)	(0.75; 0.25; 0.20)	(0.75; 0.25; 0.20)	(0.9;0.1;0.1)	(0.10;0.90;0.90)
E9	(0.10;0.90;0.90)	(0.10;0.90;0.90)	(0.9;0.1;0.1)	(0.10;0.90;0.90)	(0.10;0.90;0.90)	(0.10;0.90;0.90)
E16	(0.35;0.75;0.80)	(0.50;0.5;0.50)	(0.10;0.90;0.90)	(0.35;0.75;0.80)	(0.75;0.25;0.20)	(0.10;0.90;0.90)
E23	(0.75;0.25;0.20)	(0.35;0.75;0.80)	(0.10;0.90;0.90)	(0.75;0.25;0.20)	(0.35;0.75;0.80)	(0.50;0.5;0.50)
E27	(0.9;0.1;0.1)	(0.10;0.90;0.90)	(0.9;0.1;0.1)	(0.35;0.75;0.80)	(0.10;0.90;0.90)	(0.75;0.25;0.20)
E34	(0.35;0.75;0.80)	(0.35;0.75;0.80)	(0.10;0.90;0.90)	(0.9;0.1;0.1)	(0.75;0.25;0.20)	(0.35;0.75;0.80)
E39	(0.50;0.5;0.50)	(0.50;0.5;0.50)	(0.9;0.1;0.1)	(0.35;0.75;0.80)	(0.9;0.1;0.1)	(0.75;0.25;0.20)
E42	(0.75;0.25;0.20)	(0.9;0.1;0.1)	(0.10;0.90;0.90)	(0.50;0.5;0.50)	(0.75;0.25;0.20)	(0.10;0.90;0.90)
E62	(0.35;0.75;0.80)	(0.50;0.5;0.50)	(0.50;0.5;0.50)	(0.35;0.75;0.80)	(0.75;0.25;0.20)	(0.75;0.25;0.20)
E70	(0.10;0.90;0.90)	(0.50;0.5;0.50)	(0.35; 0.75; 0.80)	(0.10;0.90;0.90)	(0.35; 0.75; 0.80)	(0.35; 0.75; 0.80)
E80	(0.50;0.5;0.50)	(0.50;0.5;0.50)	(0.50;0.5;0.50)	(0.9;0.1;0.1)	(0.35; 0.75; 0.80)	(0.10;0.90;0.90)

Table 6: Neutrosophic relative frequency.

INDICATORS	(0.9;0.1;0.1)	(0.75; 0.25; 0.20)	(0.50;0.5;0.50)	(0.35; 0.75; 0.80)	(0.10;0.90;0.90)
Economic	0.1667	0.3333	0.5000	0.8333	1,0000
Staff	0.0833	0.0833	0.5833	0.7500	1,0000
Family	0.2500	0.4167	0.5833	0.6667	1,0000
Political	0.1667	0.4167	0.5000	0.8333	1,0000
Cultural	0.1667	0.5000	0.5000	0.8333	1,0000
Institutional	0.0000	0.2500	0.4167	0.5833	1,0000

Table 7: Cut-off points and criteria scale.

N - Average.	SVNN
-1.13	Useful
-0.84	Not very useful
-1.17	Useful
-1.18	Useful
-1.22	Useful
-0.39	Not very useful

Round 3: Refining the assessments

Based on the responses from Round 2, a summary of the assessments is produced and areas of agreement and disagreement are identified. Clarifications are sought and experts are asked to reconsider their previous responses if they deem it necessary, especially in areas where there was significant disagreement.

Questions:

1. Given the variety of responses on the impact of economic factors, would you like to modify your assessment?
2. Are there any new considerations you would like to add regarding the impact of personal factors?
3. Given the variety of opinions on family factors, do you think your assessment needs to be adjusted?
4. How would you evaluate the importance of political factors after seeing the responses of other experts?
5. Based on the feedback received, would you modify your perception of cultural factors?
6. Are there arguments that lead you to change your assessment of institutional factors?

Answers:

Experts review their previous assessments and adjust them based on discussions and arguments presented by other participants.

Round 4 (and subsequent rounds if necessary): Consensus

If significant discrepancies still exist, another round is held focusing on the points of disagreement. Responses are obtained and the experts reach a consensus.

Conclusion of the Delphi process

At the end of the Delphi process [17], a final report is produced that summarizes the consensus reached, highlighting the factors of school dropout considered most critical by the panel of experts, as well as their interrelations and impacts. This report can serve as a basis for decision-making and the development of solutions aimed at minimizing school dropout.

It is found that the factor that most influences school dropout and the right to education is the institutional problem, which brings with it a lack of quality in education, inclusion and low efficiency of the educational system in general.

Considering that the institutional problem, characterized by the lack of educational quality, insufficient inclusion and limited efficiency of the educational system, is identified as the main factor in school dropouts, the following solutions are proposed to effectively address this problem.

Solutions to address the institutional problem:

Improving Educational Infrastructure and Resources: Investing in modernizing school infrastructure and providing adequate educational resources. In addition, including educational technology that facilitates interactive and modern learning, accessible to all students.

Continuous teacher training and development: Implement continuous professional development programs for teachers, focused on inclusive and effective teaching methodologies, diversity management in the classroom, and updates on disciplinary content to ensure quality education.

Inclusion and student support policies: Develop and implement inclusion policies that address the needs of all students, especially those belonging to vulnerable groups. This includes support and tutoring programs, curricular adaptations for special needs, and strategies to encourage active student participation in their educational process.

Strategies for applying the AHP method to solve the problem:

1. Promotion of Educational Inclusion Policies and Programs (PP)
2. Implementation of Educational Quality Improvement Programs (PI)
3. Strengthening Teacher Training and Professional Development (FF)
4. Optimization of Management and Efficiency of the Educational System (OG)

Table 8. Neutrosophic AHP paired matrix [18]

Strategies	PP	PI	FF	OG
PP	**1**	〈(4,5,6);0,80,0,15,0,20 〉	〈(6,7,8);0,90,0,10,0,10 〉	〈(6,7,8);0,90,0,10,0,10 〉
PI	1/ 〈(4,5,6);0.80,0.15,0.20 〉	**1**	〈(2,3,4);0,30,0,75,0,70 〉	〈(4,5,6);0,80,0,15,0,20 〉
FF	1/ 〈(6,7,8);0,90,0,10,0,10 〉	1/ 〈(2,3,4);0.30,0.75,0.70 〉	**1**	〈(2,3,4);0,30,0,75,0,70 〉
OG	1/ 〈(6,7,8);0,90,0,10,0,10 〉	1/ 〈(4,5,6);0.80,0.15,0.20 〉	1 /(2,3,4);0.30,0.75,0.70 〉	**1**

Table 9. Criterion weights using the Neutrosophic AHP method.

Strategies	PP	PI	FF	OG	Weight
PP	0.67	0.77	0.62	0.44	0.62
PI	0.13	0.15	0.26	0.31	0.22
FF	0.10	0.05	0.09	0.19	0.11
OG	0.10	0.03	0.03	0.06	0.05

It is concluded that the inclusion and equity strategy is the one that has the greatest weight, so specific policies and programs must be implemented that promote inclusion and equity in the educational system, targeting vulnerable groups and those at risk of dropping out of school. This may include scholarship programs, psychosocial support, tutoring and mentoring services, and curricular adaptations for students with special educational needs. The objective is to ensure that each student has access to quality education and learning opportunities appropriate to their circumstances.

A. Discussion

To address the problem of school dropouts in Ecuador, it is necessary to implement policies and programs that focus on:

- Ensure universal access to free, quality education.
- Implement scholarship or incentive programs to help low-income families keep their children in school.
- Raise community awareness about the importance of education and change cultural perceptions that may affect school attendance.
- Create safe and violence-free school environments.
- Provide support and educational opportunities for teenage mothers.
- Both the government and society must work together to address this issue and ensure that all children and adolescents in Ecuador have equal opportunities to access and remain in the education system.

To prevent or reduce school dropout among children and adolescents in Ecuador, it is necessary to implement a series of recommendations and strategies that address the various causes of the problem. Below are some key recommendations:

1. Establish scholarship and financial aid programs targeted at low-income students and families to cover education-related expenses such as school supplies, uniforms, and transportation. This will help reduce the economic barrier that leads many children to leave school to work.
2. Ensure equal access to educational resources, including textbooks, teaching materials and technology, especially in rural and marginalized communities, to facilitate the learning process and encourage school retention.
3. Implement school feeding programs that ensure adequate nutrition for students. Poor nutrition can affect academic performance and class attendance, and such a program can be an incentive for children to attend school regularly.
4. Conduct awareness campaigns on the importance of education and the risks associated with school dropout. Involve the community and parents in the educational process, highlighting their role in supporting and monitoring their children's education.

5. Provide training for teachers to identify warning signs and risk factors in students who may be at risk of dropping out of school. It is also critical that teachers are trained to offer emotional and academic support to students who need it.
6. Implement tutoring and mentoring programs between students of different grades or between students and community professionals. This can foster motivation and a sense of belonging to the school, while providing a space for support and guidance for students.
7. Develop strategies to prevent and combat child labour, such as creating support programmes for families and promoting alternatives for generating income that do not affect children's school attendance.
8. Implement policies and programs that address the needs of children and adolescents with disabilities, indigenous populations, migrants and other vulnerable groups, ensuring that they have access to quality education and feel integrated into the educational system.
9. Establish systems to monitor and follow up on student attendance and academic performance to identify dropout patterns and take preventive actions in a timely manner.
10. Promote collaboration between the government, educational institutions, non-governmental organizations, private enterprise and civil society to design and implement comprehensive strategies that address school dropout from multiple perspectives.

5. Conclusion

Every public agenda highlights the role of education as a tool for economic development or as a channel for progress, because education is one of the issues that unite public policies throughout the world and is the key to achieving full employment and the eradication of poverty and inequality, and laying the foundations for sustained economic growth. Therefore, it is necessary to maintain inclusion as a fundamental premise, to share and build new educational proposals based on prior knowledge regarding what makes it difficult for children and adolescents to remain in the classroom and, at the same time, to help them carry out a comprehensive analysis of the possibilities to eliminate school dropout.

The use of the Neutrosophic Delphi and AHP methods highlights the importance of implementing inclusive educational policies in the prevention and reduction of school dropout in Ecuador. Educational policies and programs must focus on these key aspects to achieve inclusive and equitable education for all children and adolescents in the country. Therefore, a sustained commitment is required from governments, educational institutions, teachers, families and the community in general, as well as the willingness to invest in the long term to achieve significant and lasting changes in the educational system.

Considering education as a universal right of the population implies that everyone who aspires to study can do so and that the system itself includes it. However, in practice it is known that there are many obstacles to complying with this precept. The responsibility of making a truly inclusive education possible must be shared by all and not fall solely on educational institutions. Providing resources is a responsibility that no government should shirk. It is necessary to create Educational Institutions that approve and value differences and recognize equity and equal opportunities. This implies changes and modifications of content, approaches, structures and strategies based on a common vision.

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Neutrosophic cognitive map for the analysis of substances subject to control

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Abstract. This study addresses the central problem of analyzing the commercialization of controlled substances using a neutrosophic cognitive map approach, a tool that has been little explored in this context. The need to investigate this phenomenon lies in the increasing complexity and ambiguity of controlled substances markets, where factors such as regulation, consumer behavior, government policies, and the evolution of supply and demand, generate a highly uncertain and dynamic environment. Despite previous research in the field of commercialization of these substances, traditional approaches fail to capture the interaction and combined effects of the multiple elements that influence the market. This study fills that gap by integrating the concept of neutrosophic to more accurately represent the relationships between these factors, taking into account the inherent imprecision in data and human behavior. The methodology used in this work is based on the construction of a neutrosophic cognitive map, which not only organizes and visualizes the interactions between the different elements of the commercialization process but also allows for modeling the uncertainty and nonlinear interactions between them. The results reveal that this approach offers a richer and more dynamic understanding of the controlled substances market, highlighting the key factors that contribute to fluctuating supply and demand and providing a useful tool for formulating more effective policies. The contribution of the study is significant, as it not only offers a new theoretical perspective on the analysis of complex markets but also provides a methodology that can be applied to other fields facing problems of uncertainty, such as the regulation of other controlled products or decision-making in highly complex contexts.

Keywords: Neutrosophic Adjacency Matrix, Neutrosophic Cognitive Mapping, Neutrosophic Modeling, Decision Making in Complex Contexts; Cognitive Analysis; Drug Trafficking.

1. Introduction

The history of substance use dates back to ancient times, initially linked to mystical and religious practices. Over the centuries, various cultures, such as the Sumerian, Chinese, Indian, Greek and Roman, used substances such as poppies and marijuana in religious and medicinal contexts. However, today, the marketing of controlled substances has evolved to become a global problem, associated with organized crime and related crimes such as homicide and money laundering. [1]

In Ecuador, social and cultural changes, together with economic factors, have led to an increase in the commercialization of these substances, especially in impoverished areas and among young people. Organized crime deploys complex structures that favor micro-trafficking and illicit association.

The definition of controlled substances addresses the legal prohibition or medical and pharmaceutical restriction on their commercialization. Various entities, such as the World Health Organization and the National Agency for Regulation, Control and Health Surveillance, provide perspectives on these substances.

At the international level, the fight against trafficking and consumption of prohibited substances involves organizations such as the UN and the OAS, highlighting bilateral and multilateral cooperation. In Guayaquil, drug-related problems affect the quality of life, generating insecurity and violence [2] , particularly among young people.

Statistics show a worrying increase in the consumption of controlled substances in Guayaquil since January 2021. Despite government efforts and international cooperation, the magnitude of consumption persists, especially in marginalized areas of the city.

The concept of controlled substances encompasses addictive narcotics and psychotropic drugs that directly affect mental and intellectual activities by reducing the activity of the nervous centers. When consumed, these substances cause psychological dependence, sleep, narcosis, numbness, loss of consciousness and anxiety, which leads to accelerated deterioration of moral limitations and loss of motor skills. They are considered high-risk and are regulated by governments to prevent their cultivation [3] , illegal sale and consumption.

The classification of narcotic substances, according to the National Agency for Regulation, Control and Health Surveillance, includes narcotics, psychotropics, chemical precursors and specific chemical substances. From the point of view of their chemical structure, they are divided into depressants, stimulants, hallucinogens, opioids, inhalants and cannabis.

Depressants act as sedatives, weakening nerve functions, and include the use of alcoholic beverages. Stimulants accelerate activity in the central nervous system, generating alertness and energy, as in the case of cocaine and amphetamines. Hallucinogens alter the perception of reality, time, and space, inducing fictitious visual and auditory experiences. Opioids include illicit heroin, synthetic opioids, and prescription painkillers. Inhalants are absorbed by inhalation and enter the bloodstream and are found in household products such as paints and adhesives. Finally, cannabis can cause mental disorders, including psychosis. [4]

The sale of controlled substances, especially narcotics, is considered a wilful crime. This crime requires that individuals know the availability of the drug and a subjective element, which is the intention to sell narcotics, which is why it is linked to illicit drug trafficking activities.

The marketing of these substances influences various contexts, sometimes driven by consumption needs or poverty conditions. Additionally, the participation of minors in the criminal chain is highlighted [5], contributing to the disproportionate increase in associated criminal activities. [6]

It is important to note that illicit drug trafficking involves several stages, from the manufacturing of raw materials to final distribution. The marketing of controlled substances represents the last phase of this illicit activity, which allows the drug to reach the final consumer. This process includes preparation, transportation, storage and distribution before reaching the marketing stage.

Microtrafficking refers to the problem related to the sale of small quantities of narcotics or controlled substances. It involves the commercialization of drugs on a small scale, with minimal seizures and concealment in distribution for direct and immediate consumption. This phenomenon poses an important challenge with social, political, economic and security implications, since it constitutes a threat to Ecuador due to its systematic expansion at the level of consumption and its commercial appeal in developing countries.

Citizen security refers to the guarantee of quality of life and human dignity, based on freedom as a fundamental right and access to social opportunities in globalized markets. This concept is based on public trust in the power of the State, which, respecting the principle of legality, aims to protect citizens from social dangers. Citizen security is also linked to the fight against poverty and drug addiction, since high levels of these issues indicate insecurity.

The Constitution establishes that it is the primary duty of the State to guarantee peace, social security, progress and development of society in the various social and economic aspects. Citizen security becomes essential to combat the increase in crime and protect national heritage, as it is a key element to maintain the stability of the State. Therefore, the present study aims to comprehensively analyze the

phenomenon of the increase in the commercialization of controlled substances in Guayaquil, using an approach based on cognitive maps to merge information from various sources, with the purpose of providing a deep and systematic understanding of the problem.

2. Related work

Neutrosophic cognitive maps

Based on the above elements, in this particular work the use of Neutrosophic Cognitive Maps (NCMs) is proposed, considering the advantages that this technique offers compared to other soft-computing techniques , in terms of interpretability , scalability, knowledge aggregation, dynamism and its capacity to represent feedback and indeterminacy relationships[7].

NCMs were introduced in 2003. NCMs are an integration of Fuzzy Cognitive Maps (FCMs) introduced by Kosko in 1986 and Neutrosophic Sets (NSs) introduced by Smarandache in 1995. This technique overcomes the inability of traditional FCMs to represent indeterminacy. The inclusion of indeterminacy establishes that neutrality and ignorance are also forms of uncertainty. It shows that FCMs constitute a technique that has received increasing attention due to its possibilities to represent causality. A set of definitions necessary to work with NCMs is presented below . First, let us formally state the original definition of neutrosophic logic as shown in [8] .

Definition 1:$Let\ N\ =\ \{(T,I,F)\colon T,I,F\in[0,1]\}$

Let C be a *neutrosophic set of evaluations* . v: $P\rightarrow$N is a mapping of a set of propositional formulas to N, i.e. each sentence $p\in P$is associated with a value in N, as given in Equation 1, meaning that p is T% true, I% indeterminate and F% false [9].

$$\mathrm{v\,(p)\ =\ (T,I,F)} \tag{1}$$

Therefore, neutrosophic logic is a generalization of fuzzy logic, based on the concept of neutrosophy [10, 11, 12].

Definition 2: (See Let K be the ring of real numbers. The ring spanned by $K\cup I$is called *a neutrosophic ring* if it involves the indeterminacy factor, where I satisfies $I2\ =\ I, I+I\ =\ 2I$and in general, $I+I+\ldots+I\ =\ nI, if\ k\in K, then\ k.I\ =\ kI, 0I\ =\ 0$. The neutrosophic ring is denoted by $K(I)$, which is spanned by $K\cup I, i.e., K(I)=<K\cup I>, where\ <K\cup I>$denotes the ring spanned by K and I.

Definition 3: A *neutrosophic matrix* is a matrix $A\ =\ [a_{ij}]ij$i = 1, 2, …, m and j = 1, 2, …, , n; $m,n\geq1$such that each $a_{ij}\in K(I)$, where K(I) is a neutrosophic ring.

Note that an element of the matrix can have the form $a+bI$, where "a" and "b" are real numbers, while I is the indeterminacy factor. The usual operations on neutrosophic matrices can be extended from classical matrix operations.

For example, $\begin{pmatrix}-1 & \mathrm{I} & 5\mathrm{I}\\ \mathrm{I} & 4 & 7\end{pmatrix}\begin{pmatrix}\mathrm{I} & 9\mathrm{I} & 6\\ 0 & \mathrm{I} & 0\\ -4 & 7 & 5\end{pmatrix}=\begin{pmatrix}-21\mathrm{I} & 27\mathrm{I} & -6+25\mathrm{I}\\ -28+\mathrm{I} & 49+13\mathrm{I} & 35+6\mathrm{I}\end{pmatrix}$.

Furthermore, a *neutrosophic graph* is a graph that has at least one indeterminate edge or one indeterminate node. The *neutrosophic adjacency matrix* is an extension of the adjacency matrix of classical graph theory. $a_{ij}=0$means that nodes i and j are not connected, $a_{ij}=1$means that these nodes are connected and $a_{ij}=I$, which means that the connection is indeterminate (it is unknown whether it is connected or not). Fuzzy set theory does not use such notions [13,14,15].

On the other hand, if indeterminacy is introduced into a cognitive map as it is called, then this cognitive map is called a *neutrosophic cognitive map* , which is especially useful in the representation of causal knowledge. It is formally defined in Definition 4.

Definition 4. A *neutrosophic cognitive map* (NCM) is a neutrosophic directed graph with concepts such as policies and events, among others, as nodes and causalities or indeterminacies as edges. It represents the causal relationship between concepts.

The measures described below are used in the proposed model, they are based on the absolute values of the adjacency matrix:

- The out-degree ( ) is the sum of the row elements in the $v_i$neutrosophic adjacency matrix . It reflects the strength of the outgoing relationships (c_{ij}) of the variable.

$$od(v_i) = \sum_{i=1}^{n} c_{ij} \quad (2)$$

- The in-degree (v_i) is the sum of the elements in the column. It reflects the strength of the relationships ($_{ij}$) that come out of the variable.

$$id(v_i) = \sum_{i=1}^{n} c_{ji} \quad (3)$$

- The total centrality (total degree td (v_i)), is the sum of the in-degree and the out-degree of the variable.

$$td(v_i) = od(v_i) + id(v_i) \quad (4)$$

Static analysis is applied using the adjacency matrix, taking into consideration the absolute value of the weights. Static analysis in Neutrosophic Cognitive Maps (NCM), see, initially contains the neutrosophic number of the form (, where I = indeterminacy). It requires a $a + bI$deneutrosophication process as proposed in, where $I \in [0,1]$and are replaced by their maximum and minimum values.

Finally, the average of the extreme values is calculated using Equation 5, which is useful for obtaining a single value. This value contributes to the identification of the characteristics to be addressed, according to the factors obtained, for our case study.

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (5)$$

$$A \succ B \Leftrightarrow \frac{a_1 + a_2}{2} > \frac{b_1 + b_2}{2} \quad (6)$$

3. Methods

This study adopted a qualitative-quantitative approach, applying statistical tools and qualitative methods, including cognitive maps [7] , to characterize the phenomenon. A non-experimental grounded theory design was used, with a descriptive and analytical-explanatory scope.

The theoretical methods included analytical-synthetic, inductive-deductive and systems approaches. At the empirical level, documentary analysis, interviews and surveys were used. Additionally, cognitive maps were implemented as a tool to merge information from various sources [16, 8], thus providing a comprehensive understanding of the problem of the increase in the commercialization of controlled substances in Guayaquil. The population consisted of 5 lawyers with a master's degree in human rights and more than 5 years of experience, and 10 citizens between 18 and 60 years of age from neighborhoods with a high incidence of micro-trafficking .

4. Results

In the analysis of the surveys applied to the citizens of Guayaquil regarding the commercialization of controlled substances, the following significant results were obtained:

- Awareness of the problem: 100% of respondents said they were aware of the marketing of controlled substances, which indicates that it is a widely recognized problem in Guayaquil society. However, it is worth noting that not everyone reports it, which has contributed to the surprising expansion of the phenomenon.
- Presence in Sectors: 100% indicated that the commercialization of these substances is present in their respective sectors, suggesting a widespread incidence in several neighborhoods of Guayaquil.

- Substances marketed: 40% indicated that in their sectors the main market is "H" (heroin), followed by 20% cocaine, 20% crack and 20% marijuana. The high prevalence of "H" is notable, associated with its high addictive potential and dangerous composition.
- Causes of the increase: 50% attributed the increase in sales to organized crime, 20% to poverty, another 20% to family problems and 10% to social deterioration. This reflects the perception of Guayaquil as a city affected by organized crime in conflictive neighborhoods.
- Responsibility for the increase: 50% believe that the responsibility lies with the family, 40% with the administration of justice and 10% with society. This suggests the perception that the family plays a crucial role and its deterioration contributes to the problem.
- Impact on Citizen Security: 100% believe that the sale of controlled substances affects citizen security in Guayaquil, thus linking this phenomenon not only to organized crime but also to other crimes, contributing to the city being considered one of the most violent in Ecuador.

  These results underline the urgency of addressing the marketing of controlled substances in Guayaquil and highlight the need for action at both the family level and in the justice system to counteract the impacts on public safety.

The investigation included interviews with lawyers specializing in human rights to learn about the situation regarding the sale of controlled substances in Guayaquil. The most relevant findings are the following:

- Problem definition: Respondents agree that the marketing of controlled substances refers to the small-scale sale of these substances for direct consumption. This criminal phenomenon is perceived as a rapidly growing problem that has negatively affected human development over time.
- Current legislation: The interviewees consider that the Ecuadorian government has made adequate efforts to punish this crime. They highlight laws such as the Organic Law for the Comprehensive Prevention of the Socioeconomic Phenomenon of Drugs and the Regulation and Control of the Consumption of Substances Classified as Subject to Control, as well as article 220 of the COIP, which establishes the quantities of possession of these substances.
- Impact and Associated Crimes: The marketing of controlled substances is considered a profound social problem that leads to crimes such as robbery, rape, theft, organized crime, conspiracy, corruption, etc. Additionally, it is perceived as a serious crime that threatens people's health and peace of mind.
- Violated rights: Several rights have been identified as violated, such as the right to health, criminal justice, non-discrimination, the right to childhood and especially the right to public safety. Society stigmatizes consumers, affecting their integration.
- Causes of the increase: The interviews highlight underlying causes such as the increase in organized crime, lack of job opportunities due to population growth in Guayaquil, poverty, lack of education and family disintegration.
- Suggestions for prevention: It is suggested that the problem be recognized in a comprehensive manner and that policies be reviewed so that prevention campaigns go beyond the dissemination of information. In addition, it is recommended that unemployment be addressed, that COIP sanctions be modified to prevent criminals from hiding behind consumption, and that more in-depth campaigns be carried out.

By addressing these aspects, a holistic view of the commercialization of controlled substances in Guayaquil can be obtained, providing the basis for more informed and effective strategies and policies.

Once the status of the elements under analysis has been examined, potential factors are extracted through the process approach. To do this, the factors that influence the process approach of the commercialization of controlled substances in Guayaquil are analyzed .

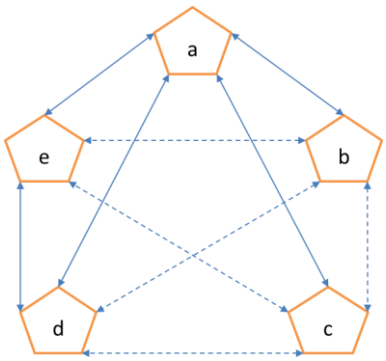
Potential factors through the process approach.

Situation I: Marketing of controlled substances in Guayaquil (responsibility of society).

A. Organized crime
B. Socioeconomic factors
C. Citizen awareness
D. Citizen perception
E. Barriers to reporting

Situation II: Marketing of controlled substances in Guayaquil (responsibility of local authorities)
F. Laws and regulations
G. Security Statistics
H. Government actions
I. Communication
J. Opinions on the causes

By analyzing the negative criteria obtained from the information extraction, it was decided to evaluate through the NCMs method which strategies have the potential to mitigate the negative effects. These factors, called elements, are denoted with the following code (a, b, c, d, e) for Situation I (Figure 1) and (f, g, h, i, j) for Situation II (Figure 2), following the previous order in the table. The group of experts evaluates the causal relationships between each of the aforementioned elements evaluated in surveys and interviews. An adjacency matrix and the graph that represents it were obtained.
Situation I:



deneutrosophicized adjacency matrix and the means of the corresponding extreme values for the NCM.

$$my = \begin{bmatrix} 0 & 1 & 0.7 & 0.8 & 1 \\ 1 & 0 & 0.5 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0 & 0.5 & 0.5 \\ 0.9 & 0.5 & 0.5 & 0 & 0.8 \\ 1 & 0.5 & 0.5 & 0.2 & 0 \end{bmatrix} \quad N(E) = \begin{bmatrix} 0 & 1 & 0.7 & 0.8 & 1 \\ 1 & 0 & I & I & I \\ 0.6 & I & 0 & I & I \\ 0.9 & I & I & 0 & 0.8 \\ 1 & I & I & 0.2 & 0 \end{bmatrix}$$

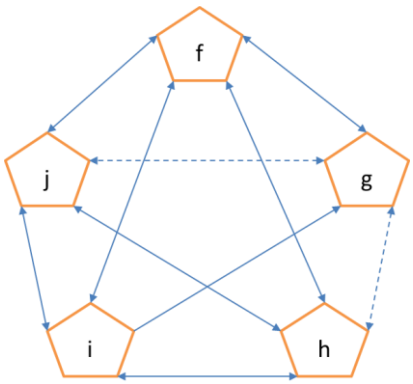
	td .
to	1.5217
b	1.0870
do	0.9348
d	1.0217
my	1.0870

The following partial conclusions can be drawn from the above:

- When A is activated, all other nodes are activated, that is, the state of crime levels in society influences the other identified vertices, so it has a direct influence due to the causal relationship with the indices (if A increases, then B, C, D and E increase in the same way as A).
- The relationships with A and the rest of the nodes are bidirectional, confirming the causal relationship in both directions and magnitude visualized.
- A strong relationship of indeterminacy is observed in B with A, C, D and E bidimensionally. Result: the order of importance of the factors will be as follows: A ≻ B ≻ E ≻ D ≻ C.

Priority factor to analyze: Mitigate the potential impacts of organized crime on the marketing of controlled substances in Guayaquil.

Situation II:



deneutrosophic adjacency matrix and the means of the corresponding extreme values of the NCM.

$$my = \begin{bmatrix} 0 & 0.2 & 0.8 & 0.9 & 0.3 \\ 0.3 & 0 & 0.5 & 0 & 0.5 \\ 1 & 0.5 & 0 & 0.9 & 0.7 \\ 0.8 & 1 & 0.6 & 0 & 0.6 \\ 1 & 0.5 & 1 & 0.2 & 0 \end{bmatrix} \quad N(E) = \begin{bmatrix} 0 & 0.2 & 0.8 & 0.9 & 0.3 \\ 0.3 & 0 & I & 0 & I \\ 1 & I & 0 & 0.9 & 0.7 \\ 0.8 & 1 & 0.6 & 0 & 0.6 \\ 1 & I & 1 & 0.2 & 0 \end{bmatrix}$$

	td .
F	1.1522
gram	0.7609
I	1.3043
Yo	1.0870
I	1.0435

The following partial conclusions can be drawn from the above:

- When H is activated, all other nodes are activated, meaning that government actions aimed at mitigating crime in society can contribute to a calm state of opinions about the cause. This makes all other problems identified at the other vertices have a direct relationship with node H, and the solution at this node positively affects the others.
- The relationships with H and the other nodes are bidirectional, confirming the causal relationship in both directions and magnitude, with a level of indeterminacy between the HG nodes.
- There is an indeterminate relationship between GH and GJ. In addition, a unidirectional relationship is observed from I to G.

Result: the order of importance of the factors will be as follows: H ≻ F ≻ I ≻ J ≻ G.

Priority factor to analyze: Implement governmental and legal actions that contribute to mitigating the impact of the commercialization of controlled substances in Guayaquil.

INTEGRATED STRATEGIES FOR THE SITUATION

Therefore, the information obtained through cognitive analysis allows for improving the effectiveness of the actions undertaken by local authorities in the prevention and control of the commercialization of controlled substances in Guayaquil. Table 1 presents the proposed strategies (see Table 1) and a university project to support community integration in the reduction of the commercialization of substances in vulnerable communities.

Table 1: Strategies to strengthen prevention and control of the marketing of controlled substances in Guayaquil. Source: Prepared by the authors.

No.	Strategies	Action points	Scope
1.	Development of personalized preventive strategies.	Demographic profile analysis.	Use demographic information to tailor preventive strategies to specific groups. Identify particular needs in areas with higher prevalence
		Differentiated educational programs.	Create preventive education programs adapted to age and socioeconomic situation. Incorporate specific approaches to address the challenges identified in each sector.
2.	Implementation of awareness campaigns	Using visual data.	Create visual campaigns based on cognitive maps to raise awareness in the community. Highlight critical areas and negative impacts of marketing.
		Local testimonies and experiences.	Use resident testimonies to convey the reality of the problem. Generate empathy and awareness about the consequences of commercialization.
3.	Strengthening citizen security	Coordination with security forces.	Establish effective collaboration protocols between local police and other security forces. Facilitate communication for a rapid and coordinated response.
		Surveillance at strategic points.	Implement surveillance systems in areas identified as critical. Use technology, such as security cameras, to monitor suspicious activity.
4.	Economic and employment strategies.	Job creation programs.	Develop programs that address unemployment rates in critical sectors. Create legitimate economic opportunities as an alternative to commercialization.
		Partnership with companies and NGOs .	Collaborate with local businesses and non-governmental organizations to promote employment initiatives. Promote investment in affected areas to generate sustainable employment.
5.	Strengthening rehabilitation and social support.	Improving rehabilitation centres.	Evaluate and improve existing rehabilitation centers. Ensure that programs are effective and address community needs.
		Psychosocial support.	Implement psychosocial support services for affected individuals and families. Address the underlying issues that contribute to marketing.
	Continuous evaluation and adjustment of strategies.	Monitoring key indicators.	Establish success indicators and continuously monitor their evolution. Adjust strategies according to the results obtained and changes in trends.

No.	Strategies	Action points	Scope
		Periodic evaluation meetings.	Organize regular meetings between local authorities, experts and community representatives. Evaluate progress and make strategic adjustments as necessary.
6.	Continuous evaluation and adjustment of strategies	Monitoring key indicators.	Establish success indicators and continuously monitor their evolution. Adjust strategies according to the results obtained and changes in trends.
		Periodic evaluation meetings.	Organize regular meetings between local authorities, experts and community representatives. Evaluate progress and make strategic adjustments as necessary.
7.	Collaboration with the community.	Creation of community committees.	Establish community committees to involve residents in decision-making. Promote active participation and collaboration with the authorities.
		Rewards and protection for whistleblowers.	Implement programs that protect those who report illegal activities. Offer incentives to encourage citizen cooperation

University Project: Implementation of Awareness Campaigns in Vulnerable Communities
Scope: Develop and implement awareness campaigns using cognitive analysis to improve perception and understanding of the risks associated with the commercialization of substances in vulnerable communities.
Duration: 12 months
General Objective: To reduce the marketing of substances in vulnerable communities through awareness campaigns based on cognitive analysis.
Specific objectives :

- Design specific awareness campaigns based on cognitive analysis.
- Evaluate the impact of campaigns on community perception and behavior.
- Provide educational resources to improve understanding of risks.

Stages:

I. Campaign planning and design.
II. Execution and dissemination of campaigns.
III. Impact assessment through surveys and cognitive analysis.
IV. Creation of additional educational resources.

Resources:

- Staff specialized in communication and psychology.
- Media (posters, brochures, social media).
- Surveying equipment.
  Expected impact:
- Improved risk perception.
- Reduce marketing to specific communities.
  Funding: Seek local sponsorship and support from government entities interested in prevention.

Results:

- Campaign impact and effectiveness report.
- Educational resources for continuous distribution.

Benefits to achieve:

- Increased community awareness.
- Active collaboration in prevention.

Role and participation of study centres:

- Design and execution of campaigns.
- Impact assessment through cognitive analysis.

The results obtained from the comprehensive study on the sale of controlled substances in Guayaquil reveal cognitive patterns that contribute to the increase in this phenomenon in the city. Through the application of surveys and interviews, key factors were identified, including the influence of organized crime, persistent poverty in vulnerable sectors, and social disintegration. Cognitive maps, as a central tool of the methodological approach, allowed a clear and detailed visualization of the interconnection of these elements.

Regarding the classification of substances, the results indicate a significant prevalence of the sale of highly addictive substances, particularly those known as "H": cocaine, crack cocaine and marijuana. The sale of these substances is not limited to individual transactions, but shows a direct relationship with organized crime, exacerbating problems of public safety in specific areas of the city. Additionally, it was identified that the consumption of these substances has a direct impact on the mental and physical health of individuals, contributing to the complexity of the problem.

The magnitude of the problem is evident when considering the variety of crimes arising from the trafficking of controlled substances, including robbery, rape, burglary, organized crime, criminal association, corruption and other violent crimes. These results underscore the urgent need to address not only the trafficking itself, but also related crimes that have a significant impact on the safety and quality of life of citizens.

5. Discussion

The findings of this study raise questions about the effectiveness of the measures currently implemented by local authorities in Guayaquil to combat the commercialization of controlled substances. The consensus among those interviewed on the need to review and strengthen government policies in this area underlines the importance of a more solid and focused intervention. The results indicate that, despite existing efforts, the magnitude of the problem persists, highlighting the need for more in-depth and specific strategies.

It is essential to highlight the interrelationship between drug trafficking and organised crime. Criminal groups have found in this illicit activity a means to maximise their income and perpetuate other criminal activities [17] . Territoriality in drug trafficking and disputes between gangs underline the need to address not only the sale of drugs but also the root of the problem, which includes the lack of economic opportunities and social disintegration in the affected areas.

The analysis of rights violations, particularly in the areas of health, access to justice and citizen security, highlights the complexity of the problem and the need for a comprehensive approach that goes beyond punitive measures. Suggestions from interviewees, such as reviewing the sanctions established in the COIP and addressing underlying causes, such as poverty and unemployment, point to specific areas for future interventions.

6. Conclusion

The research has highlighted the urgent need for comprehensive strategies to address the sale of controlled substances in Guayaquil. The comprehensive approach, supported by the integration of information through cognitive maps, reveals that the problem is multidimensional and that any mitigation effort must address not only the sale of substances but also its roots in organized crime, poverty and social disintegration.

The results demonstrate the need to review and strengthen existing government policies. The consensus among interviewees on the limited effectiveness of current measures underlines the urgency of a thorough review. The implementation of more effective and targeted policies, supported by cognitive analysis, can provide a more solid basis for combating the growing drug trade in the city.

Active community participation and the involvement of academic institutions are essential elements in the fight against this phenomenon. The proposed strategies and project should encourage collaboration between local authorities, universities and civil society. Awareness raising, education and citizen participation are essential to overcome current challenges and ensure that solutions are sustainable in the long term.

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Insertion and use of multicriteria methods for neutrosophic judicial evaluation

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Abstract. This study addresses the growing need to integrate multicriteria methods into judicial evaluation, specifically through a neutrosophic approach, to improve the accuracy and reliability of legal decision-making. Today, the judicial system faces critical challenges related to the interpretation of evidence, the consideration of multiple factors, and the uncertainty inherent in complex cases. Although various approaches have been used in judicial evaluation, few effectively address the ambiguity and indeterminacy present in the opinions of judges and experts, leading to fragmented or biased analysis. This study seeks to close that gap by applying neutrosophic multicriteria methods, which allow for a more balanced and comprehensive evaluation, incorporating all dimensions of the judicial process, from evidence to contextual factors. The methodological approach adopted is based on the use of neutrosophic multicriteria tools, which allow for the integration and analysis of various criteria through a flexible and robust framework. The results obtained reveal that this method not only improves the accuracy of judicial assessments but also provides a clearer view of the uncertainty associated with legal decisions. The main theoretical implications of this study lie in the possibility of applying a neutrosophic framework to judicial assessment, providing a tool capable of capturing the complexity of disputed cases. On a practical level, the findings are crucial for the development of judicial decision support systems, suggesting that the implementation of this approach could contribute to greater objectivity and consistency in judicial decisions, as well as to an improvement in the transparency of the judicial process.

Keywords: Multicriteria methods, judicial evaluation, neutrosophy, judicial decision-making, uncertainty, evidence analysis, support systems.

1. Introduction

The principle of immediacy is a fundamental concept within the criminal justice system of many countries. It refers to the direct proximity of the judge or court to the parties involved in a trial, which implies that the judge must directly witness the presentation of evidence and testimony to form an impartial opinion on the case. In other words, the judge must be physically present during the trial to observe first-hand the evidence and arguments presented by both parties [1] .

In white-collar crimes, which are usually financial, corruption, or high-profile crimes committed by people with power or influence, there may be a situation where the accused is not physically present during the trial. This can happen for a variety of reasons, such as the accused being in another country

and unable to be extradited or having delicate health conditions that prevent him from attending the trial [2] .

The absence of the accused at the trial may raise problems in relation to the principle of immediacy, since the judge will not be able to directly observe the reactions and expressions of the accused, which could affect his ability to assess the credibility of the testimonies, and the strength of the evidence presented. In addition, the absence of the accused may hinder the conduct of a fair trial, since it deprives the defense of the opportunity to question witnesses or present arguments in real time [3] .

However, it is important to note that the absence of the accused does not necessarily automatically imply a violation of the principle of immediacy. In some judicial systems, mechanisms may be established to ensure that the trial is conducted fairly and transparently, even in the absence of the accused. This may include allowing the defense to be represented by a lawyer or allowing communications and participation by the accused through technological means such as videoconferencing [2] .

In any event, the absence of the accused in economic or any other type of crime is a complex issue that must be carefully addressed to ensure that the fundamental rights of the accused are respected, and that a fair and equitable trial is conducted. Judicial systems must strive to strike a balance between the need for justice and respect for the fundamental principles of due process of law [4] .

The principle of immediacy is guaranteed through a series of procedural rules, such as the obligation of the judge to be present at all stages of the proceedings, the prohibition of submitting written evidence and the obligation of witnesses to appear before the court and testify. This is a fundamental principle of the Ecuadorian judicial system. It is essential to ensure that trials are fair and equitable and that judges have the necessary information to issue fair sentences [5] .

Below are some examples of how the principle of immediacy allows the judge to assess first-hand the evidence and testimony presented:

- ✓ The judge can observe the body language of witnesses and determine whether they are being truthful or not.
- ✓ The judge may ask witnesses questions to clarify their testimony.
- ✓ The judge may compare the testimonies of different witnesses to determine who is more credible.
- ✓ The judge may examine the evidence to determine if it is relevant to the case.

These crimes are often committed in the business or professional sphere and can have a significant impact on society. Some characteristics of white-collar crimes are [2]:

- ✓ They are committed by individuals with power and position.
- ✓ They occur in the business or professional sphere.
- ✓ They can have a significant impact on society.
- ✓ They are often difficult to research and prove.

Some examples of white-collar crimes are:

These are the most common types of white-collar crimes:

- ✓ Fraud
- ✓ Bribery
- ✓ Corruption
- ✓ Money laundering
- ✓ Abuse of power
- ✓ Abuse of confidence
- ✓ Tax evasion

White-collar crime is a serious problem in Ecuador, as it can cause economic, social and political damage. Ecuadorian authorities must take measures to prevent and combat these crimes. The research aims to develop a judicial management system that maximizes the application of the principle of immediacy while ensuring procedural efficiency, optimal use of resources and accessibility to justice.

1. Methodology.

To carry out the investigation, the steps are as follows:

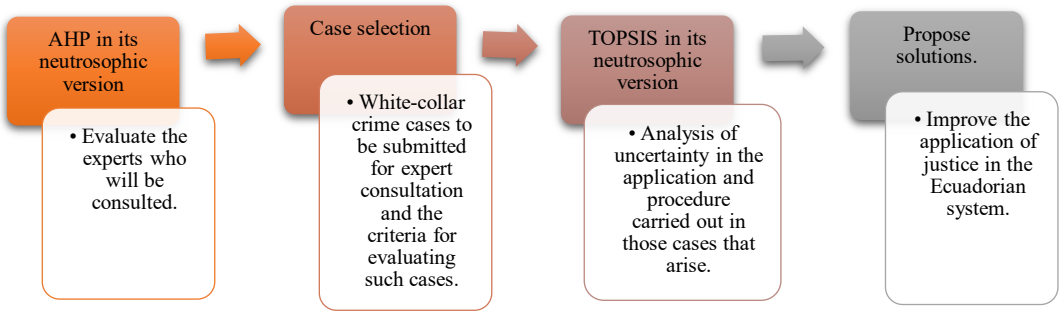

Figure 1. Prosed method

Analytic Hierarchy Process (AHP): This method models the problem resulting in the creation of a representative hierarchy of the decision-making scheme in question [6] . The importance levels or weights of the criteria are estimated by making pairwise comparisons between them. The theory of the AHP technique within a neutrosophic framework can be found in [7] . For this method triangular neutrosophic numbers will be used, see [8-10] .

TOPSIS: In the case of TOPSIS, the selection is based on identifying the alternative that is closest to the ideal solution and farthest from the worst solution [9 , 11 , 12] . This method has evolved into Neutrosophy, so in this paper linguistic terms will be linked to Single Valued Neutrosophic Numbers (SVNN). This allows experts to make their evaluations in linguistic terms, which is more natural. For more details, see [13,14].

2. Application and results

A. Experts: A judge, a prosecutor and two lawyers were selected for the consultation.
Vectors of importance were determined based on the specific responsibilities and roles of these actors within the criminal justice system. While the judge acts as an impartial arbiter making decisions based on the law and evidence presented during the trial, and the lawyer defends the interests of his or her client (whether the accused or the plaintiff), the prosecutor is the driving force behind the prosecution, playing an active role in combating white-collar crime from the initial investigation to the eventual conviction. This combination of prosecutorial responsibility, specialized knowledge, authority to decide on prosecution, and the duty to protect the public interest justifies the higher level of importance assigned to the prosecutor in the process of evaluating a white-collar crime case.
Expert weights: Lawyer 1 = (0.19444,0.17307,0.17); Lawyer 2 = (0.08333,0.09230,0.01); Prosecutor = (0.41667,0.48076,0.49); Judge = (0.30556,0.32692,0.33)

B. Alternatives

Table 1. Alternatives

Alternative 1	Alternative 2	Alternative 3
Odebrecht: In 2017, the Brazilian company Odebrecht pleaded guilty to bribery of public officials in several countries, including Ecuador. The bribes were paid to obtain government contracts and the total amount of the bribes is estimated to be over $788 million.	**National Assembly:** In 2018, it was revealed that several Ecuadorian legislators had received bribes to vote in favor of certain laws. The bribes were paid in exchange for votes to approve laws that benefited private companies. The case involving the National	**Panama Papers:** In 2016, a journalistic investigation revealed that several Ecuadorian politicians and businessmen had used companies in Panama to evade taxes. The Panama Papers case has damaged Ecuador's image and led

Alternative 1	Alternative 2	Alternative 3
The Odebrecht case has had a significant impact in Ecuador, as it has led to the dismissal of several public officials and the investigation of others.	Assembly has damaged the image of Ecuadorian democracy and led to the dismissal of several legislators.	to the investigation of several public officials.

These white-collar crime cases in Ecuador were selected for neutrosophic analysis because the principle of immediacy in these cases is particularly illustrative due to the complexity and covert nature of these crimes. These crimes, often perpetrated in corporate or financial contexts by high-status individuals, require a detailed understanding of personal transactions and interactions for detection and prosecution.

C. Criteria

1. Maximizing Immediacy: Ensuring that the judge has maximum direct contact with the parties, witnesses and evidence for a better understanding and assessment of the facts.
2. Procedural Efficiency: Reduce case resolution times, minimizing delays without compromising the quality of justice.
3. Resource optimization: Efficient use of available human and material resources, including courtrooms, information technologies and judicial staff.
4. Accessibility: Ensure that all parties have equal access to the justice system, regardless of their geographic location, socioeconomic status or physical capabilities.
5. Adaptability and resilience: The ability of the system to adapt to unforeseen changes, such as increased demand for judicial services or crises.

Table 2. TOPSIS method according to expert 1 evaluation.

	Maximizing immediacy	**Procedural efficiency**	**Resource optimization**	**Accessibility**	**Adaptability and resilience**
Odebrecht	(0.70, 0.25, 0.30)	(0.90, 0.10, 0.10)	(0.50, 0.50, 0.50)	(0.60, 0.35, 0.40)	(0.30, 0.75, 0.70)
National Assembly	(0.50, 0.50, 0.50)	(0.80, 0.15, 0.20)	(0.20, 0.85, 0.80)	(0.70, 0.25, 0.30)	(0.10, 0.90, 0.90)
Panama Papers	(0.60, 0.35, 0.40)	(0.40, 0.65, 0.60)	(0.90, 0.10, 0.10)	(0.50, 0.50, 0.50)	(0.80, 0.15, 0.20)

Table 3. TOPSIS method according to expert evaluation 2.

	Maximizing immediacy	**Procedural efficiency**	**Resource optimization**	**Accessibility**	**Adaptability and resilience**
Odebrecht	(0.53, 0.23, 0.33)	(0.89, 0.12, 0.16)	(0.62, 0.52, 0.50)	(0.66, 0.38, 0.38)	(0.29, 0.77, 0.80)
National Assembly	(0.48, 0.51, 0.52)	(0.75, 0.11, 0.11)	(0.21, 0.85, 0.86)	(0.73, 0.30, 0.13)	(0.08, 0.92, 0.86)
Panama Papers	(0.58, 0.34, 0.41)	(0.37, 0.68, 0.67)	(0.87, 0.17, 0.03)	(0.57, 0.47, 0.52)	(0.75, 0.22, 0.26)

Table 4. TOPSIS method according to expert evaluation 3.

	Maximizing immediacy	Procedural efficiency	Resource optimization	Accessibility	Adaptability and resilience
Odebrecht	(0.64, 0.24, 0.27)	(0.93, 0.10, 0.12)	(0.41, 0.45, 0.50)	(0.54, 0.49, 0.41)	(0.27, 0.80, 0.73)
National Assembly	(0.49, 0.48, 0.54)	(0.77, 0.16, 0.19)	(0.15, 0.99, 0.68)	(0.71, 0.23, 0.38)	(0.08, 0.87, 0.81)
Panama Papers	(0.59, 0.37, 0.51)	(0.38, 0.64, 0.59)	(0.86, 0.03, 0.13)	(0.53, 0.47, 0.48)	(0.70, 0.09, 0.15)

Table 5. TOPSIS method according to expert evaluation 4.

	Maximizing immediacy	Procedural efficiency	Resource optimization	Accessibility	Adaptability and resilience
Odebrecht	(0.73, 0.28, 0.34)	(0.93, 0.00, 0.12)	(0.49, 0.59, 0.52)	(0.66, 0.39, 0.36)	(0.26, 0.66, 0.65)
National Assembly	(0.48, 0.48, 0.48)	(0.83, 0.20, 0.13)	(0.14, 0.82, 0.83)	(0.65, 0.19, 0.38)	(0.04, 0.91, 0.92)
Panama Papers	(0.65, 0.42, 0.44)	(0.34, 0.70, 0.52)	(0.79, 0.19, 0.18)	(0.46, 0.59, 0.54)	(0.78, 0.22, 0.19)

Table 6. Distances between negative and positive solutions.

Alternative	s_i^+	s_i^-	$\widetilde{\rho_j}$	order
Odebrecht	1.141224	1.19552	0.511618	2
National Assembly	1.758339	0.881627	0.333954	3
Panama Papers	1.021049	1.725995	0.62831	1

Alternatives are ranked based on CP, with higher values being better. Based on this criterion, alternatives are ranked as follows:

1. **National Assembly** : It is considered the least preferable due to its lower CP in this aggregate analysis, indicating that this case had the least application of the proposed criteria.
2. **Panama Papers** : It has an intermediate CP, ranking second.
3. **Odebrecht** : Has the highest CP, indicating that it is the most preferable of the three according to the experts' aggregate evaluations.

This result shows how aggregate analysis can vary from individual assessments, highlighting the importance of considering multiple perspectives in decision-making.

3. Discussion.

The expert with the highest weighted value was the prosecutor, which is justified based on the following elements:

- ✓ Prosecutor's Responsibility: The prosecutor plays a central role in the investigation and prosecution of white-collar crimes. He or she is responsible for gathering and presenting the evidence necessary to prove the defendant's guilt beyond a reasonable doubt. This task involves a thorough understanding of the legal and financial complexities associated with these crimes, as well as the ability to build a compelling case that can withstand scrutiny in court.
- ✓ Specialized Knowledge: White collar crimes often involve sophisticated financial maneuvers, tax evasion, fraud, embezzlement, and other forms of corporate and government corruption.

The prosecutor, because of his or her experience in prosecuting these types of crimes, possesses specialized knowledge that is crucial to identifying and understanding the tactics employed by criminals, giving him or her a unique position in the evaluation process.

- ✓ Prosecutor's decision: The prosecutor has the discretion to decide whether there is sufficient evidence to proceed with formal prosecution and to bring charges against the accused. This decision depends not only on the availability of evidence, but also on strategic considerations and the public interest, which requires an informed and experienced judgment about the strength of the case.
- ✓ Protecting the public interest: Prosecutors represent the State and the general public, seeking to administer justice and maintain confidence in financial and government institutions. Their role is essential to deter future economic crimes by effectively prosecuting offenders, thereby contributing to the integrity of the economic and government system.

The application of the TOPSIS neutrosophic method in the present analysis illustrates an approach to decision-making in contexts characterized by high levels of uncertainty and indeterminacy. In this scenario, the Odebrecht, National Assembly and Panama Papers alternatives (A1, A2 and A3 respectively) were evaluated under a set of weighted criteria, to determine the most suitable alternative based on the calculation of the positive neutrosophic proximity to the ideal solution (CP). The essence of this approach lies in its ability to effectively incorporate and manage the ambiguity inherent in expert preferences and judgments, offering a robust framework for multi-criteria analysis[14,15].

In the specific case of our analysis, alternative 1, Odebrecht, turned out to be the most preferable, having the highest CP value. This result indicates its optimal proximity to the ideal positive neutrosophic solution, highlighting its superiority across the spectrum of criteria considered. The attribution of this preference is not arbitrary, but rather reflects an aggregate consensus based on expert evaluations, mitigating the impact of subjectivity by applying neutrosophic logic.

On the other hand, the National Assembly was positioned as the least preferable alternative, attributable to its lower CP. This finding suggests that A2 deviates significantly from the neutrosophic ideal solution, reflecting deficiencies or areas for improvement in the context of the evaluated criteria. Panama Papers, with an intermediate CP, is located at an equilibrium point between Odebrecht and the National Assembly, denoting moderate compatibility with the positive ideal solution and, therefore, relative viability as a decision option.

The case of Ecuador's National Assembly in 2018, where several legislators were implicated in receiving bribes in exchange for voting in favor of laws that benefited private companies, raises a number of critical considerations under the lens of specific criteria commonly associated with governance and effective institutional performance. The involvement of these legislators in acts of corruption not only undermines the integrity of democratic institutions but also violates fundamental principles that should guide the conduct of public servants. Below, we analyze how this case transgressed the following criteria:

- ✓ **Maximizing immediacy (C1):** This criterion, which emphasizes the importance of rapid and effective responses to society's needs and challenges, was clearly violated. The legislators involved prioritized personal and third-party interests over collective well-being, delaying or denying immediate responses to the needs of the population. Corruption in this context contributes to public distrust and cynicism, eroding faith in the ability of institutions to act quickly and effectively for the benefit of the population .
- ✓ **Procedural efficiency (C2):** Efficiency in legislative processes involves the enactment of laws and policies in a fair, transparent manner and in the public interest. In the case of the National Assembly, this principle was compromised, as legislative processes were manipulated to favour private interests. Procedural efficiency was not only violated in terms of the quality and impartiality of the legislation passed, but also in the public perception of the Assembly as an efficient and trustworthy institution.

- ✓ **Resource Optimization (C3):** Public resources, including time and legislative capacity, should be used as effectively as possible to promote the general welfare. However, this case illustrates a diversion of resources towards the pursuit of corrupt objectives, which affects the efficient allocation of those resources for legitimate purposes and undermines the integrity of legislative decisions .
- ✓ **Accessibility (C4):** Accessibility in this context refers to the ability of the population to influence the legislative process and access their representatives so that their voices are heard. Corruption undermines this principle by closing channels of communication and public participation, privileging instead those who are willing to pay bribes to obtain exclusive access and legislative favors .
- ✓ **Adaptability and resilience (C5):** An institution's adaptability and resilience are manifested in its ability to resist corruption and adapt to the changes necessary to preserve its integrity. In this case, the involvement of legislators in corrupt practices reveals a systemic failure in control, accountability and transparency measures, which calls into question the institution's ability to adapt and strengthen itself in the face of such challenges.

It is essential to address these challenges and seek solutions that ensure due process and justice in cases where the accused is absent. To do so, the regulations must be adapted and the technologies available in the Ecuadorian judicial system must be used. It is suggested that these criteria be considered for the selection and analysis of judicial proceedings in which the principle of immediacy will be applied. For example, the court could implement a hybrid system of hearings, combining in-person sessions for cases where immediacy is critical, with virtual hearings in situations where efficiency and accessibility are prioritized.

4. Conclusion

The principle of immediacy is a fundamental pillar of the Ecuadorian judicial system, which aims to guarantee a fair and equitable trial by allowing the judge to directly witness the evidence, testimony and expressions of the parties involved in the process. However, in cases of white-collar crimes, in which individuals with power, influence and resources are involved, the absence of the accused can pose challenges to the effective application of this principle. The absence of the accused in court affects the judge's ability to directly observe the reactions and expressions of the accused, which can make it difficult to assess his credibility and to properly evaluate the evidence presented. Additionally, the lack of direct participation of the accused can lead to biased judicial decisions or decisions based solely on the version presented by the accusing party.

Some possible solutions include allowing defendants to appear by videoconference, appointing a legal representative to defend defendants in their absence, and allowing the submission of evidence and testimony in writing. These measures can help ensure the effective participation of defendants in the trial and facilitate their defense, even if they are not physically present. The negative impact of the National Assembly case on the image of Ecuadorian democracy is a wake-up call on the urgent need to strengthen institutions, ensure accountability, and reinforce control mechanisms to prevent corruption. This analysis not only highlights violations of the fundamental principles of democratic governance, but also underlines the urgency of adopting structural reforms that promote a culture of integrity, transparency, and public service.

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Optimizing Tourism Offers with COPRAS-SVNS for Stress Reduction and Sustainability Promotion

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Abstract. This study focused on the evaluation of tourist packages in the Pichincha region, Ecuador, designed to combine sustainability and stress reduction. Its main objective was to identify the most effective alternatives using multicriteria decision methods, incorporating neutrosophic logic as a key approach. Eight tourist packages were analyzed, integrating activities such as reforestation, meditation, and rural tourism with organic gardens, all aimed at environmental sustainability and the emotional well-being of participants. Using the neutrosophic correlation method, the most relevant criteria for subsequent analysis were selected. The application of the COPRAS-SVNS method determined that the "Eco-destinations with electric bicycle" package represented the best option, as it optimally balanced the objectives of sustainability and stress reduction. The study emphasized the relevance of neutrosophic logic in addressing the uncertainty and ambiguity inherent in evaluating complex tourist alternatives.

Keywords: Sustainable tourism, stress reduction, multicriteria decision-making, neutrosophic logic, COPRAS-SVNS.

1 Introduction

Tourism has undergone significant evolution since its beginnings when it was primarily limited to travel for recreational purposes. Today, this concept has acquired new dimensions, including the pursuit of physical and emotional well-being, connection with nature, and the promotion of sustainable practices [1], [2]. As people seek experiences that allow them to disconnect from the fast-paced rhythm of modern life, tourism emerges as a means to achieve tranquility, relaxation, and, at the same time, contribute to environmental conservation [3].

Stress, understood as the body's response to situations perceived as threatening or challenging, has been the subject of numerous studies due to its adverse effects on health. It has been demonstrated that prolonged stress negatively impacts various areas of human well-being, including mental, physical, and social health [4]. Its relationship with disorders such as anxiety, depression, hypertension, and even autoimmune diseases has been extensively documented [5]–[7]. To mitigate these effects, various approaches such as meditation, physical exercise, and connection with nature have proven to be effective tools for stress reduction. Thus, tourism not only serves as a means for recreation but also as a strategy for stress management, promoting physical and emotional recovery for individuals.

In this context, previous studies in the Ecuadorian region of Pichincha have identified the close relationship between the type of tourism consumed in the area and the levels of stress that tourists experience when enjoying these services. This raises the need to optimize tourism offerings so that not only the momentary pleasure or enjoyment is considered, but also its capacity to contribute to sustainability

and stress reduction.

This challenge requires the use of effective decision-making tools that allow for the selection of tourism packages that best combine these two elements. Multicriteria decision-making (MCDM) methods are tools that enable the evaluation of alternatives based on multiple criteria [8], making them a powerful resource for addressing the complexity of problems involving multiple competing factors, such as the design of tourism packages that balance sustainability and well-being.

However, traditional decision-making methods are not always suitable for dealing with the uncertainty inherent in real-world problems [9]–[11]. The available information is often incomplete, ambiguous, or imprecise, making it difficult to formulate decisions that faithfully reflect reality. In this regard, fuzzy set theory, introduced by Zadeh in the 1960s, has stood out as a tool for modeling imprecision in information. However, this theory has limitations when addressing indeterminacy and ambiguity in complex situations. Due to these limitations, a new approach known as Neutrosophy emerged, developed by Florentin Smarandache in 1995. Neutrosophy is a branch of mathematical philosophy that allows modeling not only uncertainty but also indeterminacy and falsehood in a unified space, making it a more robust tool for decision-making in complex environments [12].

Neutrosophy has proven especially useful in solving multicriteria problems, as it allows for the incorporation of vagueness and indeterminacy into the evaluation of alternatives, which is essential for tackling the uncertainties arising in the field of sustainable tourism. By combining Neutrosophy with multicriteria decision-making methods, decision-makers can introduce greater flexibility into the process, allowing for a more comprehensive and realistic analysis of the available alternatives. In practice, this involves creating more accurate evaluation models that consider both the quantitative and qualitative aspects of tourism alternatives [13].

The use of multicriteria decision-making methods and Neutrosophy enables a more comprehensive and detailed evaluation of tourism offerings, considering diverse criteria. In this regard, the purpose of this work is to identify and evaluate tourism packages in the Pichincha region that combine sustainability and stress reduction. Through this, it is expected to select the optimal alternative that enhances the user experience and promotes responsible tourism practices. By conducting detailed analysis using multicriteria decision-making methods supported by Neutrosophy, the goal is to provide a tool to improve the tourism offering, optimizing customer experience, and promoting more responsible and sustainable practices in the industry.

2 Methods

2.1 Preliminaries

Definition 1. Let X represent a space of points (or objects), with a generic element denoted as x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, i.e., $T_A(x): X \rightarrow]0^-, 1^+[$, $I_A(x): X \rightarrow]0^-, 1^+[$ and $F_A(x): X \rightarrow]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$. Due to the complexity of applying neutrosophic sets in practice, the single-valued neutrosophic set (SVNS) was introduced as a simplified form for real-world scientific and engineering applications. The following definitions provide the formalization of SVNSs.

Definition 2. Given a set X of objects with elements denoted by x, an SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, where $T_A(x), I_A(x), F_A(x) \in [0,1]$. An SVNS A can be expressed as $A = \{x, T_A(x), I_A(x), F_A(x) \mid x \in X\}$, and the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, satisfies $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 3. The complement of an SVNS A is denoted by Ac and is given by $Ac = \{x, F_A(x), 1 - I_A(x), T_A(x) \mid x \in X\}$

Definition 4. An SVNS A i is considered a subset of another SVNS $B, A \subseteq B$ if and only if $T_A(x) \leq$

$T_B(x), I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$ for every x in X.

Definition 5. Two SVNSs A and B are equal, written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$

Definition 6. For any two SVNSs A and B in the universe $X = \{x_1, x_2, \ldots, x_n\}$, their correlation coefficient is defined as:[14]

$$M(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[\phi_i(1-\Delta T_i) + \varphi_i(1-\Delta I_i) + \psi_i(1-\Delta F_i)] \quad (1)$$

Where the parameters are defined as:

$$\phi_i = \frac{3-\Delta T_i - \Delta T_{max}}{3-\Delta T_{min} - \Delta T_{max}}, \qquad \varphi_i = \frac{3-\Delta I_i - \Delta I_{max}}{3-\Delta I_{min} - \Delta I_{max}}, \qquad \psi_i = \frac{3-\Delta F_i - \Delta F_{max}}{3-\Delta F_{min} - \Delta F_{max}}$$

The differences are calculated as:

$$\Delta T_i = |T_A(x_i) - T_B(x_i)|, \qquad \Delta I_i = |I_A(x_i) - I_B(x_i)|, \qquad \Delta T_i = |T_A(x_i) - T_B(x_i)|,$$

and

$$\Delta T_{min} = min_i|T_A(x_i) - T_B(x_i)|, \quad \Delta F_{min} = min_i|F_A(x_i) - F_B(x_i)|, \quad \Delta I_{max} = max_i|I_A(x_i) - I_B(x_i)|,$$
$$\Delta I_{min} = min_i|I_A(x_i) - I_B(x_i)|, \quad \Delta T_{max} = max_i|T_A(x_i) - T_B(x_i)|, \quad \Delta F_{max} = max_i|F_A(x_i) - F_B(x_i)|,$$

for any $x_i \in X$ and i = 1, 2, . . ., n

However, the differences of importance are considered in the elements in the universe. Therefore, it is necessary to take the weight of the element. $x_i (i = 1, 2, \ldots, n)$ into account. In the following, a weighted correlation coefficient between SVNSs is introduced.

Definition 7. Let w_i be the weight for each element x_i $(i = 1, 2, \ldots, n), w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then the following weighted correlation coefficient between the SVNSs A and B can be defined:

$$M_w(A,B) = \frac{1}{3}\sum_{i=1}^{n} w_i[\phi_i(1-\Delta T_i) + \varphi_i(1-\Delta I_i) + \psi_i(1-\Delta F_i)] \quad (2)$$

Definition 8. For two SVNS numbers $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ Their sum is defined as:

$$A + B = (T_A + T_B - T_A t_B, I_A I_B, F_A F_B) \quad (3)$$

Definition 9. The product of $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ is given by:

$$A * B = (T_A T_B, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B) \quad (4)$$

Definition 10. Let $A = (T_A, I_A, F_A)$ be a SVN number and $\lambda \in \mathbb{R}$ an arbitrary positive real number, then:

$$\lambda A = \left(1 - (1 - T_A)^{\lambda}, {I_A}^{\lambda}, {F_A}^{\lambda}\right), \lambda > 0 \quad (5)$$

Definition 11. For two SVNSs A= $\{A_1, A_2, \ldots, A_n\}$, and B= $\{B_1, B_2, \ldots, B_n\}$ (i= 1,2,…,m) the separation measure based on normalized Euclidean distance is:

$$q_n(A,B) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left(\left(T_A(x_i) - T_B(x_i)\right)\right)^2 + \left(\left(I_A(x_i) - I_B(x_i)\right)\right)^2 + \left(\left(F_A(x_i) - F_B(x_i)\right)\right)^2}$$

$(i = 1,2, \ldots, n)$ (6)

Definition 12. Let $A = (T_A, I_A, F_A)$ be a single-valued neutrosophic number, a score function is mapped $\tilde{N}_A$ into the single crisp output $S(\tilde{N}_A)$ as follows:

$$S(\tilde{N}_A) = \frac{3+T_A-2I_A-F_A}{4} \quad (7)$$

where $S(\tilde{N}_A) \in [0,1]$. This score function ensures consistency within the interval for SVNS evaluations.

2.2 Decision-making method using the correlation coefficient of SVNSs [14]

In problems of decision-making involving multiple attributes and single-valued neutrosophic information, the performance of an alternative A_i $(i = 1,2,\dots,m)$ for a given attribute C_j $(j = 1,2,\dots,n)$ is characterized as $A_i = \{C_j, T_{Ai}(C_j), I_{Ai}(C_j), F_{Ai}(C_j) | Cj \in C, j = 1,2,\dots,n\}$. Where $T_{Ai}(C_j)$, $I_{Ai}(C_j)$, $F_{Ai}(C_j)) \in [0,1]$ and $0 \leq T_{Ai}(C_j), I_{Ai}(C_j), F_{Ai}(C_j) \leq 3$ for $Cj \in C, j = 1,2,\dots,n, and\ i = 1,2,\dots,m.$

For simplicity, the set of values $T_{Ai}(C_j)$, $I_{Ai}(C_j)$, $F_{Ai}(C_j)$ is expressed as a single-valued neutrosophic value (SVNV)$d_{ij} =< t_{ij}, i_{ij}, f_{ij} >$ $(i = 1,2,\dots,m;\ j = 1,2,\dots,n)$. This value typically originates from expert evaluations of the alternative $A_i$with respect to a criterion C_j. Consequently, a single-valued neutrosophic decision matrix can be constructed as $D = (d_{ij})_{mxn}$.

In multiple-attribute decision-making, the concept of an ideal point serves as a theoretical benchmark to evaluate alternatives, even though such an ideal alternative A^* can be defined as:

$d_j^* =< t_j^*, i_j^*, f_j^* >=< 1,0,0 > (j = 1,2,\dots,n)$

The weighted correlation coefficient between an alternative A_i $(i = 1,2,\dots,m)$ and the ideal alternative A^* is given by:

$$M_w(A_i, A^*) = \frac{1}{3}\sum_{j=1}^{n} w_j\left[\phi_{ij}\left(1 - \Delta t_{ij}\right) + \varphi_{ij}\left(1 - \Delta i_{ij}\right) + \psi_{ij}\left(1 - \Delta f_{ij}\right)\right] \quad (8)$$

where the parameters are defined as follows:

$$\phi_{ij} = \frac{3 - \Delta t_{ij} - \Delta t_{i\,max}}{3 - \Delta t_{i\,min} - \Delta t_{i\,max}},$$

$$\varphi_i = \frac{3 - \Delta i_{ij} - \Delta i_{i\,max}}{3 - \Delta i_{i\,min} - \Delta i_{i\,max}},$$

$$\psi_i = \frac{3 - \Delta f_{ij} - \Delta f_{i\,max}}{3 - \Delta f_{i\,min} - \Delta f_{i\,max}},$$

$$\Delta t_{ij} = \left|t_{ij} - t_j^*)\right|,$$

$$\Delta i_{ij} = \left|i_{ij} - i_j^*)\right|,$$

$$\Delta f_{ij} = \left|f_{ij} - f_j^*)\right|,$$

$$\Delta t_{i\,min} = min_j\left|t_{ij} - t_j^*\right|,$$

$$\Delta i_{i\,min} = min_j\left|i_{ij} - i_j^*\right|,$$

$$\Delta f_{i\,min} = min_j\left|f_{ij} - f_j^*\right|,$$

$$\Delta t_{i\,max} = max_j\left|t_{ij} - t_j^*\right|,$$

$$\Delta i_{i\,max} = max_j\left|i_{ij} - i_j^*\right|,$$

$$\Delta f_{i\,max} = max_j\left|f_{ij} - f_j^*\right|,$$

for $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$.

Using the correlation coefficients $M_w(A_i, A^*)$ $(i = 1,2,\dots,m)$, alternatives can be ranked, and the best alternative(s) can be identified.

2.3 COPRAS-SVNS

The concept of a linguistic variable proves to be highly beneficial in addressing decision-making problems characterized by intricate and multifaceted content. The value of such a variable is defined as an element within its term set, and these linguistic values can be represented as single-valued neutrosophic numbers (SVNNs).

Within the COPRAS-SVNS method, the decision-making framework involves k decision-makers, m alternatives, and n criteria. The k decision-makers assess the importance of the m alternatives based on the n criteria and evaluate the performance of these criteria using linguistic terms, which are subsequently transformed into SVNNs. Table 1 illustrates the importance weights assigned to linguistic terms, expressed through SVNNs.

Table 1: Linguistic variable and SVNSs. Source:[15]

Linguistic terms	**SVNNs**
Extremely good (EG)/ 10 points	(1.00, 0.00, 0.00)
Very very good (VVG)/ 9 points	(0.90, 0.10, 0.10)
Very good (VG)/ 8 points	(0.80, 0.15, 0.20)

Linguistic terms	SVNNs
Good (G) / 7 points	(0.70, 0.25, 0.30)
Medium good (MG) / 6 points	(0.60, 0.35, 0.40)
Medium (M) / 5 points	(0.50, 0.50, 0.50)
Medium bad (MB) / 4 points	(0.40, 0.65, 0.60)
Bad (B) / 3 points	(0.30, 0.75, 0.70)
Very bad (VB) / 2 points	(0.20, 0.85, 0.80)
Very very bad (VVB) / 1 point	(0.10, 0.90, 0.90)
Extremely bad (EB) / 0 points	(0.00, 1.00, 1.00)

The COPRAS-SVNS method for group decision-making is executed through the following sequential steps[16,17]:

- **Determine Expert Importance**: When decisions are made by a panel of experts, the contribution of each expert to the final decision is quantified. Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ represent the importance vector of the k experts, where $\lambda_k \geq 0$ and $\sum_{k=1}^{K} \lambda_k = 1$.
- **Evaluate Alternatives**: Each expert evaluates the m alternatives with respect to the n criteria using linguistic terms (refer to Table 1). The resulting decision matrix for an expert k is denoted as:

$$X^k = \begin{bmatrix} x^k{}_{11} & x^k{}_{12} \dots & x^k{}_{1n} \\ x^k{}_{22} & x^k{}_{22} \dots & x^k{}_{2n} \\ \vdots & \vdots & \vdots \\ x^k{}_{m1} & x^k{}_{m2} \dots & x^k{}_{mn} \end{bmatrix} \quad (9)$$

- **Calculate Criterion Weights**: The aggregated weights of the criteria are derived as:

$$\mathrm{w}_j = \lambda_1 \mathrm{w}_j^{(1)} \cup \lambda_2 \mathrm{w}_j^{(2)} \cup \dots \cup \lambda_k \mathrm{w}_j^{(k)} = \left(1 - \prod_{k=1}^{K} \left(1 - T_j{}^{(w_k)}\right)^{\lambda_k}, \prod_{k=1}^{K} \left(I_j{}^{(w_k)}\right)^{\lambda_k}, \prod_{k=1}^{K} \left(F_j{}^{(w_k)}\right)^{\lambda_k}\right) \quad (10)$$

- **Construct Aggregated Decision Matrix**: The aggregated weighted decision matrix. $\tilde{X}$ is given by:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} \dots & \tilde{x}_{1n} \\ \tilde{x}_{22} & \tilde{x}_{22} \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} \dots & \tilde{x}_{mn} \end{bmatrix} \quad (11)$$

where each element is determined as:

$$\tilde{x}_{ij} = \lambda_1 \mathrm{x}_{ij}^{(1)} \cup \lambda_2 \mathrm{x}_{ij}^{(2)} \cup \dots \cup \lambda_k \mathrm{x}_{ij}^{(k)} = \left(1 - \prod_{k=1}^{K} \left(1 - T_j{}^{(x_k)}\right)^{\lambda_k}, \prod_{k=1}^{K} \left(I_j{}^{(x_k)}\right)^{\lambda_k}, \prod_{k=1}^{K} \left(F_j{}^{(x_k)}\right)^{\lambda_k}\right) \quad (12)$$

- **Determine Weighted Decision Matrix**: The weighted decision matrix can be expressed as $D = \left[d_{ij}\right], d = 1,2, \dots, m; j = 1,2, \dots, n$, where $d_{ij} = \tilde{x}_{ij} * w_j$. A single element of the weighted decision matrix can be calculated as

$$d_{ij} = T_{ij}^{\tilde{x}} T_j^w, I_{ij}^{\tilde{x}} + I_j^w - I_{ij}^{\tilde{x}} I_j^w, F_{ij}^{\tilde{x}} + F_j^w - F_{ij}^{\tilde{x}} F_j^w \quad (13)$$

- **Summation for Benefit Criteria**: For criteria to be maximized $L_+ = \{1,2, \dots, L_{max}\}$ calculate the benefit index:

$$P_{+i} = \sum_{j=1}^{L_{max}} d_{+ij} \quad (14)$$

- **Summation for Cost Criteria**: For criteria to be minimized $L_- = \{1,2, \dots, L_{min}\}$ calculate the cost index:

$$P_{-i} = \sum_{j=1}^{L_{min}} d_{-ij} \quad (15)$$

- **Determine Minimum Cost Value**: Identify the smallest P_{-i} among all alternatives.

- **Calculate Score Values**: The aggregated benefit and cost indices are used to derive the score values. Q_i for each alternative. At the beginning, the score values are calculated from the aggregated values for benefit and the cost $S(P_{+i})$ and $S(P_{-i})$ by using equation (7). The score values of the alternatives can be expressed as

$$Q_i = S(P_{+i}) + \frac{S(P_{-min}) \sum_{i=1}^{L_{min}} S(P_{-i})}{S(P_{-min}) \sum_{i=1}^{L_{min}} \frac{S(P_{-min})}{S(P_{-i})}} \quad (16)$$

- **Optimality Criterion**: Identify the optimal alternative based on the maximum Q_i:

$$K = max_i Q_i; i = 1,2, \dots, m \quad (17)$$

- **Rank Alternatives**: Higher Q_i values correspond to higher-ranking alternatives, indicating their priority.

3 Results

The study analyzed eight sustainable tourism packages, selected for their ability to integrate environmental sustainability with benefits in stress reduction. These packages included activities designed to promote ecological awareness and personal well-being. Among them, the Reforestation and Meditation package combines reforestation days with outdoor meditation sessions, promoting ecosystem restoration and mental relaxation. The Rural Tourism with Organic Gardens package highlights local agricultural practices and healthy eating, while the Digital Detox in Nature Reserves focuses on digital rest. Other options included Eco-destinations with Electric Bicycles, Wellness Retreats in Eco-Lodges, Exploration of Local Fauna and Flora, Cultural Routes with Sustainable Traditions, and Environmental Clean-Up Camps. Each package makes unique contributions to sustainability and emotional well-being.

To evaluate the alternatives, the experts involved in the study identified various relevant elements for analysis. Through a brainstorming process, six potential criteria were initially generated. However, it was deemed necessary to prioritize those elements that could have a more significant impact on the quality of the experience offered to potential customers of the tourism packages. For this purpose, the neutrosophic correlation method was implemented, which allowed for refining and selecting the most relevant criteria, thus ensuring an approach focused on maximizing the effectiveness and relevance of the alternatives studied.

In this sense, the initial evaluations from the experts provided the data for the development of the method. Table 1 presents the main results obtained.

Table 1: Method output. Source: Own elaboration

Criteria evaluated	φij			μij			ψij			Mw
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
Package cost	1.20	1.20	1.20	1.08	1.08	1.08	1.04	1.04	1.08	0.67
Environmental sustainability	1.14	1.14	1.14	1.08	1.08	1.08	1.04	1.04	1.08	0.72
Diversity of activities	1.20	1.20	1.20	1.08	1.08	1.08	1.04	1.04	1.08	0.66
Community participation	1.40	1.40	1.40	1.09	1.09	1.09	1.08	1.08	1.09	0.56
Logistics and accessibility	1.47	1.47	1.47	1.14	1.14	1.14	1.08	1.08	1.09	0.55
Impact on stress reduction	1.20	1.20	1.20	1.08	1.08	1.08	1.04	1.04	1.09	0.69

Subsequently, based on the criteria obtained as the most relevant in the previous analysis, the evaluation of the initially considered tourism package options was conducted. Using the experts' evaluations and following the logic of the COPRAS-SVNS method, the necessary transformations were made to obtain the decision matrix. The equation (12) was then applied, which allowed for generating the weighted decision matrix, and the results are summarized in Table 2.

Table 2: Weighted decision matrix. Source: Own elaboration

Tour packages	Package cost	Environmental sustainability	Impact on stress reduction
Reforestation and meditation	(0.53;0.47;0.43)	(0.5;0.5;0.47)	(0.28;0.8;0.83)
Rural tourism with organic gardens	(0.53;0.47;0.43)	(0.4;0.6;0.59)	(0.3;0.76;0.78)
Digital disconnection in nature reserves	(0.75;0.25;0.23)	(0.62;0.38;0.36)	(0.53;0.47;0.41)
Eco-destinations with electric bicycles	(0.57;0.43;0.38)	(0.58;0.42;0.39)	(0.54;0.46;0.42)
Wellness retreats in eco-lodges	(0.59;0.41;0.39)	(0.49;0.51;0.51)	(0.54;0.46;0.42)
Exploring local flora and fauna	(0.53;0.47;0.43)	(0.48;0.53;0.49)	(0.43;0.58;0.55)
Cultural routes with sustainable traditions	(0.61;0.39;0.34)	(0.54;0.46;0.42)	(0.48;0.53;0.49)
Environmental clean-up camps	(0.57;0.43;0.38)	(0.5;0.5;0.47)	(0.43;0.58;0.55)

Once the relevant information was obtained, the coefficients established by the method for selecting the tourist packages were calculated. During this analysis, it was identified that criteria 2 and 3 correspond to benefit criteria, so their objective was to be maximized. On the other hand, criterion 1 was classified as a cost criterion, so it was considered more beneficial to minimize it. The results derived from this analysis and calculation of the data are presented in Table 3.

Table 3: Weighted decision matrix. Source: Own elaboration

Tour packages	Pi+	Pi-	S(P+)	S(P-)	Q
Reforestation and meditation	(0.62, 0.40, 0.39)	(0.53, 0.47, 0.43)	0.59	0.54	1.24
Rural tourism with organic gardens	(0.69, 0.46, 0.46)	(0.53, 0.47, 0.43)	0.65	0.54	1.30
Digital disconnection in nature reserves	(0.82, 0.18, 0.15)	(0.75, 0.25, 0.23)	0.83	0.76	1.29
Eco-destinations with electric bicycles	(0.81, 0.19, 0.16)	(0.57, 0.43, 0.38)	0.82	0.58	1.41
Wellness retreats in eco-lodges	(0.77, 0.24, 0.22)	(0.59, 0.41, 0.39)	0.77	0.59	1.36
Exploring local flora and fauna	(0.70, 0.31, 0.27)	(0.53, 0.47, 0.43)	0.70	0.54	1.35
Cultural routes with sustainable traditions	(0.76, 0.25, 0.20)	(0.61, 0.39, 0.34)	0.76	0.62	1.33
Environmental clean-up camps	(0.71, 0.29, 0.26)	(0.57, 0.43, 0.38)	0.72	0.58	1.32

After applying the method, the results indicated that the tourism package with the highest Q value was "Eco-destinations with electric bicycles," suggesting that this option might be the most suitable for combining sustainability and stress reduction in the Pichincha region, according to the experts. The values close between the packages, such as "Wellness retreats in eco-lodges" and "Exploration of local fauna and flora," show that several alternatives offer similar benefits in terms of enhancing the user experience and promoting responsible practices. However, the difference in Q values reflects a slight preference for packages that integrate more direct physical and ecological well-being practices.

4. Conclusion

This study allowed for a detailed analysis of several tourism packages in the Pichincha region, aiming to identify those that best combine environmental sustainability with stress reduction. Using multicriteria decision-making methods supported by neutrosophic logic, various alternatives integrating ecological and well-being practices were evaluated. This approach addressed the uncertainty inherent in the available information, providing a robust framework for selecting the most suitable option based on multiple criteria. The results showed that the "Eco-destinations with electric bicycles" package stood out as the optimal option, highlighting the effectiveness of this method in discerning between alternatives offering similar benefits in terms of sustainability and well-being.

The study made a significant contribution to improving the tourism offerings in Pichincha, providing an efficient tool to optimize the user experience and promote responsible practices. The combination of Neutrosophy with multicriteria decision-making methods proved essential for handling the ambiguity and indeterminacy present in tourism decision-making. This methodology not only enabled more precise evaluation but also expanded the possibilities for analysis in future studies on sustainable tourism, offering a path toward creating more balanced offerings adapted to the contemporary needs of tourists.

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Plithogenic Hypothesis on the Influence of Electronic Commerce on Retail Sales Dynamics in Shopping Centers

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Abstract. The present research addresses the growing argument on the ways in which e-commerce is changing dynamics in shopping centers by offering a plitogenic hypothesis that looks into the intricacies between the two worlds. The present research goes into duality of these selling environments and their coexistence in a backdrop of challenging traditional models of buying which digital platforms are. Despite the rich literature on electronic commerce and retail, most of the studies have isolated perspectives on the subject, avoiding interrelations that properly emerge from online commerce and physical shopping centers. The study attempts to fill that theoretical gap by applying an innovative approach to analyzing how these two selling models interact while at the same time interacting in a world that changes quickly in customer behavior. In a mixed-method approach, that is, it combines both qualitative and quantitative analysis, this paper uses a plitogeni [c] approach, done to capture the complexities and contingencies of decisions involving consumer purchases. This indicates that the two do not really oppose each other but are two realities that exist in relation to the other; sometimes they negate each other and sometimes they do not under the conditions of convenience or experience. It is one research study that extended not only the theoretical understanding of the relationships between these two worlds of retail but gave tremendous practical implications, such as the design of hybrid strategies using the strengths of both channels. In short, the paper highlights the importance of rethinking business dynamics in a setting dominated by hybridity of platforms, with the ultimate aim being increasing effectiveness both online and at the point of physical sale.

Keywords: E-commerce, sales dynamics, shopping malls, Plithogenic hypothesis, interaction, sales models, consumer behavior , Falsifiability of a Hypothesis, Multivalued Logics, Plithogenic Statistics.

1. Introduction

E-commerce has profoundly altered the dynamics of global retail, impacting both physical stores and shopping centers. In the last ten years, the rapid growth of online sales platforms has transformed the dynamics among consumers, items, and retailers. In this context, investigating the impact of e-commerce on conventional retail sales is crucial for comprehending how shopping malls can adjust to the contemporary digital landscape. Notwithstanding the significance of this subject, prior research has predominantly been disjointed and constrained, resulting in a substantial void in the examination of the interconnections between the two sales models. This study aims to bridge the conceptual and empirical gap by examining the influence of e-commerce on sales dynamics in shopping malls, specifically on hybrid consumption patterns that integrate both physical and digital experiences.

Historically, shopping malls have been seen as the nucleus of retail, representing an evolution of standalone establishments that enabled consumers to procure a diverse array of things in a single location. In recent decades, e-commerce has emerged as an alternative to this model, originally concentrating on the sale of technological and electronic products. Nonetheless, the emergence of platforms like

Amazon, Alibaba, and other retail behemoths has expanded digital commerce to include a broader array of products, ranging from apparel to food, so transforming the in-store shopping experience. The reactions of shopping malls have been diverse; some have embraced digital trends by introducing mobile applications and online shopping services, whereas others persist in their difficulties to remain pertinent amid virtual competition. Nevertheless, research to far has been inadequate in examining the interaction and mutual influence of online and physical sales modes, particularly with customer purchase decisions.

This research aims to examine the impact of e-commerce boom on sales dynamics in shopping malls. Some research indicate that e-commerce may diminish physical sales, while others propose that both models can survive complementarily. The absence of unanimity in the literature prompts a critical inquiry: how does e-commerce influence purchasing decisions at shopping malls, and to what degree do customers embrace a hybrid shopping experience that integrates both physical and digital elements? The significance of this issue is to its implications for the viability of the traditional shopping mall model and its capacity to adapt to contemporary consumer needs. The primary aim of this study is to investigate the correlation between e-commerce and conventional retail sales in shopping malls, using a plithogenic hypothesis to elucidate the interaction between these two channels. Through quantitative and qualitative research, we aim to comprehend the variables that drive customers to amalgamate both purchase models, as well as the techniques employed by shopping malls to integrate the physical and digital realms. Secondly, the study seeks to offer strategic advice to assist shopping centers in adapting to this evolving landscape, fostering synergy between both sales channels and enhancing the consumer experience [4][5].

This study will utilize the concepts of platform hybridization and retail adaptability to demonstrate how shopping malls can evolve into hybrid locations where online and physical commerce are complimentary rather than opposing elements. As consumers get increasingly adept at utilizing technology in their purchasing decisions, expectations for the integration of both channels rise, necessitating an inventive response from merchants to sustain relevance in a saturated market.

This study's results will offer a novel perspective on the evolution of retail malls, emphasizing the tactics that optimally align the confluence of two realms. This study will specifically analyze the impact of technologies such mobile commerce, online ordering systems, and the incorporation of shopping platforms into the physical shopping mall experience [7]. These studies aim to make a substantial contribution to retail theory and business practice by suggesting methods to enhance competitiveness in this evolving scenario. This study aims to analyze the impact of e-commerce on physical sales dynamics and to provide shopping malls with tangible solutions to reformulate their business approaches and maintain competitiveness in a progressively digital market. This study's findings are anticipated to enhance the academic discourse on the future of retail and provide a foundation for subsequent research in the sector.

2. Preliminary

2.1. Electronic Commerce in Sales Dynamics

E-commerce has profoundly influenced worldwide sales dynamics, providing unparalleled ease and altering the manner in which retailers interact with their audiences. The proliferation of the Internet, the ascendance of mobile devices, and enhancements in logistics have resulted in a transformation of consumer behavior and purchase patterns. The ease of internet purchasing has drawn millions of consumers, compelling companies to modify their product marketing techniques. Nonetheless, e-commerce poses obstacles, including intense online competition, necessitating businesses to allocate resources towards digital marketing methods such as personalization, audience segmentation, and recommendation systems[8].

Consumer trust is essential in e-commerce, necessitating that firms invest in secure platforms and data protection protocols to guarantee the security of customer transactions. The emergence of e-com-

merce has influenced conventional retail patterns, resulting in a transition towards omni-channel purchasing experiences[9,10].

E-commerce has democratized access to global markets, enabling small and medium-sized firms (SMEs) to expand their audience and foster innovation. Nonetheless, logistics, an essential element of e-commerce, poses issues for enterprises. E-commerce has facilitated the emergence of novel monetization models, including subscription services, digital goods sales, and affiliate platforms[11].

Digital marketing, augmented by social media, influencers, and tailored content, has transformed the sales environment, enabling organizations to engage with their target audience more directly and accurately. This shift in purchasing behavior necessitates that companies adjust to a more discerning and knowledgeable consumer demographic that prioritizes quality, sustainability, and brand integrity[11].

2.2. Plithogenic Probability

Neutrosophic data, also known as indeterminate, are defined by their lack of clarity and incomplete nature, which makes them prone to the presence of partial unknowns and contradictory information [12-15]. These data can be classified as quantitative (metric), qualitative (categorical), or a combination of both. A relevant aspect of this type of data is plithogenic variables [16], which are used to describe relationships or correlations between neutrosophic variables.

On the other hand, a neutrosophic variable [17, 18], whether a function or operator, has the ability to handle neutrosophic data in its values, its arguments, or both. This type of variables is essential when facing complex problems, which, due to their multidimensional nature, require several measurements and observations. This need is particularly highlighted in scientific research, where obtaining data from diverse sources is crucial. Furthermore, neutrosophic variables can exhibit different types of relationships, such as dependence, independence, partial dependence, partial independence, or partial indeterminacy, common phenomena in many scientific fields [19].

A Plithogenic Set [20, 21] is a non-empty set Pwhose elements within the domain of discourse U($P \subseteq U$) are characterized by one or more attributes $A_1, A_2, \cdots, A_m$, where m is at least 1. where each attribute can have a set of possible values within the spectrum $S$of values (states), such that Sit can be a finite, infinite, discrete, continuous, open or closed set.

Each element $x \in P$is characterized by all possible values of the attributes found within the set $V = \{v_1, v_2, \cdots, v_n\}$. The value of an attribute has a degree of membership $d(x, v)$to an element $x$of the set.P, based on a specific criterion . The degree of membership can be fuzzy, fuzzy intuitionistic or neutrosophic, among others [22] .

That means,

$$\forall x \in P, d: P \times V \to \mathcal{P}([0,1]^z) \quad (1)$$

Where$d(x, v) \subseteq [0,1]^z$ and $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$.$z = 1$ (the diffuse degree of belonging), $z = 2$(the intuitionistic diffuse degree of belonging) or $z = 3$ (the neutrosophic degree of belonging).

Plithogenic [23,24], derived from plithogenic variable analysis , represents a multidimensional probability (" plitho " meaning "many" and synonymous with "multi"). It can be regarded as a probability comprised of subprobabilities, each of which delineates the behavior of a particular variable. The event being analyzed is presumed to be affected by one or more variables, each characterized by a probability distribution function (PDF).).

Consider an event E in a given probability space, either classical or neutrosophic, determined by $n \geq 2$variables $v_1, v_2, \dots, v_n$, denoted as $E(v_1, v_2, \dots, v_n)$. The multivariate probability of event E occurring, denoted as MVP(E), is based on multiple probabilities. Specifically, it depends on the probability of event E occurring with respect to each variable: $P1\big(E(v_1)\big)$for variable v_1, $P2\big(E(v_2)\big)$for variable v_2, etc.

It is therefore $MVP\big(E(v_1, v_2, \dots, v_n)\big)$ represented as $\Big(P1\big(E(v_1)\big), P2\big(E(v_2)\big), \dots, Pn\big(E(v_n)\big)\Big)$. Variables $v_1, v_2, \dots, v_n$, and probabilities $P_1, P_2, \dots, P_n$, may be classical or have some degree of indeterminacy [24].

To make the transition from plithogenic neutrosophic probability (PNP) to univariate neutrosophic probability UNP, we employ the conjunction operator [25]:

$$UNP(v_1, v_2, \dots, v_n) = v_1 \wedge_{i=1}^{n} v_n \quad (2)$$

$\wedge$ In this context, it is a neutrosophic conjunction (t-norm). If we take$\wedge_p$ as the plithogenic conjunction between probabilities of the PNP type, where $(T_A, I_A, F_A) \wedge_p (T_B, I_B, F_B) = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$, such that $\wedge$is the minimum t-norm of fuzzy logic and $\vee$the maximum t-norm [26, 27].

a. Formulate the hypothesis
   Start by explicitly stating the hypothesis you intend to examine. Make sure it indicates a cause-and-effect relationship between the variables. For example, "More study time leads to higher test scores."
b. Identify key variables

   Determine the independent variable, representing the cause, and the dependent variable, denoting the effect, inside your hypothesis. This facilitates the precise targeting of your study inquiries towards the specific link to be examined.Formulate specific research questions
   Break the hypothesis down into precise research questions phrased as "Does X cause Y?" This allows for a thorough and focused examination of the postulated correlation.
c. Perform sentiment analysis on scientific literature.

   To perform sentiment analysis on a research paper and quantify the occurrences of “Yes”, “Possibility/Indeterminacy”, and “No”, a sentiment analysis tool for scientific statements is needed. In this instance, we employ Consensus Meter methods to classify the assertions into three unique categories: Positive (affirmative), Indeterminate (possibility or indeterminacy), and Negative (negative).
d. Formulate neutrosophic probabilistic hypotheses

   Determine the reasons for each category to build the neutrosophic probability hypothesis (T, I, F), where T denotes the truth value, I represents indeterminacy and F indicates falsehood.
e. Calculate the plithogenic neutrosophic probability (PNP)

   Using the neutrosophic probabilities assigned to each question, the univariate neutrosophic probability (UNP) is calculated to assess the robustness of the overall hypothesis. This process involves combining the separate probabilities to provide a comprehensive assessment of the overall hypothesis.

   $$UNP(v_1, v_2, \dots, v_n) = (Min(t_1, t_n, \dots, t_n), Max(i_1, i_n, \dots, i_n), Max(f_1, f_n, \dots, f_n)) \quad (3)$$

   Where:

   $T_1, T_2, \dots, T_n$: are the probability values of truth for each question.
   $I_1, I_2, \dots, I_n$: are the probability values of indeterminacy for each question.
   $F_1, F_2, \dots, F_n$: are the probability values of falsehood of each question
   a. Analyze the validity of the general hypothesis.

In this case, the negation of NPH is represented as [28]:

$$(T, I, F) = (F, I, T) \quad (4)$$

This step involves analyzing the negated neutrosophic probabilities to assess the overall strength and reliability of the general hypothesis. By assessing the levels of falsity, uncertainty, and veracity, one can determine the degree to which the hypothesis is valid, ambiguous, or incorrect based on the scientific literature.

3. Case study

The method used in the study is based on [25]. In today's digital age, e-commerce has disrupted various industries, changing the way consumers interact with brands and retailers. Shopping malls, traditionally physical sales locations, have had to adapt to new market trends. The introduction of e-commerce platforms in these shopping malls is changing retail sales dynamics, affecting both customer interaction and the operational efficiency of stores. This research focuses on analyzing the influence of e-commerce on these dynamics, using a plithogenic approach to assess the probabilities surrounding the implementation of online sales platforms in shopping malls.

The plithogenic hypothesis proposed for this study is that e-commerce, when implemented in shopping malls, influences the frequency of purchases, consumer behavior, and the operational efficiency of retail stores. This phenomenon can be explained through neutrosophic and plithogenic variables that describe the correlations between various variables, such as the accessibility of online platforms, the level of consumer satisfaction, and the increase in sales.

Hypothesis Statement:

The implementation of e-commerce significantly influences retail sales dynamics within shopping malls, improving both product availability and consumer accessibility to more diverse options. This study seeks to assess how e-commerce affects purchasing decisions in shopping malls, considering the impact of factors such as competitiveness, convenience, and product exclusivity.

Key Variables:

- **Independent Variable (Cause):** Implementation of electronic commerce in shopping centers.
- **Dependent Variable (Effect):** Retail sales dynamics in shopping centers.

Research Questions:

- **Q1:** Does the increased implementation of e-commerce increase sales within shopping malls?
  - *Variable:* Sales volume in shopping centres.
- **Q2:** Does e-commerce improve accessibility and convenience for consumers in shopping malls?
  - *Variable:* Accessibility and convenience in the purchasing process.
- **Q3:** Does e-commerce create greater competition between brands within shopping malls?
  - *Variable:* Level of competitiveness between brands in the shopping center.
- **Q4:** Does the presence of e-commerce in shopping centres allow for greater exclusivity in the products offered?
  - *Variable:* Exclusivity of products available in shopping centers.

Sentiment Assessment on Research Questions:

Consensus Meter is used to assess the positions of the scientific literature regarding each of the research questions. The articles are classified into three categories: Positive, Indeterminate and Negative, and a neutrosophic probability is assigned to each of the questions.

Ask	Positive	Indeterminacy	Negative	Neutrosophic Probability
P1	[26, 27, 28]	[29]	[30]	(0.80, 0.10, 0.10)
P2	[31, 32]	[33]	[34]	(0.75, 0.15, 0.10)
P3	[35,36]	[37, 38]	[39]	(0.70, 0.20, 0.10)
P4	[40,42,43]	[44]	[45]	(0.77, 0.13, 0.10)

Calculation of the Univariate Neutrosophic Probability (UNP):

Now, we calculate the univariate neutrosophic probability (UNP) using the neutrosophic probabilities of each of the investigated questions.

Where the neutrosophic probabilities of each question are as follows:

- **P1:** (0.80, 0.10, 0.10)
- **P2:** (0.75, 0.15, 0.10)
- **P3:** (0.70, 0.20, 0.10)
- **P4:** (0.77, 0.13, 0.10)

The calculation of the univariate neutrosophic probability (UNP) is performed as follows:

UNP (v1 , v2 , … , vn) = (min (t1 , t2 , … , tn) , max (i1 , i2 , … , in) , max (f1 , f2 , … , fn))

UNP=(min(0.80,0.75,0.70,0.77),max(0.10,0.15,0.20,0.13),max(0.10,0.10,0.10,0.10))

Given:

- **t** : t1=0.80,t2=0.75,t3=0.70,t4=0.77
- **i** : i1=0.10,i2=0.15,i3=0.20,i4=0.13
- **f** : f1=0.10,f2=0.10,f3=0.10,f4=0.10

Steps:

1. **First value (minimum of t)** :
   min (0.80,0.75,0.70,0.77)=0.70
   Second value (maximum of i) :
   max (0.10,0.15,0.20,0.13)=0.20
2. **Third value (maximum of f)** :
   max (0.10,0.10,0.10,0.10)=0.10

Bottom line:

UNP =(0.70,0.20,0.10)

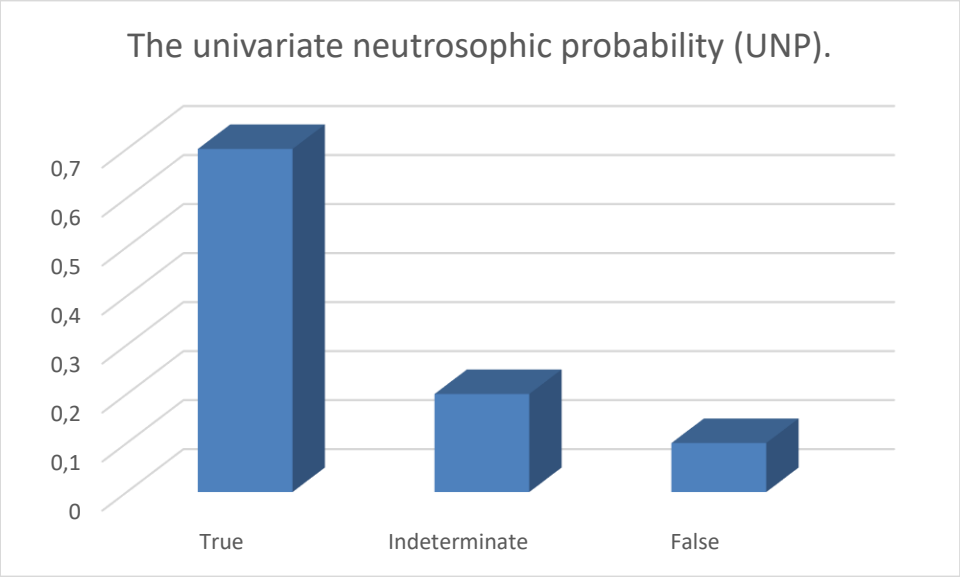

Figure 1: Univariate neutrosophic probability (UNP).

Interpretation of the Result:

- **0.70:** There is a 70% probability that the hypothesis is true, suggesting a high probability that e-commerce will positively influence sales dynamics in shopping centers.
- **0.20:** There is a 20% probability that there is uncertainty about the relationship between e-commerce and retail sales. This value reflects the uncertainty about the total impact of the factors involved, such as competition and accessibility.
- **0.10:** There is a 10% chance that the hypothesis is false, which indicates that, although low, there is a small possibility that e-commerce will not have significant effects on the sales dynamics of shopping centers.

Denial of the Hypothesis:

For the negation of the hypothesis, we calculate the negative neutrosophic probabilities, which results in the values:

$(T,I ,F)=(0.10,0.20,0.70)(T, I, F) = (0.10, 0.20, 0.70)(T,I,F)=(0.10,0.20,0.70)$

Interpretation of Denial:

- **0.10:** There is a 10% chance that the negation of the hypothesis is true, implying that excluding e-commerce would not have a significant impact on retail sales.
- **0.20:** There is 20% uncertainty about the validity of the denied hypothesis, reflecting the presence of uncertain factors affecting sales dynamics.
- **0.70:** There is a 70% probability that the negation of the hypothesis is false, which implies that it is highly likely that e-commerce will influence retail sales dynamics in shopping malls.

Detailed Analysis of the Results:

This research indicates that e-commerce substantially enhances shopping mall sales, with a 70% likelihood that the initial hypothesis is valid. Nonetheless, there exists a 20% uncertainty over specific aspects that may fluctuate based on the deployment of e-commerce in each particular mall, including consumer adaption and competition with local shops.

The negation of the hypothesis, however, possesses a substantial falsehood value of 70%, hence reinforcing the notion that e-commerce, overall, positively impacts sales dynamics. The 20% indeterminacy indicates that not all retail centers would see uniform benefits due to differences in their characteristics and marketing strategy.

In conclusion, while the hypothesis demonstrates a significant likelihood of accuracy, additional study encompassing diverse situations and varieties of shopping centers is advisable to mitigate uncertainty and elucidate the impact of e-commerce on retail sales.

4. Conclusion

The results of this study suggest that, overall, e-commerce seems to have a positive impact on sales within shopping centres, with a considerable probability that the original hypothesis is true. While 70% of the data supports this idea, it cannot be denied that there is a degree of uncertainty of 20% that could be related to unpredictable factors, such as the particularities of each shopping centre, variations in consumer behaviour or local competition. In any case, the results are still interesting and open many doors to explore how e-commerce continues to influence the sales landscape. From a practical perspective, this finding is important for retailers and designers of commercial strategies, as it could imply that investing in digital solutions is not only a viable option, but a necessity to remain competitive in an increasingly digitalised market. However, as with everything, there are nuances. Perhaps the impact of e-commerce is not the same in all contexts, and integration strategies must be careful and adapted to local circumstances. What works in a shopping center with a very young and connected audience might not be as effective in one with a more traditional customer base.

One of the major contributions of this study is the application of a neutrosophic probability approach to analyse such complex and variable phenomena as e-commerce. Although this is not a new methodology, its use in this specific context provides a useful framework for understanding the uncertainty inherent in situations of high variability. This undoubtedly brings something new to the field of study and offers researchers more sophisticated tools to address similar problems. However, like any analysis, this study is not without limitations. Although the results indicate a clear trend, the small margin of indeterminacy (20%) shows that we do not yet have a definitive answer. Differences between shopping centres, types of consumers and business models may influence the results, limiting the widespread applicability of the findings. Furthermore, the approach used could benefit from a more detailed analysis of contextual variables. For future research, it would be interesting to explore the influence of e-commerce in shopping centres in different regions, with different demographic and economic profiles. Furthermore, incorporating complementary methodologies, such as Big Data analytics or real-time behavioral studies, could offer deeper and more accurate insight into how e-commerce is truly transforming retail sales. All in all, this field has a lot to offer and it seems like we are just scratching the surface of what could be an even more fascinating and revealing area of study.

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Application of Neutrosophic Multi-Criteria Decision-Making Techniques for Assessing Violations of Child Support Rights

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Abstract. The study addresses a crucial challenge in the analysis of rights violations by applying neutrosophic methods. Specifically, it investigates how multi-criteria assessment tools can provide a more precise and nuanced approach to addressing rights violations, a topic that remains of global relevance, given the complexity and diversity of situations that occur in different contexts. The existing literature, although abundant in human rights studies, often lacks approaches that integrate the uncertainty and ambiguity inherent in these cases, limiting the effectiveness of conventional analyses. This work fills that gap by implementing a neutrosophic approach that captures these dynamics more effectively and adequately. Using the neutrosophic-based multi-criteria assessment methodology, the study employs advanced techniques to manage and model the indeterminacy present in human rights violations. The analysis undertaken reveals that this approach allows for a more complete and robust assessment of the circumstances of rights violations, by integrating the divergent perceptions of the experts and actors involved. The findings underline how this approach improves decision-making in scenarios where situations are not entirely clear or definitive. The study's contribution lies in offering a more precise and practically applicable methodology for human rights assessment, allowing for a more nuanced and reliable analysis that can influence policies and corrective actions at a global level.

Keywords: Multicriteria Evaluation, Neutrosophy, Uncertainty, Ambiguity, Advanced Analysis, Decision Making, Divergent Perception, Corrective Policies, Robust Analysis, Human Rights, Neutrosophical Approach.

1. Introduction

Child support is recognized as a fundamental right throughout the world and strict disciplinary sanctions have always been applied to those who fail to meet their obligations. Various sanctions are used to ensure that these rights are not violated, and in Latin America similar sanctions are applied to those who have child support obligations as part of their efforts to protect the best interests of the child. In the case of Ecuador, it was stated that both child support payers and pension administrators have rights and obligations. However, there have been cases of misappropriation of funds by the administration, which constitutes a violation of this right [10].

A judgment is a final decision issued by a judge or court to resolve a legal dispute and conclude a case. It is an order intended to end a dispute and represents a final decision to resolve a dispute between two parties [11]. Alimony is defined as an amount paid periodically to cover the living expenses of the beneficiary. These funds are supposed to be used only for the support of the beneficiaries, but the lack of transparency in their management causes serious problems.

The problem is the lack of an efficient system that ensures that costs are only passed on to the beneficiaries. This creates uncertainty as to whether the funds are used to meet the needs of the beneficiaries, such as food, clothing, housing, healthcare and education [12]. This situation is further facilitated by the legal system's inability to determine the true purpose of the payment. A real solution to this problem would be to make it mandatory for judges, when making decisions on alimony, to submit detailed monthly reports on the use of funds received by public servants. This will ensure the correct distribution of pensions and give contributors peace of mind about the purpose of their contributions. In addition, legal reforms and more effective monitoring mechanisms are needed to ensure transparency and accountability in the administration of alimony to protect the rights of beneficiaries and ensure their well-being [13].

2. Related work

To achieve the proposed objectives, the study used the multi-criteria decision-making method (MDMC). MDMC allows to develop procedures that take very complex real situations and make decisions with simplifications in certain situations. In this way, the original problem reaches a solvable state [1], [2] .

Several MDMCs have been developed to solve problems arising in various areas of life and society. [3]–[9] However, traditional methods often use explicit values to evaluate alternatives. Due to the complexity of the environment and human subjectivity, MDMC problems often involve uncertainty, so the information provided to solve them is often confusing or verbal.

To make the information used clearer, the basic elements of neuroscience theory and the methodology used are first described. The results obtained and the conclusions drawn from the research are then applied and presented.

Decision making typically involves the use of human language or, commonly, linguistic parameters. Arguments simply represent words or concepts used in human language. Therefore, this linguistically variable approach is a suitable way for decision makers to express their evaluation. Basic levels can be expressed using linguistic variables. Linguistic variants can be edited in SVNS, as shown in Table 1.

Table 1: Parameters and Single-Digit Neural Networks [SVNN]. Source: [13]

Definition	SVNS
Extremely preferred [ExP]	[1,O ,O]
Very Very Preferred [VVP]	[O.9, O.1, O.1]
Highly preferred [VP]	[0,8,0 ,15,0,20]
Preferred [P]	[0.70,0.25,0.30]
Equally preferred [EP]	[0.50,0.50,0.50]
Not preferred [NP]	[0.35, 0.75 , 0.80]
Very little preferred [VNP]	[0.20,0.85,0.80]
Very little preferred [VVNP]	[0,10,0,90,0,90]
Extremely Not Preferred [ENP]	[O,1,1]

A set of options $G = [G1, G2, \dots, Gn]$ and $A = [A1, \dots, Am]$ a collection of properties. Let be $W = [w1, w2, \dots, wn]$the weight of the attribute where $0 \leq wj \leq 1$ y $\sum_{j=1}^{n} w_j = 1$. Let aij with i = 1, 2...m and j = 1, 2,... , n be the value of the selection attribute Ai associated with the attribute Gj. Then A = [aij] m×n =

$\langle(T_{ij}, I_{ij}, F_{ij})\rangle_{mxn}$ the SVNN matrix. Here, Tij, Iij and Fij are the membership degree, the degree of uncertainty in membership and the degree of uncertainty, respectively. The diagnosis procedure is as follows [15, 16]:

The first step is to find a decision option to evaluate. In the second step, the weights of the decision makers are determined. The logic of the method allows each decision maker to receive unique and different ratings from the other decision makers. This is because each rating is given based on each expert's level of knowledge about the decision issue under discussion. The relative weight of each determinant is considered a language variable and an input to the SVNN, which is then defined by Equation 1. In the third step, the language scores provided by the experts are converted to the SVNN. From the clear and precise individual matrices obtained according to the expert judgment, the individual neutrosophic decision-making matrix is constructed, as shown in Table 1. The initial relationship matrix is obtained for options A = [A1 ,... , Am] and features G = [G1 , G2 ,..., Gn], where each aij , i = 1, 2 ... m , j = 1, 2,.., n are the values of selection attribute Ai with attribute G. A = [aij] m × n = 〈 [Tij,Iij,Fij] 〉 _ mxn is the SVNN matrix, where Tij , Iij , and Fi are the orders of. Equation [2] is used to calculate the member score, the undetected member score, and the non-member score. In the fifth step, the information about the solution is combined. This means that we normalize A = [aij] m × n to B = [bij] m × n . If the decision is a cost factor, the decision information must be shifted to an additional set using equation [17, 18], but if the decision is an efficiency factor, it cannot be shifted.

To give more context to the topic, the decision-making process is crucial in all areas, from personal life to business and academic environments. In academic settings, students are faced with decisions ranging from course selection to choosing research topics. Adopting systematic approaches, such as the method described, can help students make informed and balanced decisions, taking into account various factors and expert opinions.

Furthermore, the use of matrices and equations provides a quantitative framework for evaluating options, which can be especially useful in situations where decisions are influenced by multiple variables. However, it is important to note that while methods can provide structure and clarity, it is also critical to consider the specific context and individual needs when making decisions. Therefore, the application of these methods must be complemented by careful analysis and a deep understanding of the problem at hand. Step 6: Use [5] to build the preference function $P_j[B_i, B_r]$ to replace Bi by Br with feature G j [19] .

$$P_j(B_i, B_r) = \begin{cases} 0 \,, d \le p \\ \frac{d-p}{q-p} \,, p < d < q \\ 1 \,, d \ge q \end{cases} \qquad [1]$$

Step 7 : Calculate the relative weight of attribute w jr , which is the relative weight of G_r with respect to G_j . here

$$w_{jr} = \frac{w_j}{w_r} = [j, r = 1,2, \dots, n] \qquad [2]$$

Step 8: Determine priority power π [B_{yo}, Br] of circuit Bi with respect to Br using the following formula:

$$[1] \quad \pi(B_i, B_r) = \frac{\sum_{j=1}^{n} w_{jr} P_j(B_i, B_r)}{\sum_{j=1}^{n} w_{jr}} \qquad [3]$$

Step 9: Calculate the input $\Phi^+[B_i]$, the output $\Phi^-[B_i]$ and the net input $\Phi[B_i]$ as follows:

$$\Phi^+(B_i) = \frac{\sum_{r=1}^{m} \pi(B_i,B_r) - \min\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_i,B_r)\}}{\max\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_i,B_r)\} - \min\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_i,B_r)\}} \qquad [4]$$

$$\Phi^-(B_i) = \frac{\sum_{r=1}^{m} \pi(B_r,B_i) - \min\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_r,B_i)\}}{\max\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_r,B_i)\} - \min\limits_{1\le i\le m} \{\sum_{r=1}^{m} \pi(B_r,B_i)\}} \qquad [5]$$

$$\Phi(B_i) = \Phi^+(B_i) - \Phi^-(B_i) \qquad [6]$$

Step 1O: Sort all options by value $\Phi(B_i)$. The higher the value, $\Phi(B_i)$, the better the option.

Systematic process

To find alternatives to evaluate, we analyze child support payments, trying to identify and understand fundamental violations of the rights of the beneficiaries.

1. **Obligation to Pay [VIOL1]** : One of the most common violations is that a person who owes child support payments fails to meet their financial obligations in a regular and timely manner. This can be due to a variety of reasons, including unemployment, lack of reported income, or simply refusing to comply with court orders.
2. **Unilateral Change of Agreement [VIOL2]** – In some cases, the obligor may attempt to unilaterally change the terms of a child support agreement without the consent of the payee or due process of law. This may include attempts to cut alimony or suspend benefits without court permission.
3. **Lack of Access to Financial Information [VIOL3]** : Recipients have the right to complete and up-to-date information about the income and assets of the person obligated to pay child support to ensure that the proper amount is paid. Lack of financial transparency on the part of the debtor can make it difficult to determine a fair amount of support.
4. **Deprivation of visitation or access to children [VIOL4]** : When child support includes custody of a child, unreasonable deprivation of visitation or access to a child may be considered a violation of the rights of the recipient's child.
5. **Failure to Comply with a Court Order [VIOL5]** – This includes any action that directly violates a court order related to child support, such as failure to comply with a mediation agreement, failure to comply with a court-ordered payment plan, or failure to comply with a court-ordered filing of documents. Financial statements required by the court.

The other evaluation criteria provide guidance for use in the decision-making process and serve as a source of principles for evaluating controls. In this way, alternatives to a problem can be analyzed from the same perspective. This study was designed using four evaluation criteria, which were presented to decision-makers and validated. During the analysis, the experts agreed to give each criterion the same weight in terms of importance [each weight value w = 0.25].

For the purposes of this survey the following criteria will be used:

1. **Receipt of Child Support** : Assessing whether the recipient has active access to child support, including receiving agreed-upon payments on a regular and timely basis.
2. **Reasonable Amount Obligation** – Determines whether the amount of child support is sufficient and reasonable to meet the needs of the recipient and any dependent children [if applicable].
3. **Right to financial information** – Make sure the recipient has the right to maintain up-to-date information about the income and assets of the person who owes child support to ensure the proper amount is paid.
4. **Custody and Visitation** : Assesses whether the recipient's custody and visitation rights are being respected under the child support agreement.

Conclusion

Table 2 shows the scores given to the decisions based on their relative importance to the issue under consideration.

Table 2: Classification assigned to decision makers according to their importance. Source: Home.

Deciders	Linguistic assessment	SVNN	Numeric value
Decider 1	very important	[0.9;0. 1;0.1]	0.23
Decider 2	Moderately important	[0.5;0. 5;0.5]	0.15

Deciders	Linguistic assessment	SVNN	Numeric value
Decider 3	very important	[0.9;0. 1;0.1]	0.22
Decider 4	very important	[0.9;0. 1;0.1]	0.20
Decider 5	Important	[0.75;0. 25;0.20]	0.2

The decision maker evaluates the specific alternatives individually based on each criterion or attribute selected for evaluation using Equation [2], which is then transformed to obtain an overall decision matrix of the alternatives shown in Table 3.

Table 3: *Reference for option decisions* . Source: Own elaboration

	Accessibility	**Job stability**	**Equal opportunities**	**Reasonable adjustments**
VIOL1	[0.61434; 0.38576 ; 0.35486]	[0.67439; 0.33571 ; 0.38374]	[0.7636; 0.3374 ; 0.3081]	[0.7357; 0. 3743; 0.3519]
VIOL2	[0.55653; 0. 44347; 0.43667]	[0.5;0. 5;0.5]	[0.56731;0. 43369;0.41301]	[0.5;0. 5;0.5]
VIOL3	[0.68696;0. 31304;0.3988]	[0.54397;0. 47088;0.45555]	[0.47187; 0. 54413; 0.5515]	[0.6034;0. 4096;0.3789]
VIOL4	[0.69071;0. 30939;0.39533]	[0.61633; 0.38377 ; 0.35344]	[0.47187; 0.54413 ; 0.5515]	[0.5673; 0.4337 ; 0.413]
VIOL5	[0.5; 0. 5; 0.5]	[0.55653; 0.44347 ; 0.43667]	[0.5; 0. 5; 0.5]	[0.7445; 0.3555 ; 0.3555]

This calculation can be performed using the linear function proposed in [4]. For this case it is assumed that q = 1, p = 0, obtaining the matrices from P_1 to P_4.

$$P_1 = \begin{array}{c|ccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_1 & 0.0000 & 0.0000 & 0.0187 & 0.0199 & 0.0000 \\ B_2 & 0.0339 & 0.0000 & 0.0436 & 0.0438 & 0.0000 \\ B_3 & 0.0000 & 0.0000 & 0.0000 & 0.0013 & 0.0000 \\ B_4 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_5 & 0.0484 & 0.0345 & 0.0671 & 0.0683 & 0.0000 \end{array}$$

$$P_2 = \begin{array}{c|ccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_1 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_3 & 0.0731 & 0.0000 & 0.0103 & 0.0493 & 0.0344 \\ B_2 & 0.0619 & 0.0000 & 0.0000 & 0.0390 & 0.0143 \\ B_4 & 0.0339 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_5 & 0.0476 & 0.0000 & 0.0000 & 0.0347 & 0.0000 \end{array}$$

$$P_3 = \begin{array}{c|ccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_1 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_2 & 0.0683 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_3 & 0.1198 & 0.0515 & 0.0000 & 0.0000 & 0.0225 \\ B_4 & 0.1198 & 0.0515 & 0.0000 & 0.0000 & 0.0225 \\ B_5 & 0.0973 & 0.0290 & 0.0000 & 0.0000 & 0.0000 \end{array}$$

$$P_4 = \begin{array}{c|ccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_1 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ B_2 & 0.0827 & 0.0000 & 0.0364 & 0.0290 & 0.0815 \\ B_3 & 0.0463 & 0.0000 & 0.0000 & 0.0000 & 0.0451 \\ B_4 & 0.0537 & 0.0000 & 0.0074 & 0.0000 & 0.0525 \\ B_5 & 0.0012 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{array}$$

Using Equation [6] the general preference index is obtained as shown in the matrix and Πfrom there the inputs, outputs and net flow for each option are obtained as shown in Table 4.

$$\Pi = \begin{vmatrix} & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_1 & 0.000 & 0.000 & 0.005 & 0.005 & 0.000 \\ B_2 & 0.062 & 0.000 & 0.022 & 0.030 & 0.026 \\ B_3 & 0.057 & 0.013 & 0.000 & 0.010 & 0.020 \\ B_4 & 0.049 & 0.013 & 0.002 & 0.000 & 0.019 \\ B_5 & 0.049 & 0.013 & 0.017 & 0.023 & 0.000 \end{vmatrix}$$

Table 4: *Inputs, outputs and net substitute flows . Note:* Source: Personal information.

VIOL1	Oh	1,000	- 1,000
VIOL2	1	O.OOO	1,000
VIOL3	O.691	OO36	O.654
VIOL4	O.555	O.167	O.388
VIOL5	O.7O3	O.15O	O.553

In this analysis, positive and negative trends represent levels of strength and lightness relative to other options. Unilateral contract modifications, where a promisor can attempt to change the terms of a child support agreement without the recipient's consent or due process, carry more risk and harm than other standards... This leads to a lack of access to financial information, lower preferences than others, and defaults.

When analysing the negative flow, it is observed that non-payment is the crime with the greatest disadvantage compared to others. In this case, all regulations demonstrate the same level of risk aversion. These findings highlight the importance of prioritising unilateral contractual changes and payment issues, as well as the need to improve access to financial information to ensure transparency and accountability in the administration of child support. It is important to implement measures to protect the rights of beneficiaries and ensure that the funds provided to support them are used appropriately.

3 Conclusion

In summary, the legal reviews by many authors highlight the importance of establishing systems for monitoring maintenance administrators and ensuring transparency in the management of these resources. Experts agree that this approach not only protects the interests of the alimony recipient, but also promotes a balanced relationship between the parties by protecting the rights and peace of mind of the payer. The importance of the design and methods used in this study is reflected in the collective responses of lawyers who support the implementation of a monitoring system. This bill provides a possible solution to the problem by ensuring adequate control over the use of funds allocated for the maintenance and well-being of alimony recipients. The validity of the reform proposals was underlined by the support of commentators and lawyers. Through surveys and interviews, a consensus was reached on the need to establish a control system for cost administrators. This program will not only strengthen the rights of beneficiaries, but will also provide security and transparency to payers about the purpose of the funds provided. Ultimately, the implementation of this system will help improve the effectiveness of the judicial system and ensure greater well-being for all parties involved in the collection of child support.

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The Tourism Infrastructure of Rural Areas and Its Relationship with Visitor Satisfaction at the Tourist Destination Incorporating Neutrosophic Analysis of Uncertainty

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Abstract: This study examined how several predictor variables, such as lodging, transportation, natural resources, telecommunications, and cultural values, related to how satisfied people were with tourist destinations in rural areas. Data from 647 travelers was evaluated using a multiple linear regression technique. The findings showed that visitor destination satisfaction correlates significantly with all predictor variables. In particular, the biggest influence was found in cultural values, which were followed by transportation and natural resources. These results provide a thorough grasp of the elements influencing visitor happiness at tourist destinations, giving managers useful information for enhancing the experience of tourists. This study developed gender-specific predictive models to account for unique group characteristics, generating tailored predictions with enhanced precision. The results highlight differences in prediction intervals and neutrosophic uncertainty, emphasizing the impact of gender on model performance and accuracy. It is advised that more research be done to confirm and expand on these findings and investigate additional potential factors that may affect visitor happiness.

Keywords: Infrastructure, Tourism, Satisfaction, Tourist destination, Neutrosophic uncertainty, Indeterminacy modeling

1. Introduction

This study aims to identify the critical elements of tourism infrastructure that influence visitor satisfaction in rural tourism destinations, thereby promoting sustainable tourism, enhancing local economic development, safeguarding natural and cultural heritage, and improving the overall tourism experience for the benefit of local communities and the tourism sector.

Rural tourism possesses the capacity to enhance the local economy and significantly contribute to job creation and revenue generation in rural regions. They often possess a varied collection of natural and cultural assets, and by improving their tourism infrastructure, sustainable tourism may be promoted that respects local traditions and the environment while aiding in the protection and preservation of these resources [1, 2].

Given that several tourist destinations are predominantly located in metropolitan or coastal regions, sometimes overlooking the potential of rural areas, a robust rural tourism sector might mitigate the rural exodus to urban centers by generating employment opportunities and fostering economic growth [3,4].

The infrastructure of the tourist sector is a crucial element of the visitor experience. Comprehending the infrastructure attributes that hold the most significance for tourists can facilitate the formulation of strategies to enhance visitor satisfaction and encourage repeat visits.

The quality of tourism infrastructure in rural areas is garnering growing attention in tourism research, as it is crucial for attracting and retaining tourists and ensuring the long-term sustainability of local communities.

Consequently, visitor satisfaction is a crucial aspect for the ongoing development of these rural tourist locations. Tourism infrastructure strongly influences travelers' perceptions and satisfaction levels by providing the necessary conditions for an enjoyable journey [7,8]. A well-developed infrastructure can boost accessibility, accommodation quality, digital connectivity, authentic cultural experiences, and the enjoyment of natural resources. These infrastructural enhancements not only elevate tourist satisfaction but also foster economic and sustainable development in rural regions. [5,6]

Accessibility is a pertinent aspect in examining happiness and quality in rural tourism locations and their infrastructure, as it affects the ease with which tourists may arrive at and navigate within a site. Accessibility can be assessed using characteristics such as transportation connectivity, proximity to large population centers, and the presence of sufficient transportation infrastructure. A research in Indonesia examining rural tourism development indicates that enhanced road infrastructure and expanded public transportation can markedly improve accessibility and, consequently, visitor satisfaction at tourism destinations [8]. In rural regions, where distances can be extensive and transportation infrastructure may be inadequate, accessibility is a vital element in facilitating a favorable tourism experience.

Accommodation is a fundamental component of the tourism infrastructure in rural areas, significantly influencing visitor satisfaction and the entire tourist experience during their stay. Van Trung and Mohanty [9] assert that housing quality can be evaluated based on factors including room comfort, cleanliness, staff friendliness, and the provision of supplementary services. Research by Zhang et al. [10] indicates that diverse accommodation alternatives, including boutique hotels and cottages, enhance tourist satisfaction and elevate perceptions of tourism infrastructure quality in rural settings.

Telecommunications are crucial as they comprise the accessibility and quality of services, including internet connectivity and mobile phone coverage. In the digital era, travelers anticipate remaining connected even while exploring remote locales. The lack of telecommunication services might create a negative perception of the place, whereas robust infrastructure in this area greatly enhances the visitor experience and overall satisfaction level [11].

Cultural values encompass the traditions, practices, and heritage of a destination, serving as essential components in attracting and fulfilling travelers seeking authentic and meaningful experiences. In the rural regions of Guayas, cultural diversity and local celebrations provide a unique allure that, if adequately kept and promoted, may greatly enhance visitor happiness.

Natural resources are a primary allure of rural tourism destinations, encompassing landscapes, natural parks, rivers, and other environmental components that tourists appreciate and relish. The sustainable and careful maintenance of these resources is crucial for maintaining their appeal and ensuring visitor happiness. In rural regions, natural resources should be utilized in a manner that safeguards the environment and fosters sustainable tourism [13].

These considerations are especially pertinent in the context of the rural regions of Guayas, Ecuador, where the measurement apparatus will be utilized.

Hypotheses

H1: Sufficient accessibility, encompassing road quality and public transportation availability, will enhance satisfaction with the rural tourism location.

The diversity and caliber of lodging options in rural regions will correlate positively with overall tourist satisfaction and the perception of the destination's tourism infrastructure.

The accessibility and connectivity of telecommunication services will greatly enhance satisfaction with the rural tourist destination.

Local cultural values will significantly influence tourist satisfaction in rural destinations.

The natural charms of the tourist location would enhance visitor pleasure in rural regions.

2. Materials and Methods

A questionnaire was created for this study, grounded in a theoretical framework utilized in prior research regarding satisfaction with tourism services and infrastructure. The questionnaire was developed based on the works of various esteemed authors in the field [9; 11, 12], thereby ensuring the validity and reliability of the measurement equipment.

The survey was executed over multiple cantons of the Guayas province of Ecuador, namely Daule, Salitre, Balzar, Colimes, Palestina, and Santa Lucía. The selected cantons are part of the "Rice Route," an agricultural area distinguished for its rice cultivation and sustainably exploitable natural resources. The selection of Guayas province is warranted by its position as one of the largest and most diverse provinces in Ecuador, facilitating the development of a socio-economic model centered on sustainable tourism.

A convenience sampling strategy was utilized, focusing exclusively on tourist visitation in the specified localities. This method facilitated the acquisition of a sample that accurately represents the attributes of the visiting demographic in each examined canton.

The surveys were administered over three successive weekends, as this timeframe corresponds to the peak influx of visitors in these rural regions. Survey locations were deliberately positioned at the most renowned tourist attractions in each canton to guarantee that the majority of respondents were tourists rather than locals. Prior to presenting the questionnaire, each participant was apprised of the study's goal, and their informed consent to partake in the survey was secured.

The constructed questionnaire comprised multiple parts intended to assess various facets of tourism infrastructure and overall satisfaction with the destination. The measures were designed on six-point Likert scales to mitigate central tendency bias among participants. Participants indicated their degree of agreement or pleasure for certain statements pertaining to accessibility (ACC), accommodations (ALJ), telecommunications (TLC), cultural values (VCL), natural resources (RNT), and satisfaction with the visited tourist destination (SDT).

The gathered data were examined utilizing descriptive statistics and regression methods to discern patterns and relationships between infrastructure characteristics and tourist satisfaction. The analysis utilized Jamovi statistical software, version 2.3.28 [14]. Focus was directed towards pinpointing areas for enhancement in tourism infrastructure that could facilitate the establishment of a sustainable socio-economic framework in the region.

$$Y = \beta_0 + \beta_1 ACC + \beta_2 ALJ + \beta_3 RNT + \beta_4 TLC + \beta_5 VCL \quad (1)$$

Where:
SDT: Satisfaction with the tourist destination.
ACC: Accessibility.
ALJ: Accommodations.
TLC: Telecommunications.
VCL: Cultural values.
RNT: Natural resources.
β0: Intercept.
β1, β2, β3, β4, β5: Coefficients representing the impact of each variable.

ϵ: Error term.

Prediction intervals were established and gender disparities analyzed by independently training specific models for men and women, considering their unique characteristics. Prediction intervals were determined using statistical techniques that established the lower and upper limits for each estimate [13]. Furthermore, neutrosophic representations [14, 15, 16] were utilized to model the uncertainty inherent in the predictions, facilitating the detection and quantification of indeterminacy in the outcomes. This methodology facilitated a comparative investigation of the accuracy and variability between the models for males and females, emphasizing significant disparities in their prediction tendencies.

3. Results

The results obtained from the collected data are presented below, aiming to provide a detailed and comprehensive overview of the studied variables. During the data collection process, 697 surveys were gathered, of which 647 were deemed valid for subsequent analysis, excluding those that were incomplete or contained errors. First, the demographic data of the participants are outlined. Then, the key findings related to the stated objectives are described, highlighting significant patterns, trends, and relationships. These results serve as the foundation for the subsequent discussions and conclusions of the study.

In Table 1, it can be observed that 43% of the respondents were men, while the remaining 57% were women. Additionally, demographic data regarding age reveal a higher representation in the 26 to 35 age group, followed by the 18 to 25 age group. The extreme age groups, those under 18 and over 66 years old, show a lower number of respondents

Table 1. Respondent demographics.

Category	Description	Frequency
Gender	Male	42.66%
	Female	57.34%
Age	under 18	4.48%
	18-25	21.95%
	26-35	28.75%
	36-45	21.17%
	46-55	12.52%
	56-65	5.87%
	over 66	5.26%
Purpose of the trip	Rest	28.13%
	Business	6.65%
	Others	19.17%
	Walk	28.90%
	Vacation	8.96%
	Family Visit	8.19%

The demographic data related to the respondents' reasons for visiting the tourist destination are also presented. These results show that the majority visited the destination primarily for relaxation (28%) and leisure (29%). In contrast, the least common reason was business, accounting for only 7%. The category "other," which may include various unspecified reasons, also holds a significant share at 19%. Visits for

vacations and to see family are relatively common as well, at 9% and 8%, respectively. This information is essential for understanding the main factors that attract visitors to this tourist destination.

Table 2 presents the statistical results of six variables: Accessibility (ACC), Accommodation (ALJ), Telecommunications (TLC), Cultural Values (VCL), Natural Resources (RNT), and Satisfaction with the Tourist Destination (SDT). The data include the minimum value, first quartile, median, mean, third quartile, and maximum value for each variable.

Table 2. Descriptive Data of the Construct Variables.

	SDT	**TRN**	**ALJ**	**TLC**	**VCL**	**RNT**
N	647	647	647	647	647	647
Lost	0	0	0	0	0	0
Average	4.4162	3.8667	4.2940	2.5008	4.2883	4.0947
Median	4.7500	4.2500	4.7500	2.0000	5.0000	4.7500
Standard deviation	1.2581	1.2414	1.3791	1.2072	1.4929	1.3563
Minimum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Maximum	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000

To gain a better understanding of the relationships between the variables, Figure 1 presents the scatterplot matrix of the variables.

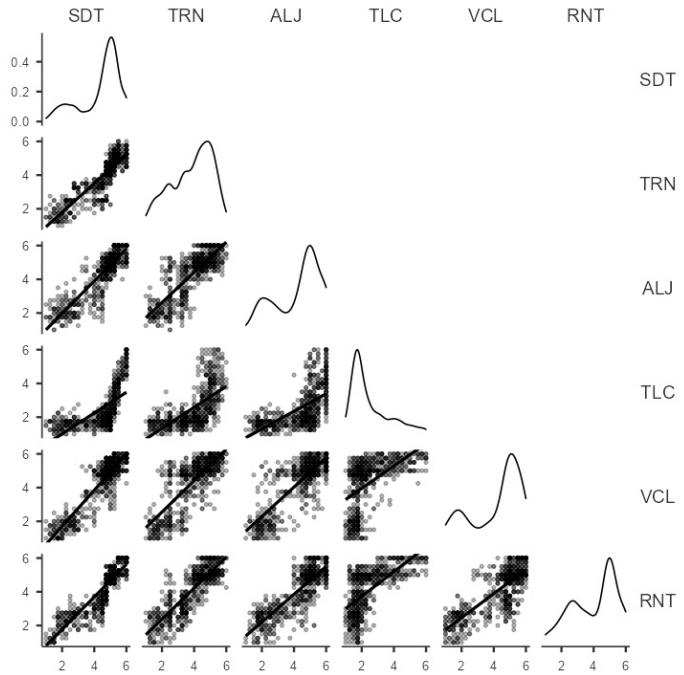

Figure 1. Scatterplot matrix of variables

Similarly, Table 3 presents the results of the correlation matrix between variables. These results indicate that the highest correlation is observed between the variables SDT and RNT (0.906), while the lowest correlation is found between TLC and VCL (0.633). The strong correlation between SDT and ACC (0.897) suggests a significant relationship between these two variables

Correlation indicators greater than 0.7 suggest a strong linear relationship between the variables involved. In the context of the presented correlation matrix, it is noteworthy that the variables Natural Resources, Accessibility, Accommodation, Telecommunications, and Cultural Values exhibit a strong and significant correlation with the variable Satisfaction with the Tourist Destination. This highlights how the variables are interconnected, which is essential for understanding the interdependencies and potential mutual influences within the study's context

Table 3. Descriptive Data of the Construct Variables

	SDT	TRN	ALJ	TLC	VCL	RNT
SDT	—					
TRN	0.8701	—				
ALJ	0.8774	0.8080	—			
TLC	0.6424	0.6413	0.5952	—		
VCL	0.9011	0.7742	0.8072	0.5371	—	
RNT	0.8927	0.8426	0.8384	0.6504	0.7869	—

Table 4 presents the values from the linear regression applied to the data, revealing a strong relationship between the dependent variable and the independent variables. The high R-squared value (0.9221) indicates a good model fit, meaning that 92.21% of the variability in the dependent variable is explained by the model.

Overall, the performance indicators of the multiple linear regression model demonstrate its strong explanatory power and accuracy in predicting the dependent variable, SDT, based on the independent variables ALJ, TLC, VCL, and RNT. The high correlation (R), elevated coefficient of determination (R^2), adjusted R^2, and reasonably low values for AIC, BIC, and RMSE all indicate that the model is robust and well-suited for the analyzed data.

Table 4. Descriptive Data of the Construct Variables

Model	R	R^2	R^2 corrected	AIC	BIC	RMSE
1	0.9603	0.9221	0.9215	494.7240	526.0304	0.3508

All coefficients are significant, with VCL, RNT, and TRN showing the highest values, suggesting that these variables have a substantial impact on the dependent variable. The residual analysis indicates that the errors are reasonably evenly distributed around zero, confirming that the model is well-specified.

Table 5. Coefficients of the SDT model.

Predictor	Estimator	EE	t	p
Constante	0.5176	0.0478	10.8382	< .001
TRN	0.1822	0.0231	7.8768	< .001
ALJ	0.1507	0.0212	7.1255	< .001
TLC	0.0480	0.0156	3.0864	0.002
VCL	0.3387	0.0172	19.7249	< .001
RNT	0.2378	0.0229	10.3970	< .001

$p < 0.001$ (***); $p < 0.01$ (**); $p < 0.05$ (**)

Table 5 provides and explains the results of the coefficients in the multiple linear regression model for the dependent variable SDT and the predictor variables TRN, ALJ, TLC, VCL, and RNT. The table summarizes the coefficient estimates, their standard errors (SE), t-values, and corresponding p-values (Table 6).

In this work, models were trained independently for men and women, considering the unique characteristics of each group. Thereafter, personalized predictions were generated for each model, facilitating a more precise and customized study pertinent to the traits of each group. This distinct methodology guarantees enhanced representativeness and accuracy in the outcomes achieved.

Table 6. Prediction intervals and neutrosophic forms

Prediction	Prediction Interval [Lower Bound, Upper Bound] (Men)	Neutrosophic form (Men)	Prediction Interval [Lower Bound, Upper Bound] (Women)	Neutrosophic form (Women)
1	[10,15]	$10 + 15I;\ I \in [0,0.3333]$	[11.5, 16.5]	$11.5 + 16.5I;\ I \in [0,0.303]$
2	[17.5, 22.5]	$17.5 + 22.5I;\ I \in [0,0.2222]$	[19.0, 25,0]	$19.0 + 25.0I;\ I \in [0,0.24]$
3	[13.5, 18.5]	$13.5 + 18.5I;\ I \in [0,0.2703]$	[15.0, 20.0]	$15.0 + 20.0I;\ I \in [0,0.25]$

The table compares forecasts and their corresponding uncertainty, shown by prediction intervals and neutrosophic models for both men and women. This facilitates the assessment of the impact of gender on predictive modeling results and their associated uncertainty.

4. Discussions

Regarding the model coefficients, the constant represents the predicted value of SDT when all independent variables are zero. A value of 0.5176 with a significant p-value ($p < 0.001$) indicates that the constant is statistically significant.

An increase of one unit in TRN is associated with an increase of 0.1822 units in SDT, holding all other variables constant. Similarly, a one-unit increase in ALJ is linked to a 0.1507-unit rise in SDT

For TLC, a one-unit increase corresponds to a 0.0480-unit increase in SDT. Although this coefficient is smaller than those of the other predictors, the significant p-value ($p = 0.002$) confirms that TLC is a meaningful predictor of SDT

Additionally, a one-unit increase in VCL results in a 0.3387-unit increase in SDT. This coefficient is the largest among the predictors, highlighting a strong effect on SDT. Moreover, a one-unit increase in RNT is associated with a 0.2378-unit rise in SDT. The significant p-value ($p < 0.001$) underscores that RNT is a significant predictor of SDT

This study involved the individual training of models for men and women to address the distinct characteristics of each group. This methodological differentiation facilitated the creation of individualized predictions customized to the characteristics of each group, hence improving the accuracy and pertinence of the analysis.

Table 6 delineates the prediction intervals together with their respective neutrosophic representations for both genders. The prediction intervals, denoted as [Lower Bound, Upper Bound], encompass the spectrum of possible results for each forecast. The neutrosophic forms elucidate predictions by integrating the degree of uncertainty, denoted by I, within designated intervals.

The initial forecast for men spans from [10, 15], represented in neutrosophic form as $(10 + 15I;\ I \in [0, 0.3333])$. The analogous interval for women is [11.5, 16.5], represented in neutrosophic form as $(11.5 + 16.5I;\ I \in [0, 0.303])$. The results reveal a reduced uncertainty range for women in this instance, implying possible variations in variability or data structure across the groups.

The investigation demonstrates the impact of gender-specific modeling on the accuracy and uncertainty of forecasts by comparing prediction intervals and neutrosophic representations. This method highlights the necessity of customized predictive models for populations with unique traits, guaranteeing both representativeness and reliability in the results [17].

5. Conclusions

All predictors in the model (TRN, ALJ, TLC, VCL, and RNT) are statistically significant ($p < 0.05$), indicating that each has a meaningful impact on the dependent variable, SDT. Among them, VCL stands out with the highest coefficient, making it the strongest predictor in the model. These findings offer valuable insights into how each predictor contributes to the variability in SDT, paving the way for further discussions and more in-depth analyses based on these results

Equation 2 represents the substitution of the regression coefficients obtained from the analysis:

$$Y=0.5176+0.1822\cdot ACC+0.1507\cdot ALJ+0.2378\cdot RNT+0.0480\cdot TLC+0.3387\cdot VCL \quad (1)$$

This mathematical model describes the relationship between the dependent variable, SDT, and the independent variables (ACC, ALJ, RNT, TLC, and VCL). The coefficients indicate the expected change in Y (SDT) for each additional unit of the respective independent variable while keeping all other variables constant.

After analyzing the results of the linear regression model and examining the resulting mathematical model that describes the relationship between the studied variables, it is concluded that, according to the model, the variables that most positively influence satisfaction with the tourist destination are cultural values and natural resources, with the highest coefficients of 0.3387 and 0.2378, respectively. This suggests that tourist destinations with a rich cultural offering and abundant natural resources tend to achieve higher levels of satisfaction among tourists.

Accessibility and telecommunications have lower coefficients, indicating a smaller impact on tourist satisfaction, while differences in variable coefficients highlight the unique contribution of each factor. This underscores the importance of considering various aspects, such as accommodation quality, natural resource preservation, and local culture promotion, to improve the visitor experience. Despite its insights, the model has limitations, as factors like weather, pricing, and safety, which may also influence satisfaction, were not included. Future research should validate and refine the model, explore additional influencing factors, and conduct targeted analyses for different tourist types and market segments to better address their needs. These findings offer valuable guidance for destination managers to enhance tourist satisfaction, competitiveness, and sustainability.

Future research ought to concentrate on broadening the utilization of neutrosophic methodologies to additional areas inside tourist management, including the assessment of uncertainty in environmental sustainability initiatives or the evaluation of marketing strategy efficacy. Integrating neutrosophic sets with powerful machine learning methods enables the management of complicated datasets and offers more robust insights into the underlying indeterminacy of tourist behaviors and preferences. This would improve decision-making processes, especially in dynamic and unpredictable contexts.

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Enhancing the Teaching-Learning Process through Neutrosophic Statistical Analysis of Professional Competencies and Metacognitive Strategies

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Abstract. In the educational field, teaching effectiveness depends on the interaction between professional competencies and teachers' metacognitive strategies. Therefore, the present study has proposed to evaluate the relationship between professional competencies and metacognitive strategies, by using the management of indeterminacies to improve the teaching-learning process. To this end, the neutrosophic statistics method was applied to analyze data on meta-cognitive competencies and strategies in a group of teachers. Among the results, a general positive relationship was revealed between professional competencies and metacognitive strategies, with significant variations in specific areas. Finally, it has been concluded that there is a significant connection, where the variability suggests the need to strengthen training in specific competencies and metacognitive strategies to optimize the educational process.

Keywords: Neutrosophic statistics, Teaching-learning, Educational methodology, Teaching process.

1 Introduction

The educational field plays a central role in the development of society, by promoting learning and the physical and psychological growth of students [1]. Education not only focuses on the acquisition of knowledge, but also on the comprehensive development of the human being, including cognitive, social and moral aspects [2]. In this sense, teaching work at the university is not limited to teaching a profession, but also seeks to train individuals who contribute to the progress of society.

Therefore, the competency approach has emerged as a key methodology in education, particularly in the training of education professionals [3]. This approach encompasses a series of knowledge, attitudes and skills that allow individuals not only to understand their environment, but also to transform it effectively. Competencies, understood as the integration of theoretical and practical knowledge, are essential for successful professional performance, and the university has the responsibility to foster their continuous development [4].

The concept of competence is complex and multidimensional, involving not only technical knowledge, but also the ability to apply such knowledge in specific contexts [5]. Professional competences include the ability to solve problems autonomously, work in a team and adapt to the changing demands of the environment. Within this framework, university education must provide the necessary tools for students to develop these competences and apply them effectively in professional life [6].

Therefore, metacognitive strategies must be proposed that play a crucial role in the development of competencies, by allowing students to reflect on their own learning process and regulate it effectively [7]. Metacognition , which involves the knowledge and control of one's own cognitive processes, focuses on achieving meaningful learning and the development of autonomy in students. Therefore, university teachers must master these strategies to implement them in pedagogical practices and thus enhance the comprehensive development of students [8].

Finally, it has been observed that the Universidad Nacional Mayor de San Marcos, by participating in projects such as Tuning , recognizes the competencies in the formation of human capital. These competencies not only prepare students for the labor market, but also promote personal and social growth. Although the indeterminate elements that affect the integration of the variables, professional competencies and metacognitive strategies must be identified and analyzed. In this context, the present research seeks to evaluate the relationship between professional competencies and metacognitive strategies of teachers, by using neutrosophic statistics to manage indeterminacies and improve the teaching-learning process.

2 Materials and methods

2.1 Neutrosophic statistics.

Neutrosophic probability and statistics are a generalization of classical, imprecise probability and statistics [6]. The neutrosophic probability of an event E is the probability that event E occurs, the probability that event E does not occur, and the probability of indeterminacy (not knowing whether event E occurs or not) [9]. In classical probability nsup≤1, while in neutrosophic probability nsup≤3+. The function that models the neutrosophic probability of a random variable X is called the neutrosophic distribution:

Where $\vartheta_A(x), \eta_A(x), \delta_A(x)$the following condition is met$0 \leq \vartheta_A(x), \eta_A(x), \delta_A(x) \leq 3$ for all $x \in X$. So to define each neutrosophic number it is expressed in the following way $h, i, j$for the modeling of the neutrosophic methodology to be used. Therefore, the following functions are defined:

$h = \vartheta_A(x)$for membership functions to true, where ∈ {0,1}.

$i = \eta_A(x)$for membership functions to indeterminate, where ∈ {0,1}.

$j = \delta_A(x)$for membership functions to false, where ∈ {0,1}.

Therefore, the neutrosophic number defined for the study is determined as $L = (h, i, j)$, where h, i, j ∈ {0,1}and satisfies the following condition $0 \leq h + i + j \leq 3$. Thus, let B be defined as the scoring function of a neutrosophic number according to the proposal of Smarandache or Basset for deneutrosification.

Where h represents the probability that the value x occurs, j(x) represents the probability that the value x does not occur, and I(x) represents the indeterminate or unknown probability of the value x. Neutrosophic statistics is the analysis of neutrosophic events and deals with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, and neutrosophic regression among others. For this, one must operate with neutrosophic numbers belonging to the single valued neutrosophic set (SVNS), in order to be used for practical purposes [10]. Some operations between SVNS are expressed below:

Given $L_1 = \ and \ L_2 = (h_2, i_2, j_2)$, the sum between them $L_1 y\ L_2$is defined as:

$$L_1 + L_2 = (h_1 + h_2 - h_1 h_2, i_1 i_2, j_1 j_2) \quad (1)$$

Given $L_1 = (h_1, i_1, j_1)$and $L_2 = (h_2, i_2, j_2)$ the multiplication between $L_1 y\ L_2$is defined as:

$$L_1 \cdot L_2 = (h_1 h_2, i_1 + i_2 - i_1 i_2, j_1 + j_2 - j_1 j_2) \quad (2)$$

The product of a positive scalar with a SVNS, $L = (h, i, j)$c) is defined by:

$$L = (1 - (1 - h), i, j) \quad (3)$$

Finally, neutrosophic logic [11], neutrosophic sets and neutrosophic probabilities and statistics have a wide application in various research fields and constitute a novel reference for study in full development. Neutrosophic statistics includes all the techniques to summarize and describe the characteristics of neutrosophic numerical data [12].

For the development of the study, probabilities and neutrosophic statistics are used, when integrated into the analysis of the relationship between professional teaching skills and the use of metacognitive strategies, by incorporating an indeterminacy component. This allows us to address the evaluation of educational events with information that may be imprecise, incomplete or unknown. For the modeling of neutrosophic statistics, the neutrosophic random variable is identified, where L represents the lower and upper level that the variable studied can reach, in an indeterminate interval. Thus, the neutrosophic mean of the variable follows ($\bar{L}$)when formulating [14]:

$$\bar{L} = \frac{1}{n_N} \sum_{i=1}^{n_N} L_i \quad (4)$$

Where $n_N$is a neutrosophic random sample of the population studied. In this study the sample consisted of (20-40) teachers, to whom validated questionnaires were applied. A neutrosophic response scale was used to collect data on the variables studied (see table 1).

Table 1: Neutrosophic linguistic terms. Source: Own elaboration.

Criterion	SVNS
Extremely high (EA)	(1,0,0)
Very very high (MMA)	(0.91,0.06,0.14)
Very high (MA)	(0.81,0.16,0.24)
High (A)	(0.71,0.26,0.34)
Moderately high (MDA)	(0.61,0.36,0.44)
Medium (M)	(0.51,0.46,0.54)
Medium Low (MDB)	(0.41,0.56,0.64)
Bass (B)	(0.31,0.65,0.74)
Very low (MB)	(0.21,0.76,0.84)
Very very low (MMB)	(0.11,0.86,0.94)
Extremely Low (EB)	(0,0.96,1)

Once the neutrosophic mean has been defined, the variance of the neutrosophic sample is calculated. To do this, the following equation is defined [15]:

$$S_N^2 = \frac{\sum_{i=1}^{n_N} (L_i - \bar{L}_i)^2}{n_N} \quad (5)$$

The neutrosophic coefficient (NCV) is then calculated, which measures the consistency of the variable. The lower the NCV value, the more consistent the performance of the factor is than that of the other factors. To do this, the following equation is proposed [16]:

$$CV_N = \sqrt{S_N^2} \times 100 \quad (6)$$

A null hypothesis was formulated in neutrosophic terms, which proposed the nonexistence of a significant relationship between the variables, using a significance level $\alpha = 0.05$, evaluated under neutrosophic criteria . While to determine the Pearson correlation between both variables, the following equation is applied [17].

$$r = \frac{Cov\ (L^X, L^Y)}{\sqrt{\sigma_{L^X}^2 \cdot \sigma_{L^Y}^2}} \qquad (7)$$

Where to calculate the Covariance, the following formula is applied [18].

$$Cov\ (X,Y) = \frac{\sum_{i=0}^{n}(L_i^X - \bar{L}^X)(L_i^Y - \overline{L^Y})}{n-1} \qquad (8)$$

While it $\sigma$is obtained from the results obtained from neutrosophic statistics or can be replaced by for S_N^2sample analysis.

3 Results

3.1 Data collection. Description of variables.

Metacognitive strategies were applied to the teachers, both appointed and contracted, of the PAE of Physical Education of the Faculty of Education of the UNMSM. Below, the variables and dimensions are visualized, where the elements that allow the neutrosophic analysis are highlighted (see table 2).

Table 2: Variables that affect the comprehensive development of university students. Source: Prepared by the authors.

Variable	Code	Dimensions	Code	Items	
Professional skills	CP	Subject competencies	CP-A	▪	Knowledge of the subject
				▪	Curriculum management
		Teaching management skills	CP-B	▪	Teaching didactics
				▪	Teaching planning
				▪	Teaching activities
		Participatory skills	CP-C	▪	Link with the institution
				▪	Link to address
				▪	Link to context
Metacognitive strategies	EM	Expectations towards students	EM-A	▪	Learning to understand
				▪	Identification with students
				▪	Tutorial performance
		Teaching strategies	EM-B	▪	Didactic adaptation
				▪	Pedagogical communication
				▪	Motivator
		Affectivity in the classroom	EM-C	▪	Promoter of ecological climate
				▪	Social behavior
				▪	Mediator of peer interactions

3.2 Neutrosophic statistics.

3.2.1 Professional skills.

The study carried out evaluated the *professional competences* of 40 teachers, using an approach based on neutrosophic logic. To do so, three key dimensions were defined: subject competences, teaching management competences, and participatory competences, incorporating indeterminacy in the final assessment of each element (see table 3).

Table 3: Neutrosophic evaluation of each dimension belonging to the professional competencies variable. Source: Own elaboration.

Teaching	CP-A	CP-B	CP-C
1	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
2	(0.71,0.26,0.34)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
3	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
4	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
5	(0.61,0.36,0.44)	(0.91,0.06,0.14)	(0.71,0.26,0.34)
6	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
7	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
8	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
9	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
10	(0.71,0.26,0.34)	(0.91,0.06,0.14)	(0.71,0.26,0.34)
11	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.81,0.16,0.24)
12	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
13	(0.61,0.36,0.44)	(0.91,0.06,0.14)	(0.71,0.26,0.34)
14	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
15	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
16	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
17	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
18	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
19	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
20	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
21	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
22	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
23	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
24	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.71,0.26,0.34)
25	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
26	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
27	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
28	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
29	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
30	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
31	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
32	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
33	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
34	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
35	(0.81,0.16,0.24)	(0.91,0.06,0.14)	(0.81,0.16,0.24)
36	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
37	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
38	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
39	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
40	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)

Interpretation and analysis:

Knowledge of the subject (CP-A): Most teachers fall into the neutrosophic categories ranging from "Average High" to "Very High". This indicates that most teachers have a solid and varied understanding of the subjects they teach, although there is a range in the depth of their knowledge.

Use of content (CP-B): The results show a predominance in neutrosophic categories such as "Very High" and "Extremely High". This reflects that teachers organize and apply the content in a highly effective manner. This suggests that the use of content in teaching practice is adapted to the needs of

students.

Curriculum Management and Assessment (CP-C): In this dimension, most teachers are classified in neutrosophic levels ranging from "Medium High" to "Very High". This shows that, although most teachers manage curriculum planning and assessment well, there is some variability and indeterminacy in the effectiveness of their curriculum management.

Taken together, these results provide a clear view of teachers' performance in terms of knowledge, content application and curriculum management. Most of them are at high levels, reflecting a generalized competence, although with some indeterminations in the depth and effectiveness in each dimension. Therefore, it is necessary to resort to the analysis of CV_{ND} these dimensions and determine which one has the greatest impact on the variable to determine the planned solutions (see table 4).

Table 4: Determination of $\bar{L}$, S_N and CV_N of each dimension in the professional competencies variable. Source: Own elaboration.

Professional skills	L		S N	CV N
CP-A	0.76	[(0.61,0.36,0.44);(0.71,0.26,0.34)]	0.005	0.072
CP-B	0.92	[(0.81,0.16,0.24);(0.91,0.06,0.14)]	0.002	0.043
CP-C	0.78	[(0.61,0.36,0.44);(0.71,0.26,0.34)]	0.005	0.069

From the results obtained, it can be seen that the *teaching management competencies dimension* has the most consistent performance among the other dimensions with a value of 0.043. While, on the contrary, the *subject competencies dimension* contains a greater indeterminacy in the professional development process with a value of 0.072. Therefore, the results obtained from the variable should be enhanced by including the indeterminate elements that exist for the development of professionals.

Metacognitive strategies.

Finally, the variable of metacognitive strategies was evaluated in 40 teachers, by applying a neutrosophic measurement. For the comprehensive assessment, three fundamental dimensions were defined: expectations towards students, pedagogical strategies, and affectivity in the learning environment. Thus, it allowed the incorporation of indeterminacy in the final evaluation, by offering a more complete and precise vision of the educational dynamics (see table 5).

Table 5: Neutrosophic evaluation of each dimension belonging to the metacognitive strategies variable. Source: Own elaboration.

Teaching	EM-A	EM-B	EM-C
1	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
2	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
3	(0.81,0.16,0.24)	(0.75,0.25,0.1)	(0.71,0.26,0.34)
4	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
5	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
6	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
7	(0.81,0.16,0.24)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
8	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
9	(0.61,0.36,0.44)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
10	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
11	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
12	(0.81,0.16,0.24)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
13	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
14	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.61,0.36,0.44)

Teaching	EM-A	EM-B	EM-C
15	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
16	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
17	(0.81,0.16,0.24)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
18	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
19	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
20	(0.61,0.36,0.44)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
21	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
22	(0.81,0.16,0.24)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
23	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
24	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
25	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
26	(0.81,0.16,0.24)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
27	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
28	(0.71,0.26,0.34)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
29	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
30	(0.81,0.16,0.24)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
31	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.61,0.36,0.44)
32	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
33	(0.81,0.16,0.24)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
34	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
35	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.61,0.36,0.44)
36	(0.81,0.16,0.24)	(0.81,0.16,0.24)	(0.71,0.26,0.34)
37	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
38	(0.71,0.26,0.34)	(0.71,0.26,0.34)	(0.71,0.26,0.34)
39	(0.81,0.16,0.24)	(0.81,0.16,0.24)	(0.51,0.46,0.54)
40	(0.61,0.36,0.44)	(0.81,0.16,0.24)	(0.71,0.26,0.34)

Interpretation and analysis:

Expectations of Students (EM-A): Most teachers fall into the neutrosophic categories of "Average High" to "High." This suggests that, overall, teachers show a strong commitment to student expectations, albeit with some variability.

Teaching strategies (EM-B): The results indicate predominance in the neutrosophic categories "Medium High" and "High", which reflects the effective application of teaching strategies by teachers.

Classroom Affectivity (EM-C): Teachers are at neutrosophic levels of "Medium High" to "High", which shows effective management of the classroom environment and interactions with students, although with some variations in effectiveness.

Therefore, it is necessary to determine and analyze the dimensions of the CV_{ND} metacognitive strategies variables. This seeks to evaluate the levels of uncertainty and include them in the planned solutions (see table 6)

Table 6: Determination of $\bar{L}$, S_N and CV_N of each dimension in the metacognitive strategies variable. Source: Own elaboration.

Metacognitive strategies	L		S N	CV N
EM-A	0.77	[(0.61,0.36,0.44);(0.71,0.26,0.34)]	0.007	0.081
EM-B	0.85	[(0.71,0.26,0.34);(0.81,0.16,0.24)]	0.004	0.062
EM-C	0.71	[(0.61,0.36,0.44);(0.71,0.26,0.34)]	0.008	0.087

From the results obtained, it is observed that the *teaching strategies dimension (EM-B)* presents the most consistent performance among the dimensions evaluated, with a value of 0.062. In contrast, the

affectivity dimension in the classroom (EM-C) shows greater indeterminacy in the process of applying metacognitive strategies, with a value of 0.087. Therefore, the results obtained in the variable must be strengthened by integrating the indeterminate elements present to optimize the development and effectiveness of metacognitive strategies.

3.2.3 Correlation between professional competences and metacognitive strategies.

To carry out the correlation analysis, the neutrosophic values of the evaluations carried out by the teachers for the variable professional competences and metacognitive strategies must be converted (see table 7). To do this, these values are deneutrosified according to the Basset equation. In this way, the correlation between both variables can be analyzed (see table 8).

Table 7: Teachers' evaluation of the variables professional competencies and metacognitive strategies . Source: Own elaboration.

Teaching	Professional skills	Metacognitive strategies
1	(0.94,0.15,0)	(0.94,0.15,0)
2	(0.94,0.15,0)	(0.74,0.35,0.25)
3	(0.74,0.35,0.25)	(0.74,0.35,0.25)
4	(0.94,0.15,0)	(0.74,0.35,0.25)
5	(0.94,0.15,0)	(0.94,0.15,0)
6	(0.94,0.15,0)	(0.74,0.35,0.25)
7	(0.94,0.15,0)	(0.94,0.15,0)
8	(0.94,0.15,0)	(0.74,0.35,0.25)
9	(0.74,0.35,0.25)	(0.74,0.35,0.25)
10	(0.94,0.15,0)	(0.74,0.35,0.25)
11	(0.94,0.15,0)	(0.94,0.15,0)
12	(0.94,0.15,0)	(0.94,0.15,0)
13	(0.94,0.15,0)	(0.94,0.15,0)
14	(0.94,0.15,0)	(0.74,0.35,0.25)
15	(0.94,0.15,0)	(0.94,0.15,0)
16	(0.94,0.15,0)	(0.74,0.35,0.25)
17	(0.94,0.15,0)	(0.94,0.15,0)
18	(0.94,0.15,0)	(0.94,0.15,0)
19	(0.94,0.15,0)	(0.74,0.35,0.25)
20	(0.94,0.15,0)	(0.74,0.35,0.25)
21	(0.74,0.35,0.25)	(0.74,0.35,0.25)
22	(0.94,0.15,0)	(0.94,0.15,0)
23	(0.94,0.15,0)	(0.74,0.35,0.25)
24	(0.94,0.15,0)	(0.74,0.35,0.25)
25	(0.94,0.15,0)	(0.94,0.15,0)
26	(0.74,0.35,0.25)	(0.94,0.15,0)
27	(0.74,0.35,0.25)	(0.74,0.35,0.25)
28	(0.94,0.15,0)	(0.94,0.15,0)
29	(0.94,0.15,0)	(0.74,0.35,0.25)
30	(0.74,0.35,0.25)	(0.94,0.15,0)
31	(0.94,0.15,0)	(0.74,0.35,0.25)
32	(0.74,0.35,0.25)	(0.94,0.15,0)
33	(0.94,0.15,0)	(0.94,0.15,0)
34	(0.74,0.35,0.25)	(0.94,0.15,0)
35	(0.94,0.15,0)	(0.74,0.35,0.25)
36	(0.74,0.35,0.25)	(0.94,0.15,0)

Teaching	Professional skills	Metacognitive strategies
37	(0.74,0.35,0.25)	(0.74,0.35,0.25)
38	(0.94,0.15,0)	(0.94,0.15,0)
39	(0.74,0.35,0.25)	(0.94,0.15,0)
40	(0.94,0.15,0)	(0.94,0.15,0)

Table 8: Pearson correlation of the variables L^X y L^Y. Source: Own elaboration.

	S^2	$Cov(L^X, L^Y)$	r
L^X	0.0012564	-	
L^Y	0.0007477	-	0.913
-	-	0.001	

Metacognitive strategies reveals a positive and significant relationship with a correlation coefficient of 0.913. This result validates the research hypothesis and rejects the null hypothesis, indicating that as professional competences are strengthened, teachers' metacognitive strategies improve. However, the neutrosophic relationship between variables and dimensions with indeterminate elements must be included:

1. Subject knowledge (CP-A) and teaching strategies (EM-B):
   - Variability in depth of knowledge: The knowledge dimension of the topic shows that most teachers have a solid understanding, although with variations in depth. This variability can directly influence the teaching strategy. Teachers with deeper knowledge implement more effective and tailored strategies, while those with less depth would face difficulties in applying the strategies optimally.
2. Use of content (CP-B) and expectations towards students (EM-A):
   - Adaptability and engagement: A predominance of neutrosophic categories of content use indicates that teachers implement and organize content effectively. This ability to adapt to student needs is linked to expectations for students. A high level of expectations generally translates into better adaptation and use of content, as teachers adjust their approach according to student expectations and needs.
3. Curriculum management and evaluation (CP-C) and affectivity in the classroom (EM-C):
   - Efficiency of classroom management and environment management: The curriculum management and assessment dimension shows good planning and assessment with some variability in its effectiveness. This indeterminacy is related to affectivity in the classroom. Therefore, effective management of the environment and interactions with students improves the effectiveness of planning and assessment. It even facilitates a more coherent and adapted implementation of the curriculum plans.
4. Knowledge of the subject (CP-A) and expectations towards students (EM-A):
   - Depth of knowledge and engagement: Strength of subject knowledge influences expectations for students. Deeper knowledge enables teachers to set higher, more realistic expectations for students based on a thorough understanding of the topics and content. This, in turn, leads to greater clarity and consistency in the expectations set.
5. Use of content (CP-B) and affectivity in the classroom (EM-C):
   - Effective implementation and classroom environment: Effective use of content contributes to better management of the classroom environment, since teachers who implement content appropriately tend to better manage interactions and classroom climate, facilitating a positive and productive learning environment.

The dimensions analyzed show interdependent relationships with each other. Indeterminacy in the depth of knowledge, the ability to use and adapt content, curricular management, and affectivity in the classroom are all interrelated. These relationships highlight how different aspects of teaching practice mutually influence and affect student development and performance. Consequently, it reinforces the need for professional competences in optimizing teacher training, improving pedagogical intervention, and fostering socio-cognitive skills, critical reflection, and creativity.

4. Conclusion

Once the investigation was concluded, the following conclusions could be reached:

Analysis based on the neutrosophic statistical method has revealed that UNMSM teachers mostly exhibit a high level of professional competence, particularly in terms of pedagogical management. However, significant variations have been identified in the areas of subject knowledge and curricular management. These observations suggest the need for targeted interventions to strengthen training in these specific areas and thus improve the homogeneity of teaching performance.

Metacognitive strategies by teachers have shown a positive trend, especially in the adaptation of teaching strategies. However, the neutrosophic analysis also reveals a high variability in the affective dimension in the classroom. Thus, it indicates the need to improve training in the management of emotional and social aspects of the educational environment to optimize teaching effectiveness.

The correlation between professional competences and metacognitive strategies, assessed using the neutrosophic method, has shown a positive overall relationship. However, the observed variability suggests that, although there is a significant connection between the domain of the variables, efforts should be intensified to achieve a more consistent integration and optimize the impact on the educational process.

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NCC: Neutrosophic Control Charts, a didactic way to detect Cardiac Arrhythmias from reading Electrocardiograms

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Abstract: Cardiac arrhythmias are cardiovascular conditions caused by irregularities in the formation or transmission of electrical impulses. To identify these arrhythmias and determine their origin, an electrocardiogram (ECG) is used, which graphically records the electrical activity responsible for each heartbeat over a short period. This analysis can reveal stochastic changes, introducing a degree of uncertainty. To detect these changes, neutrosophic control charts provide an appropriate statistical tool to assess the stability of the ECG, anticipate potential heart failures, reduce variability in measurements, and, in some cases, make predictions. The results indicate that the proposed model is highly accurate in detecting and classifying arrhythmias and that T^2 control charts are useful for monitoring the quality of the detection process. In conclusion, the proposed hybrid neutrosophic model effectively identifies and classifies cardiac arrhythmias in clinical practice.

Keywords: Uncertainty Control charts, Hotelling's T^2, Neutrosophy, Cardiac arrhythmias.

1. Introduction

Currently, the preventive monitoring of cardiovascular diseases is of high importance, as it reduces the morbidity and mortality associated with these conditions, which are among the leading causes of death worldwide. The significance of monitoring this type of chronic disease lies in its ability to detect cardiac problems such as arrhythmias, heart attacks, or heart failure. It is essential to understand that early detection enables timely interventions, leading to improved clinical outcomes, reducing the need for invasive treatments, and lowering costs across the entire healthcare system. [1].

Early detection of abnormalities in the electrocardiogram (ECG) signal could indicate the development of a cardiac arrhythmia, which may have severe consequences if not properly detected and treated. The early diagnosis of these arrhythmias is essential for timely treatment and for improving the patient's quality of life. [2]

In [3], the ability to handle uncertainty, indeterminacy, and contradiction in medical data—common characteristics in both the diagnosis and monitoring processes of cardiovascular diseases—has been significantly supported by neutrosophy. Neutrosophy manages this uncertainty more effectively than traditional models by incorporating degrees of truth (T), falsehood (F), and indeterminacy (I) into its calculations. [4]. This is particularly useful for making more accurate clinical decisions based on undefined data.

On the other hand, advancements in medical technology have enabled the development of more precise and effective tools and methods for detecting cardiac arrhythmias, resulting in more accurate diagnoses and better patient stratification. This not only optimizes the diagnosis of cardiac arrhythmias but also contributes to a more efficient and proactive healthcare system. Additionally, the use of

statistical techniques, such as heart rate variability (HRV) analysis, and machine learning can aid in the early detection and accurate diagnosis of arrhythmias, thereby improving the management of cardiovascular diseases. Similarly, the use of statistical techniques, such as heart rate variability (HRV) analysis and machine learning, can assist in the early detection and accurate diagnosis of arrhythmias, which can enhance care in the management of cardiovascular diseases.

2. Preliminaries

This section summarizes statistical control charts and the theory of neutrosophic logic

2.1 Statistical Control Charts

In [5] and [6], the authors mention that control charts are valuable tools for improving quality, early detection of process deviations, and analyzing their underlying causes. These charts enable the early identification of process issues, saving time and money in error correction. Additionally, control charts help enhance productivity and efficiency by providing a better understanding of the process and facilitating quicker problem identification.

Hotelling's T^2 chart is a type of multivariate control chart used to monitor the simultaneous behavior of two or more variables in a process. This chart is based on the calculation of a T^2 statistic, which measures the distance between a point in multivariate space and the process mean. It enables the detection of deviations in both the mean and variance of the variables simultaneously, making it particularly useful in processes involving multiple interdependent variables. When a deviation is detected on the T^2 chart, it indicates a change in the process has occurred, and the cause should be investigated to take corrective actions. [7] and [8].

2.2 Neutrosophy

Neutrosophy is a recent philosophical discipline that focuses on the study of neutralities and their interactions with various concepts and entities, including ideas, propositions, theories, events, and entities, among others. Neutrosophy divides these concepts into three categories: A, anti-A, and neut-A, with the latter representing a neutral position between the first two. The term neutrosophy derives from the combination of "neutral" (from the Latin *neuter*, meaning "neutral") and "sophia" (from the Greek *sophia*, meaning "knowledge"). Its main theory posits that every idea tends to be balanced by ideas that are not A (not just anti-A, as Hegel suggested), ultimately achieving a state of equilibrium. Neutrosophy began to develop in 1995 [9].

Neutrosophy is used as a tool to analyze uncertainty and indeterminacy in various fields of knowledge. In the context of cardiovascular disease detection, neutrosophy has been applied to manage uncertainty. Specifically, [10] propose a technique that aids decision-making, assisting both patients and physicians in determining whether a patient has heart failure. Additionally, the technique enables users to handle the uncertainty arising from the imprecision and vagueness of the symmetric priority scales of various disease symptoms, allowing them to better understand the severity of the condition in their body.

2.3 Neutrosophic Control Charts.

Classical control charts, such as $\boldsymbol{X-R}$ o $\boldsymbol{X-S}$, charts, are highly effective for monitoring processes with controlled variability and well-defined data. They work very well if measurements are accurate and there is little doubt about the precision of the data. For this reason, traditional control charts can identify when a process is out of control. Additionally, they are advantageous because they effectively detect variations in stable and well-understood processes. [11] and [12].

However, when the data of variables related to cardiovascular problems contain uncertainty or indeterminacy, or if the data is contaminated with noise or unrecognized errors, the use of classical control charts is not recommended, as it may produce misleading results. On the other hand, integrating neutrosophic logic with control charts allows the model to capture not only the variability in the data but also the uncertainty and indeterminacy, instead of assuming that each piece of data is entirely true or false. This makes it a powerful tool for process monitoring in environments with uncertainty, such as healthcare. [13].

In [14], the authors propose a neutrosophic mean deviation (MD) control chart that can accommodate imprecise observations in the collected quality characteristic variables. The results indicated that the proposed MD chart design outperforms existing counterparts in terms of statistical power. Similarly, in [15], attribute control charts are presented for monitoring blood components under neutrosophic statistics. These are described as highly effective, appropriate, adaptable, and informative for the surveillance of blood components in uncertain environments, as evidenced in various applications. They also mention that the proposed control chart can be applied in medical sciences for monitoring various diseases.

3. Calculation of Neutrosophic Control Limits

The present research proposes a neutrosophic hybrid model to classify cardiac arrhythmias using parameters obtained from electrocardiogram (ECG) signals. The proposed model employs adjusted means and standard deviations for the neutrosophic components. Based on this, upper and lower limits can be calculated, considering the uncertainty and indeterminacy of the data, where μ_T is the mean adjusted by the value of T y σ_T is the standard deviation considering uncertainty. This is achieved using a linear interpretation formula such as:

$$\boldsymbol{Upper\ Control\ Limit\ (UCL)} = \mu_T + Z * \sigma_T$$
$$\boldsymbol{Lower\ Control\ Limit\ (LCL)} = \mu_T - Z * \sigma_T$$

The parameters obtained from the ECG, such as heart rate, P wave, QRS complex, and RR interval, are used to train the neutrosophic hybrid model. [16]

4. Neutrosophic Hybrid Model

Neutrosophy is a mathematical theory that allows processing information that is neither true, nor false, nor indeterminate. In the context of classifying the four types of cardiac arrhythmias, the uncertainty and ambiguity of the detected waves are studied using Hotelling's T^2 control charts and neutrosophic values to calculate the percentage of truth, uncertainty, and falsehood of their characteristics, as shown in (Table 1). [17]

The Neutrosophic Hybrid Model is an advanced approach that combines neutrosophic logic with other statistical, mathematical, or computational methods to address complex problems where uncertainty, indeterminacy, and vagueness are critical factors. This model is particularly useful in areas where data is incomplete, noisy, or ambiguous, such as medical analysis, artificial intelligence, and decision-making.

Table 1: Definition of Linguistic Terms

Linguistic Terms	SNVV
Extremely Important	(1.00, 0.00, 0.00)
Very Very Important	(0.90, 0.10, 0.10)
Very Important	(0.80, 0.15, 0.20)
Important	(0.70, 0.25, 0.30)
Moderately Important	(0.60, 0.35, 0.40)

Linguistic Terms	SNVV
Moderate	(0.50, 0.50, 0.50)
Moderately Insignificant	(0.40, 0.65, 0.60)
Insignificant	(0.30, 0.75, 0.70)
Very Insignificant	(0.20, 0.85, 0.80)
Very Very Insignificant	(0.10, 0.90, 0.90)
Extremely Insignificant	(0.00, 1.00, 1.00)

In a universe E, the truth membership function (Tx), indeterminacy (Ix) and falsehood (Fx) functions determine the neutrosophic set to which an element belongs in E. The notation of these functions allows the following equation to be used to describe the neutrosophic set to which it belongs.

$$\boldsymbol{X} = \frac{(\boldsymbol{T}(\boldsymbol{x}) + \boldsymbol{I}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x}))}{\mathbf{3}} \quad \textbf{(1)}$$

Donde:
$\boldsymbol{X}$: *The arrhythmia x belongs to the category of arrhythmias y representa y"*
$\boldsymbol{T}(\boldsymbol{x})$: *Represents "arrhythmia x belongs to the category of arrhythmias y"*
$\boldsymbol{F}(\boldsymbol{x})$: *Represents "Arrhythmia x does not belong to the category of arrhythmias y"*
$\boldsymbol{I}(\boldsymbol{x})$: *Represents "arrhythmia x is indeterminate with respect to the category of arrhythmias y"*

The values of the components of the membership function are associated with the linguistic characteristics (Table 1). Table 2 correspond cardiac parameters and values.

Characteristics	Value
Heart rate	0.6
QRS complex duration	0.8
Presence of P wave	0.2
Relationship between QRS complex and RR interval	0.9

Tabla 2: Cardiac Parameters and Values

To analyze the existence of out-of-control processes determined by neutrosophic linguistic characteristics, a matrix is generated with three variables, features that will allow the formula to be used:

$$\boldsymbol{MT^2} = \frac{\boldsymbol{ST^2}}{\boldsymbol{n}} \quad \textbf{(2)}$$

Where MT^2represents the mean T^2, ST^2 the sum T^2, and n is the number of T^2 observations. This formula calculates the arithmetic mean of the data in a set of T^2 observations. For the classification of cardiac arrhythmias, Hotelling's T^2 sample covariance matrix allows the detection of anomalies in electrocardiogram data by implementing the following formula.

$$\boldsymbol{S} = \frac{\mathbf{1}}{\boldsymbol{n-1}} * (\boldsymbol{X} - \bar{\boldsymbol{X}})' * (\boldsymbol{X} - \bar{\boldsymbol{X}}) \quad \textbf{(3)}$$

Where **S** represents the measure of the joint variability of the variables in the dataset to define the upper control limit T^2 using a test called **F** with a significance level of 5%. A neutrosophic composition matrix allows representing the composition of a set of elements and the value of the corresponding degree for each element, enabling classification as either standard or non-standard real subsets by applying the following:

$$\boldsymbol{VT(x,y)} = \sum (\boldsymbol{VT(x)} * \boldsymbol{CF(y)})$$
$$\boldsymbol{VF(x,y)} = \sum (\boldsymbol{VF(x)} * \boldsymbol{CT(y)})$$
$$\boldsymbol{VI(x,y)} = \sum (\boldsymbol{VI(x)} * \boldsymbol{CI(y)})$$

Where, for each degree of element **X**, the summation of the complement for the corresponding degree must be performed, and the summation is carried out over all elements Y in the combination that are not X. Subsequently, the results obtained from the neutrosophic composition matrix are graphed and analyzed, allowing a probabilistic evaluation of whether an arrhythmia belongs to a specific category, even in cases where the classification is unclear, ambiguous, or there is no presence of waves.

5. Results

To determine the accuracy in classifying cardiac arrhythmias, the experimental results showed that the neutrosophic hybrid model achieved an average classification accuracy of 82.99% for detecting four different types of cardiac arrhythmias. These results were comparable to the original values of each dataset, as shown in (Table 3). To evaluate the scalability of the neutrosophic hybrid model, the signal duration was increased from 3500 ms to 5000 ms, and the model's performance was assessed in terms of classification accuracy and processing time. The results demonstrated that the neutrosophic hybrid model was capable of handling ECG datasets with a higher number of signals without significantly reducing classification accuracy.

Table 3: Accuracy Results of the Hybrid Model

Id	Database	Arrhythmia	Records	Hybrid Model	Accuracy
1	MIT-BIH Atrial Fibrillation Database	Atrial fibrillation	14	14	100.00%
2	CU Ventricular Tachyarrhythmia Database	Ventricular tachycardia	18	13	72.22%
3	Norwegian Endurance Athlete ECG Database	Bradycardia	17	24	70.83%
4	MIT-BIH Supraventricular Arrhythmia Database	Supraventricular tachycardia	18	16	88.89%
	Total		***67***	***67***	**82.99%**

To formulate the mathematical model based on Table 3, an approach can be constructed to evaluate the overall accuracy of the hybrid model in classifying arrhythmias. This involves integrating the observations by arrhythmia type and calculating a general metric.

5.1 Mathematics Model

5.1.1 Define Variables:

- $\boldsymbol{n_i}$: Total number of records for the database i.
- $\boldsymbol{r_i}$: Number of correctly classified records for database i using the hybrid model.
- $\boldsymbol{p_i}$: Accuracy of the hybrid model for database i, calculated as:

$$\boldsymbol{p_i = \frac{r_i}{n_i}\, x\, 100} \tag{4}$$

5.1.2 Total Accuracy of the Hybrid Model:

Let $\boldsymbol{N}$ be the total number of records across all databases:

$$N = \sum_{i=1}^{k} n_i \tag{4.1}$$

Let $\boldsymbol{R}$ be the total number of correctly classified records across all databases.

$$R = \sum_{i=1}^{k} r_i \tag{4.2}$$

The total accuracy $\boldsymbol{P}$ of the hybrid model is calculated as:

$$P = \frac{R}{N}\, x\, 100 \tag{4.3}$$

5.1.3 Application with the Data:

For each database:

$$p_1 = \frac{14}{14}\, x\, 100 = 100\%$$

$$p_2 = \frac{13}{18}\, x\, 100 = 72.22\%$$

$$p_3 = \frac{17}{24}\, x\, 100 = 70.83\%$$

$$p_4 = \frac{16}{18}\, x\, 100 = 88.89\%$$

Total records($\boldsymbol{N}$) and correct records($\boldsymbol{R}$)

$$N = 14 + 18 + 24 + 18 = 74$$
$$R = 14 + 13 + 17 + 16 = 60$$

Total Accuracy($\boldsymbol{P}$):

$$P = \frac{60}{74}\, x\, 100 = 82.99\%$$

This model allows calculating the total accuracy of the hybrid model by integrating data from all databases to evaluate its performance in arrhythmia classification. Regarding its efficiency, the obtained result of 82.99% suggests that the model has a solid performance in classifying arrhythmias. This

indicates that the model can correctly handle the majority of cases, accurately identifying the corresponding type of arrhythmia.

Similarly, regarding the margin for improvement, it can be noted that 17.01% of the samples were not correctly classified, reflecting opportunities to optimize the model. Possible areas for improvement include adjustments to the neutrosophic parameters, higher quality training data, or the integration of additional features

Finally, we can conclude that this level of accuracy positions the hybrid model as a viable solution for classifying cardiac arrhythmias, especially in contexts where data uncertainty can hinder the use of traditional methods. However, it is important to continue refining and validating the model to achieve greater accuracy and minimize potential errors.

The robustness of the neutrosophic hybrid model was determined through the generation of neutrosophic triangular scales for each cardiac arrhythmia signal, as observed in (Algorithm_1 and Algorithm_2). Neutrosophic distances were generated, which allowed validating the percentage of divergence from each type of arrhythmia: Bradycardia (Tables 5 and 6), Ventricular Tachycardia (Tables 7 and 8), Supraventricular Tachycardia (Tables 9 and 10), and Atrial Fibrillation (Tables 11 and 12).

5.1.4 Definition of Variables

$\boldsymbol{T_i} =$ Truth degree of the signal i (Neutrosophic component).
$\boldsymbol{I_i} =$ Indeterminacy degree of the signal i
$\boldsymbol{F_i} =$ Falsity degree of the signal i
$\boldsymbol{D_i} =$ Neutrosophic distance of signal **i** to the reference neutrosophic scale

Algorithm 1 classifies data using a covariance measure. For each sample, it evaluates whether the covariance measure is less than 0.25, in which case the label "Ventricular" is assigned to sample iii. If cov_matrix is greater than or equal to 0.25 but less than 0.5, the label "Supraventricular" is assigned. If cov_matrix is greater than or equal to 0.5 but less than 0.75, the label "Bradycardia" is assigned. If cov_matrix is greater than or equal to 0.75, the label "Atrial Fibrillation" is assigned. In other words, the algorithm evaluates each sample iii and determines which class it belongs to base on the comatrix value. The steps are as follows:

The values obtained from (Algorithm 1) are related to heart conditions and are used in the context of classifying cardiac arrhythmias, which are abnormal heart rhythms. In other words, the algorithm classifies cardiac signals into one of four arrhythmia categories based on covariance values and performs a normalization preprocessing step. Although it is a simple approach, it can be useful as a first step in a more advanced classification workflow or as a quick tool for initial evaluations.

Algorithm 1: Training Algorithm for Cardiac Classification

```
# Loop through the sample set
for i in range(n_samples):
    # Classification based on the covariance matrix (cov_matrix)
    if cov_matrix < 0.25:
        y[i] = 1 # Class 1: Ventricular Rhythm
    elif cov_matrix < 0.5:
        y[i] = 2 # Class 2: Supraventricular Rhythm
    elif cov_matrix < 0.75:
        y[i] = 3 # Class 3: Bradycardia
    else:
        y[i] = 4 # Class 4: Atrial Fibrillation
# Data normalization (X)
X_norm = (X - np.mean (X, axis=0)) / np.std (X, axis=0)
```

a. Data normalization

After classifying all the samples, the algorithm normalizes the data in **X** so that they have a mean of zero and a standard deviation of one, resulting in the neutrosophic reference values for each cardiac arrhythmia (Table 4),

$$\mathbf{Xnorm[i,j]} = \frac{\mathbf{X[i,j]} - \boldsymbol{\mu}\mathbf{j}}{\boldsymbol{\sigma}\mathbf{j}} \quad (5)$$

Donde:
X[**i**, **j**]: Value of feature **j** for sample **i**.
μj: Mean of feature **j** for all samples.
σj: Standard deviation of feature **j** for all samples.
Each feature has a mean of 0 and a standard deviation of 1.

b. Sample classification

Each sample is classified into one of the arrhythmia categories based on the value of the covariance matrix (cov_matrix):

$$\mathrm{y[i]} = \begin{cases} 1;\ \text{si cov_matrix} < 0.25 (\text{Ventricular Tachycardia}) \\ 2;\ \text{si } 0.25 \leq \text{cov_matrix} < 0.5 (\text{Supraventricular Tachycardia}) \\ 3;\ \text{si } 0.5 \leq \text{cov_matrix} < 0.75 (\text{Atrial Fibrillation}) \\ 4;\ \text{si } \mathrm{cov_{matrix}} > 0.75 (\text{Bradycardia}) \end{cases}$$

c. Calculation of Neutrosophic Reference Values (NRV)

For each arrhythmia category ($\mathbf{k} \in \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$ The neutrosophic reference values($(\boldsymbol{T_k}, \boldsymbol{I_k}, \boldsymbol{F_k})$ are calculated as the means of the normalized values of the corresponding features.

Truth Degree ($\boldsymbol{T_k}$): Represents the mean of the features that indicate a direct relationship with the category:

$$\mathbf{T_k} = \frac{1}{n_k} \sum_{i \in k}^{n} X_{norm}[\![\text{i, Truth features}]\!]$$

Indeterminacy Degree ($\boldsymbol{I_k}$): Represents the mean of the features that introduce ambiguity or variability in the category:

$$\boldsymbol{I_k} = \frac{1}{n_k} \sum_{i \in k}^{n} X_{norm}[\![\text{i, Indeterminacy features}]\!]$$

Falsity Degree ($\boldsymbol{F_k}$)**:** Represents the mean of the features that contradict the category**:**

$$\boldsymbol{F_k} = \frac{1}{n_k} \sum_{i \in k}^{n} X_{norm}[\![\text{i, Falsity features}]\!]$$

Donde:
$\mathbf{n_k}$: Number of samples in category k.
$\mathbf{i} \in \mathbf{k}$: Samples classified in the category k.

d. Results for the Categories.

Table 4: Neutrosophic Reference Values

Cardiac arrhythmia	NRV
Ventricular tachycardia	(0.4, 0.6, 0)
Supraventricular tachycardia	(0.3, 0.3, 0.4)

Cardiac arrhythmia	NRV
Atrial fibrillation	(0.3, 0.2, 0.5)
Bradycardia	(0.2, 0.8, 0)

These values reflect the neutrosophic mean calculated for each category after normalization and classification.

e. Interpretation of the Categories According to the Table

Taquicardia Ventricular ($T = 0.4, I = 0.6, F = 0.0$):

- **High Truth (T=0.4):** Indicates a reliable classification based on specific features of the arrhythmia.
- **Moderate Indeterminacy (I=0.6):** The category has some ambiguity, which might be related to features shared with other arrhythmias.
- **No Falsity (F=0.0):** The features do not contradict the classification

Supraventricular tachycardia ($T = 0.3, I = 0.3, F = 0.4$):

- **Moderate Truth** ($T = 0.3$): The classification is supported but not as strong as in other categories.
- **Low Indeterminacy** ($I = 0.3$): The data is clearer and more consistent.
- **High Falsity** ($F = 0.4$): There are significant contradictions in the features, suggesting possible errors in classification or conflicting data.

Atrial fibrillation ($\boldsymbol{T = 0.3, I = 0.2, F = 0.5}$):

- **Moderate Truth (T=0.3):** Similar to supraventricular, the data partially support the classification.
- **Low Indeterminacy (I=0.2):** The data is quite clear and well-defined.
- **High Falsity (F=0.5):** Suggests a high degree of conflict, possibly due to similarities with other categories.

Bradycardia ($\boldsymbol{T = 0.2, I = 0.8, F = 0.0}$):

- **Low Truth (T=0.2):** The specific features of bradycardia are less defined compared to other arrhythmias.
- **High Indeterminacy (I=0.8):** The data shows significant variability or ambiguity.
- **No Falsity (F=0.0):** There are no clear contradictions in the data, suggesting that the conflicts are due to lack of definition rather than errors.

Robustness of the Hybrid Model

- This NRV-based approach demonstrates that the hybrid model is capable of handling ambiguous and conflicting data by representing the characteristics of arrhythmias in terms of truth, indeterminacy, and falsity.
- The differences in $\boldsymbol{T}, \boldsymbol{I}, \boldsymbol{F}$ between categories can help to:

- Identify areas where the data is inconsistent.
- Adjust the model to improve its accuracy.
- Prioritize key features in future optimizations.

In conclusion, we can state that the neutrosophic model based on normalization and $\boldsymbol{NRV}$ provides a more detailed and realistic representation of cardiac arrhythmias, addressing uncertainty and ambiguity in medical data. This is crucial for improving the accuracy and reliability of automated diagnoses and supporting clinical decisions with more interpretable data

On the other hand, (Algorithm 2) creates a transition matrix used in the analysis of discrete time series, particularly in the analysis of electrocardiograms (ECGs). The transition matrix represents the

probability of transitioning from one state to another in a discrete time series. In this case, the matrix represents the transition probabilities between different types of cardiac complexes in an ECG time series. This matrix has four rows (one for each type of cardiac arrhythmia) and three columns (one for each possible subsequent type of cardiac arrhythmia).

Algorithm 2: ENT Algorithm

```
# Creation of the neutrosophic matrix
# Each row corresponds to a specific cardiac signal feature
# Columns represent truth, indeterminacy, and falsehood values
neutro_matrix = np.array([
   [1 - fc, 0, fc],      # Row 1: Feature-based on 'fc'
   [1 - qrs, qrs, 0], # Row 2: Feature-based on 'qrs'
   [p, 1 - p, 0],                # Row 3: Feature-based on 'p'
   [1 - qrs_rr, qrs_rr, 0]       # Row 4: Feature-based on 'qrs_rr'
])
# Explanation:
# The matrix stores neutrosophic values: [truth, indeterminacy, falsehood].
# For each feature, the truth, indeterminacy, and falsehood are computed.
```

This algorithm performs a classification of cardiac arrhythmias by calculating the Euclidean distance between the neutrosophic matrix obtained from Algorithm 2.

General Mathematical Model

Neutrosophic Matrix:

$$\mathbf{neutro_{matrix}} = \begin{bmatrix} \mathbf{1-fc} & \mathbf{0} & \mathbf{fc} \\ \mathbf{1-qrs} & \mathbf{qrs} & \mathbf{0} \\ \boldsymbol{p} & \mathbf{1-}\boldsymbol{p} & \mathbf{0} \\ \mathbf{1-}\boldsymbol{qrs_rr} & \boldsymbol{qrs_rr} & \mathbf{0} \end{bmatrix} \quad (6)$$

Euclidean Distance: For any pair of states $\boldsymbol{i}$ y $\boldsymbol{j}$:

$$\mathbf{D}_{ij} = \sqrt{\sum_{k=1}^{3} (\boldsymbol{neutro_matrix}[i,k] - \boldsymbol{neutro_matrix}[j,k])^2} \quad (7)$$

This model allows calculating the distance between different states in a discrete ECG time series, representing how much one type of cardiac complex differs from another in terms of neutrosophic dimensions (truth, indeterminacy, and falsity). Higher values of $\boldsymbol{D_{ij}}$ indicate greater dissimilarity between states, which can aid in analyzing transitions and patterns in cardiac signals.

The elements of the matrix represent the transition probabilities from one type of cardiac arrhythmia to another. For an ECG signal with an arrhythmia of the bradycardia type, its parameters are: $\boldsymbol{fc = 0.3}$, $\boldsymbol{qrs = 0.7}$, $\boldsymbol{p = 0}$, $\boldsymbol{qrs_rr = 1}$ resulting in the values shown in Table 5. For ventricular tachycardia, its parameters are $\boldsymbol{fc = 0.7, qrs = 0.3, p = 0.7, qrs_rr = 1}$ resulting in the values shown in Table 7, for supraventricular tachycardia its parameters are: $\boldsymbol{fc = 0.8, qrs = 0.5, p = 0.3, qrs_rr = 0.3}$ resulting in the values shown in Table 9 and finally for atrial fibrillation its parameters are: $\boldsymbol{fc = 1}$, $\boldsymbol{qrs = 0, p = 1, qrs_rr = 0.2}$ resulting in the values shown in Table 11.

In Table 5 we can identify the three different measures (truth, uncertainty, and falsehood) for each of the three possible states of the variable (present, uncertain and absent). Where, the first row of values shows has a truth measure of 0.4, suggesting that the patient is likely to have bradycardia. The uncertainty measure is 0, indicating that there is no uncertainty in the statement, and the falsehood measure is 0.6, suggesting that it is unlikely that the patient does not have bradycardia.

Table 5: Neutrosophic scale pertaining to Bradycardia

Neutrosophic Triangular Scale		
True	**Uncertainty**	**Falsehood**
0.4	0	0.6
0.2	0.8	0
0.2	0.8	0
0.1	0.9	0

Table 6: Neutrosophic distances pertaining to Bradycardia

Neutrosophic distances			
Ventricular	**Supraventricular**	**Fibrillation**	**Bradycardia**
1.178	1.241	1.442	1.037

The results obtained in Algorithm 3 show that the neutrosophic matrix is more like the reference matrices for Bradycardia, as its Neutrosophic distance is equal to 1.037. The longest distance is for Fibrillation, suggesting that the neutrosophic matrix is less like the Fibrillation reference matrix.

Table 7: Neutrosophic scale for ventricular tachycardia

Neutrosophic Triangular Scale		
True	**Uncertainty**	**Falsehood**
0.1	0	0.9
0.1	0.9	0
0.5	0.5	0
0.1	0.9	0

In Table 7 we can identify the three different measures (truth, uncertainty, and falsehood) for each of the three possible states of the variable (present, uncertain and absent). Where, the third row of values (0.5, 0.5, 0) indicates a balanced measure of truth and uncertainty, suggesting that we are not sure whether the patient has ventricular tachycardia. The falsehood measure is 0, suggesting that it is unlikely that the patient does not have ventricular tachycardia.

Table 8: Neutrosophic distances pertaining to ventricular tachycardia

Neutrosophic distances			
Ventricular	**Supraventricular**	**Fibrillation**	**Bradycardia**
1.281	1.319	1.476	1.396

The results obtained in Algorithm 3 show that the neutrosophic matrix is more like the reference matrices for Ventricular Tachycardia, as its Neutrosophic distance is equal to 1.281. The longest distance is for Fibrillation, suggesting that the neutrosophic matrix is less like the Fibrillation reference matrix.

Table 9: Neutrosophic scale pertaining to supraventricular tachycardia

Neutrosophic Triangular Scale		
True	**Uncertainty**	**Falsehood**
0.1	0	0.9

Neutrosophic Triangular Scale		
True	**Uncertainty**	**Falsehood**
0	1	0
0.8	0.2	0
0.8	0.2	0

In Table 9 we can identify the three different measures (truth, uncertainty, and falsity) for each of the three possible states of the variable (present, uncertain and absent). Where, the third and fourth sets of values, the measure of belief in truth is high (0.8) and the measure of falsity is low (0), suggesting that the statement is believed to be probably true. However, there is also some uncertainty (0.2) associated with this belief.

Table 10: Neutrosophic distances pertaining to supraventricular tachycardia

Neutrosophic distances			
Ventricular	**Supraventricular**	**Fibrillation**	**Bradycardia**
1.49	1.2	1.49	1.726

The results obtained in Algorithm 3 show that the neutrosophic matrix is more like the reference matrices for Supraventricular Tachycardia, since its Neutrosophic distance is equal to 1.2. The longest distance is for Bradycardia, suggesting that the neutrosophic matrix is less like the Bradycardia reference matrix.

Table 11: Neutrosophic scale pertaining to atrial fibrillation

Neutrosophic Triangular Scale		
True	**Uncertainty**	**Falsehood**
0.1	0	0.9
1	0	0
1	0	0
1	0	0

In Table 11 we can identify the three different measures (truth, uncertainty, and falsehood) for each of the three possible states of the variable (present, uncertain and absent). Where, the fourth set of values, the measure of belief in truth is high (1), while the measure of falsity is low (0). This could be interpreted as an opinion that the statement is true. There is no uncertainty associated with this statement.

Table 12: Neutrosophic distances belonging to atrial fibrillation

Neutrosophic distances			
Ventricular	**Supraventricular**	**Fibrillation**	**Bradycardia**
1.849	1.612	1.606	2.302

The results obtained in Algorithm 3 show that the Neutrosophic matrix is more like the reference matrices for Arterial Fibrillation as its Neutrosophic distance is equal to 1.606. The longest distance is

for Bradycardia, suggesting that the Neutrosophic matrix is less similar to the reference matrix for Bradycardia.

To evaluate how interpretable the results obtained from the Neutrosophic hybrid model are, the visualization of the Neutrosophic distance process was used to verify its behavior for each type of cardiac arrhythmia (Figure 2 and Figure 3) using the Neutrosophic triangular function where the sets of classification rules and the most important features for the classification of cardiac arrhythmias were shown to be consistent with basic clinical knowledge.

Algorithm 3: Neutrosophic Rule Generation Algorithm

```
# Generate a sawtooth wave as the base for the triangular membership functions
tri = sawtooth(x * np.pi, 0.5)
# Calculate Euclidean distances for Ventricular classification
ventricular_dist = np.linalg.norm(neutro_matrix - ventricular_ref, axis=1)
tri_ventricular = np.zeros_like(tri)  # Initialize triangular function for Ventricular
for d in ventricular_dist:
    # Update triangular function with Ventricular distances
    tri_ventricular += np.maximum(0, 1 - 2 * np.abs(x - d))
# Calculate Euclidean distances for Supraventricular classification
supraventricular_dist = np.linalg.norm(neutro_matrix - supraventricular_ref, axis=1)
tri_supraventricular = np.zeros_like(tri)  # Initialize triangular function for Supraventricular
for d in supraventricular_dist:
    # Update triangular function with Supraventricular distances
    tri_supraventricular += np.maximum(0, 1 - 2 * np.abs(x - d))
# Calculate Euclidean distances for Bradycardia classification
bradycardia_dist = np.linalg.norm(neutro_matrix - bradycardia_ref, axis=1)
tri_bradycardia = np.zeros_like(tri)  # Initialize triangular function for Bradycardia
for d in bradycardia_dist:
    # Update triangular function with Bradycardia distances
    tri_bradycardia += np.maximum(0, 1 - 2 * np.abs(x - d))
# Calculate Euclidean distances for Atrial Fibrillation classification
fibrillation_dist = np. linalg.norm(neutro_matrix - fibrillation_ref, axis=1)
tri_fibrillation = np. zeros_like(tri)  # Initialize triangular function for Atrial Fibrillation
for d in fibrillation_dist:
    # Update triangular function with Atrial Fibrillation distances
    tri_fibrillation += np.maximum(0, 1 - 2 * np.abs(x - d))
```

Algorithm 3: Neutrosophic Rule Generation Algorithm generates neutrosophic rules to classify different types of cardiac arrhythmias (ventricular, supraventricular, bradycardia and fibrillation). This is achieved by calculating Euclidean distances from a Neutrosophic matrix to the corresponding references and constructing triangular membership functions for each type of arrhythmia and for each distance in the reference matrix. The construction of the triangular signal is performed by calculating the triangular wave function with the time variable x multiplied by pi and a factor of 0.5, at the end, the resulting triangular signals are used to determine the type of arrhythmia that best correlates with the input signal which returns the graph to review the behavior taken by each rule.

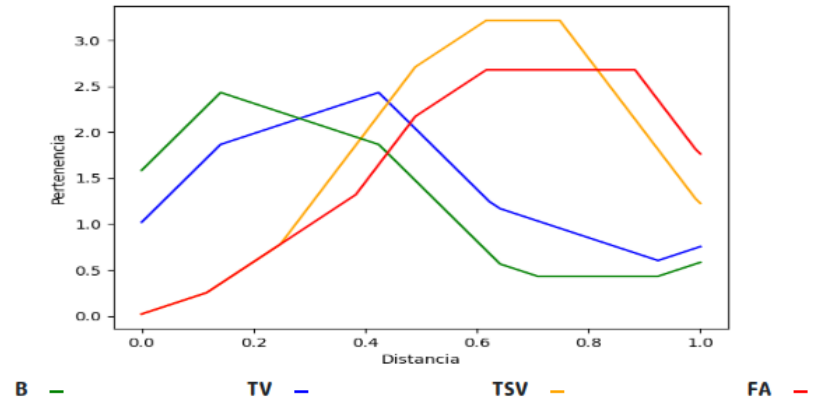

Figure 2: Behavior of neutrosophic distances for each type of cardiac arrhythmia

The results of the application of Algorithm 3 show the behavior of each neutrosophic rule generated from the ECG signal for each type of arrhythmia where B represents bradycardia, VT represents ventricular tachycardia, SVT represents supraventricular tachycardia and AF represents atrial fibrillation.

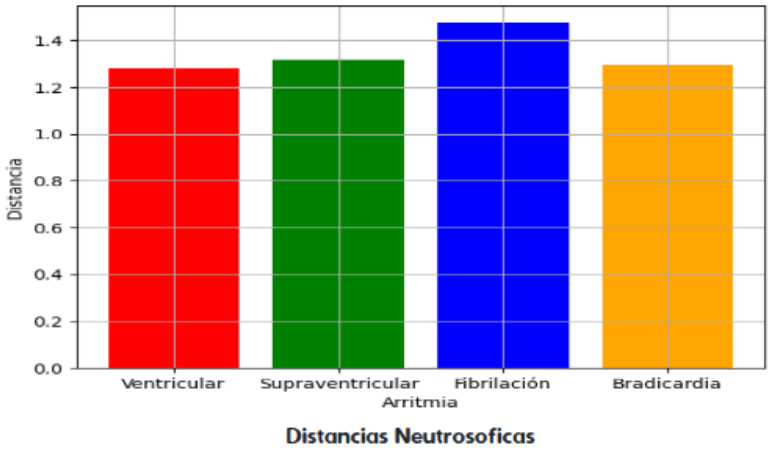

Figure 3: Results of the neutrosophic distances

Figure 3 is another way of representing the values obtained from the neutrosophic rules (Algorithm 3) and their neutrosophic distances where it is possible to interpret for each type of arrhythmia the percentage of remoteness or distance they have taking as a result the minimum value, that is, the closest as the type of arrhythmia classified.

6. Conclusión

This research work was carried out with the purpose of verifying the accuracy of the application of a hybrid model between neutrosophy and T^2 Hotelling control charts to detect and classify cardiac arrhythmias, concluding that the model has a reliable accuracy and is completely scalable by means of its important components, such as the extraction and selection of features that in fusion of the T^2 Hotelling control charts, allow an accurate classification of cardiac arrhythmias even in situations of high variability and noise in the ECG signals..

Neutrosophy offers a unique and effective way to address uncertainty and errors in medical data. While process control charts allow you to monitor process variables, classify and identify unwanted deviations. Successful implementation of such a model requires a careful approach to data selection and

preprocessing, proper identification and measurement of variables, and the establishment of appropriate thresholds and limits for control charts.
This model is also capable of handling large ECG datasets without decreasing classification accuracy, making it scalable and suitable for use in clinical settings. Interpretation of results is also accessible to the end user, increasing confidence in cardiac arrhythmia classification.

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Mixed Graph in Fuzzy, Neutrosophic, and Plithogenic Graphs

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Abstract. Graph theory examines networks consisting of nodes (vertices) and the connections (edges) between them. Mixed graphs, which combine both undirected and directed edges, provide a versatile framework for representing relationships with symmetric and asymmetric connections across various systems. In this work, we introduce and analyze the concept of mixed graphs within the contexts of Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. By exploring their properties, interrelations, and potential applications, we aim to advance the understanding of uncertain graphs.

Keywords: Mixed Graph, Neutrosophic graph, Fuzzy graph, Plithogenic Graph

1. Introduction

1.1. *Uncertain Set and Graph Theory*

Graph theory provides a fundamental framework for analyzing networks, composed of nodes (vertices) and their connections (edges). It offers insights into the structure, connectivity, and properties of diverse networks [11]. Among the various types of graphs, this paper focuses on mixed graphs, which integrate both undirected and directed edges. This integration allows the representation of both symmetric and asymmetric relationships, making mixed graphs a versatile tool for modeling complex systems. Consequently, mixed graphs have garnered significant attention in both theoretical and practical research [4, 38, 42, 45, 46, 67].

To address uncertainty in real-world scenarios, concepts like fuzzy sets have been developed [70]. A fuzzy set quantifies the degree to which an element belongs to a set using a membership function. Neutrosophic sets [50–52] further extend this idea by incorporating degrees of truth, indeterminacy, and falsity, enabling more nuanced modeling of uncertain phenomena. Below are intuitive examples illustrating these concepts in practical scenarios. Note that the following are merely intuitive examples.

Example 1.1 (Fuzzy Set)**.** Consider a scenario where A represents the set of "short buildings" and B represents the set of "tall buildings."

Let x be a building with a height of 30 meters. Its membership degree in each set might be defined as follows:

$$\mu_A(x) = 0.6 \quad \text{(short buildings)}, \quad \mu_B(x) = 0.4 \quad \text{(tall buildings)}.$$

This representation reflects that the building x exhibits characteristics of both short and tall buildings, quantifying the overlap between these categories.

Example 1.2 (Neutrosophic Set)**.** Consider the classification of weather as "Sunny," "Cloudy," or "Rainy." The weather state might not fully belong to any single category, making it suitable for representation using a Neutrosophic Set.

Let the weather condition x be characterized as follows:

- *Truth Membership Degree ($T(x)$):* The degree to which the weather is "Sunny," $T(x) = 0.7$.
- *Indeterminacy Membership Degree ($I(x)$):* The degree of uncertainty about the weather, $I(x) = 0.2$.
- *Falsity Membership Degree ($F(x)$):* The degree to which the weather is "Rainy," $F(x) = 0.1$.

Thus, the weather condition can be represented as:

$$\text{Weather} = \{(\text{Sunny}, T(x) = 0.7), (\text{Uncertain}, I(x) = 0.2), (\text{Rainy}, F(x) = 0.1)\}.$$

This paper investigates uncertain graph models, such as Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs, designed to address uncertainties in various applications. These models, collectively referred to as uncertain graphs, extend classical graph theory by incorporating different levels and types of uncertainty [3,5,6,13,16–21,23,24,44,53,57,59,60].

For example, a fuzzy graph assigns membership degrees in $[0, 1]$ to vertices and edges, representing uncertainties in connections. Neutrosophic graphs go further, assigning truth, indeterminacy, and falsity membership degrees to vertices and edges, thus capturing more complex uncertainties. These models are especially valuable in decision-making scenarios(ex. [3]). Below are examples illustrating the application of these concepts in real-world scenarios. Note that the following are merely intuitive examples.

Example 1.3 (Fuzzy Graph)**.** Consider a scenario where A represents the set of "short buildings" and B represents the set of "tall buildings."

Let x be a building with a height of 30 meters. Its membership degree in each set might be defined as follows:

$$\mu_A(x) = 0.6 \quad \text{(short buildings)}, \quad \mu_B(x) = 0.4 \quad \text{(tall buildings)}.$$

This representation reflects that the building x exhibits characteristics of both short and tall buildings, quantifying the overlap between these categories.

Now, consider a fuzzy graph $G = (V, E)$, where $V = \{x_1, x_2, x_3\}$ represents buildings and E represents connections based on architectural similarity. For instance, the edge (x_1, x_2) might have a membership degree $\mu((x_1, x_2)) = 0.8$, indicating a high degree of similarity in design, materials, or function between the two buildings. This could represent relationships such as "similar construction style" or "shared purpose."

Example 1.4 (Neutrosophic Graph)**.** Consider the classification of weather as "Sunny," "Cloudy," or "Rainy." The state of the weather might not fully belong to any single category, making it suitable for representation using a Neutrosophic Set.

Let the weather condition x be characterized as follows:

- *Truth Membership Degree ($T(x)$):* The degree to which the weather is "Sunny," $T(x) = 0.7$.
- *Indeterminacy Membership Degree ($I(x)$):* The degree of uncertainty about the weather, $I(x) = 0.2$.
- *Falsity Membership Degree ($F(x)$):* The degree to which the weather is "Rainy," $F(x) = 0.1$.

In the context of a neutrosophic graph $G = (V, E)$, let $V = \{x_1, x_2, x_3\}$, where each vertex represents a weather condition. Edges E represent relationships such as transitions between weather states. For instance, (x_1, x_2) might have the following degrees:

$$T((x_1, x_2)) = 0.5, \quad I((x_1, x_2)) = 0.3, \quad F((x_1, x_2)) = 0.2,$$

indicating the truth, indeterminacy, and falsity degrees of the transition between x_1 (Sunny) and x_2 (Cloudy). Such edges could represent the likelihood of weather transitions based on historical data or meteorological patterns.

1.2. *Our Contribution*

Research on uncertain sets and graphs, as illustrated above, is crucial for modeling real-world phenomena. However, their mathematical structures remain partially unexplored. In this work, we propose and analyze the concept of mixed graphs within the frameworks of Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. We explore their properties, relationships, and potential applications, contributing to the broader understanding of uncertain graphs. Notably, the Turiyam Neutrosophic Set is a specific case of the Quadruple Neutrosophic Set, obtained by replacing the "Contradiction" component with "Liberal" [49].

2. Preliminaries and definitions

In this section, we present a brief overview of the definitions and notations used throughout this paper.

2.1. *Basic Graph Concepts*

We outline some basic graph concepts here. Note that this is not an exhaustive list. For more foundational graph concepts and notations, please refer to [10, 11, 29, 69].

Definition 2.1 (Graph)**.** [11] A graph G is a mathematical structure consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ that connect pairs of vertices, representing relationships or connections between them. Formally, a graph is defined as $G = (V, E)$, where V is the vertex set and E is the edge set.

Definition 2.2 (Subgraph)**.** [11] A graph $H = (V_H, E_H)$ is called a *subgraph* of a graph $G = (V, E)$ if:

- $V_H \subseteq V$, i.e., the vertex set of H is a subset of the vertex set of G,
- $E_H \subseteq E$, i.e., the edge set of H is a subset of the edge set of G,
- For each edge $e \in E_H$, if $e = \{u, v\}$, then $u, v \in V_H$.

In other words, a subgraph H of G consists of a subset of the vertices and edges of G, with the condition that all edges in E_H connect vertices in V_H.

Definition 2.3 (Degree)**.** [11] Let $G = (V, E)$ be a graph. The *degree* of a vertex $v \in V$, denoted $\deg(v)$, is the number of edges incident to v. Formally, for undirected graphs:

$$\deg(v) = |\{e \in E \mid v \in e\}|.$$

In the case of directed graphs, the *in-degree* $\deg^-(v)$ is the number of edges directed into v, and the *out-degree* $\deg^+(v)$ is the number of edges directed out of v.

Definition 2.4 (Induced Subgraph)**.** (cf. [8]) Given a graph $G = (V, E)$ and a subset of vertices $S \subseteq V$, the *induced subgraph* of G on S, denoted by $G[S]$, is defined as:

$$G[S] = (S, E_S),$$

where $E_S = \{\{u, v\} \in E \mid u, v \in S\}$.

The induced subgraph $G[S]$ satisfies the following properties:

- The vertex set of $G[S]$ is exactly S.
- The edge set E_S contains all edges in G whose endpoints are both in S.

Definition 2.5 (Directed Graph)**.** [11] A *directed graph* $G = (V, A)$ is a mathematical structure consisting of:

- V: A set of vertices (or nodes),
- $A \subseteq V \times V$: A set of directed edges (or arcs), where each arc $(u, v) \in A$ has an orientation from vertex u (the tail) to vertex v (the head).

2.2. *Fuzzy, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs*

In this subsection, we examine Fuzzy Graphs, Neutrosophic Graphs, and Turiyam Neutrosophic Graphs. Fuzzy graphs are frequently discussed in comparison with crisp graphs, which represent the classical form of graphs [35, 44].

Definition 2.6. (cf. [35, 44]) A *crisp graph* is an ordered pair $G = (V, E)$, where:

- V is a finite, non-empty set of vertices.
- $E \subseteq V \times V$ is a set of edges, where each edge is an unordered pair of distinct vertices.

Formally, for any edge $(u, v) \in E$, the following holds:

$$(u, v) \in E \iff u \neq v \quad \text{and} \quad u, v \in V$$

This implies that there are no loops (i.e., no edges of the form (v, v)) and edges represent binary relationships between distinct vertices.

Taking the above into consideration, we define Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Turiyam Neutrosophic Graphs as follows. Please note that the definitions have been consolidated for simplicity.

Definition 2.7 (Unified Graphs Framework: Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Turiyam Neutrosophic Graphs)**.** (cf. [16]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, and falsity.

(1) *Fuzzy Graph [43, 44, 68]:*
   - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0, 1]$, representing the degree of participation of v in the fuzzy graph.
   - Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0, 1]$, representing the strength of the connection between u and v.

(2) *Intuitionistic Fuzzy Graph (IFG) [1, 65, 71]:*
   - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0, 1]$ (degree of membership) and $v_A(v) \in [0, 1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
   - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0, 1]$ (degree of membership) and $v_B(u, v) \in [0, 1]$ (degree of non-membership), such that $\mu_B(u, v) + v_B(u, v) \leq 1$.

(3) *Neutrosophic Graph [2, 7, 14, 30, 33, 57, 59]:*

- Each vertex $v \in V$ is assigned a triple $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where:
  - $\sigma_T(v) \in [0, 1]$ is the truth-membership degree,
  - $\sigma_I(v) \in [0, 1]$ is the indeterminacy-membership degree,
  - $\sigma_F(v) \in [0, 1]$ is the falsity-membership degree,
  - $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
- Each edge $e = (u, v) \in E$ is assigned a triple $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$, representing the truth, indeterminacy, and falsity degrees for the connection between u and v.

(4) *Turiyam Neutrosophic Graph [16, 25–27, 48]:*

- Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where:
  - $t(v) \in [0, 1]$ is the truth value,
  - $iv(v) \in [0, 1]$ is the indeterminacy value,
  - $fv(v) \in [0, 1]$ is the falsity value,
  - $lv(v) \in [0, 1]$ is the liberal state value,
  - $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
- Each edge $e = (u, v) \in E$ is similarly assigned a quadruple representing the same parameters for the connection between u and v.

2.3. *Plithogenic Graphs*

Plithogenic Graphs have recently emerged as a generalization of Fuzzy Graphs and Turiyam Neutrosophic Graphs, extending the concept to represent Plithogenic Sets [55]. These graphs are a focus of ongoing research and development [16, 34, 47, 62, 63]. The formal definition is presented below.

Definition 2.8. [63] Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A *Plithogenic Graph* PG is defined as:

$$PG = (PM, PN)$$

where:

(1) *Plithogenic Vertex Set* $PM = (M, l, Ml, adf, aCf)$:

- $M \subseteq V$ is the set of vertices.
- l is an attribute associated with the vertices.
- Ml is the range of possible attribute values.
- $adf : M \times Ml \to [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)* for vertices.
- $aCf : Ml \times Ml \to [0, 1]^t$ is the *Degree of Contradiction Function (DCF)* for vertices.

(2) *Plithogenic Edge Set* $PN = (N, m, Nm, bdf, bCf)$:

- $N \subseteq E$ is the set of edges.
- m is an attribute associated with the edges.
- Nm is the range of possible attribute values.
- $bdf : N \times Nm \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* for edges.
- $bCf : Nm \times Nm \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* for edges.

The Plithogenic Graph PG must satisfy the following conditions:

(1) *Edge Appurtenance Constraint*: For all $(x,a),(y,b) \in M \times Ml$:

$$bdf\left((xy),(a,b)\right) \leq \min\{adf(x,a), adf(y,b)\}$$

where $xy \in N$ is an edge between vertices x and y, and $(a,b) \in Nm \times Nm$ are the corresponding attribute values.

(2) *Contradiction Function Constraint*: For all $(a,b),(c,d) \in Nm \times Nm$:

$$bCf\left((a,b),(c,d)\right) \leq \min\{aCf(a,c), aCf(b,d)\}$$

(3) *Reflexivity and Symmetry of Contradiction Functions*:

$$\begin{aligned} aCf(a,a) &= 0, && \forall a \in Ml \\ aCf(a,b) &= aCf(b,a), && \forall a,b \in Ml \\ bCf(a,a) &= 0, && \forall a \in Nm \\ bCf(a,b) &= bCf(b,a), && \forall a,b \in Nm \end{aligned}$$

Example 2.9. (cf. [14]) The following examples are provided.

- When $s = t = 1$, PG is called a *Plithogenic Fuzzy Graph*.
- When $s = 2, t = 1$, PG is called a *Plithogenic Intuitionistic Fuzzy Graph*.
- When $s = 3, t = 1$, PG is called a *Plithogenic Neutrosophic Graph*.
- When $s = 4, t = 1$, PG is called a *Plithogenic Turiyam Neutrosophic Graph*.

3. Result in this paper

In this section, we present the results of this paper.

3.1. *Uncertain Mixed Graph*

We examine the concept of an Uncertain Mixed Graph, which integrates the ideas of Uncertain Graphs and Mixed Graphs. The definitions, including related concepts, are provided below.

Definition 3.1 (Mixed Graph)**.** [42,45,46,67] A *mixed graph* $G = (V, E, A)$ is a mathematical structure in graph theory consisting of three components:

- V: A set of vertices (or nodes),
- E: A set of undirected edges, where an edge $uv \in E$ (or equivalently, $[u, v]$) connects two vertices u and v without any orientation,
- A: A set of directed edges, also called arcs, where an arc $\overrightarrow{uv} \in A$ (or equivalently, (u, v)) connects two vertices u and v with a specific orientation, making u the tail and v the head.

Properties of Mixed Graphs:

(1) *Vertices and Edges:* Each vertex $u \in V$ can be connected to another vertex $v \in V$ either by an undirected edge or a directed arc. There can be multiple edges between the same pair of vertices, but for simplicity, loops (edges/arcs that start and end at the same vertex) are not considered in this definition.

(2) *Walks in Mixed Graphs:* A *walk* in a mixed graph is a sequence of vertices and edges/arcs denoted as

$$v_0, c_1, v_1, c_2, v_2, \ldots, c_k, v_k,$$

where for each index i, c_i is either an undirected edge $v_i v_{i+1}$ or a directed arc $\overrightarrow{v_i v_{i+1}}$.

(3) *Paths and Cycles:* A *path* is a walk in which no vertices, edges, or arcs are repeated, except possibly the starting and ending vertices. A *closed path* (where the start and end vertices are the same) is called a *cycle*. A mixed graph is called *acyclic* if it does not contain any cycles.

In summary, a mixed graph generalizes both undirected and directed graphs by allowing the presence of both undirected edges and directed arcs, making it a versatile model for representing complex networks with mixed types of relationships.

Proposition 3.2. *An undirected graph is a special case of a mixed graph where the set of directed edges A is empty.*

Proof. Let $G = (V, E)$ be an undirected graph, where:

- V is the set of vertices,
- E is the set of undirected edges.

Define a mixed graph $G' = (V', E', A')$, where:

- $V' = V$,
- $E' = E$,
- $A' = \emptyset$.

Since A' is empty, all edges in G' are undirected, and G' retains the same structure as G. Thus, G can be represented as a mixed graph G'. □

Definition 3.3 (Fuzzy Mixed Graph)**.** (cf. [9, 28, 39–41]) A *Fuzzy Mixed Graph* $G = (V, E, A, \sigma, \mu_E, \mu_A)$ is a mixed graph where:

- V is the set of vertices.
- E is the set of undirected edges.
- A is the set of directed edges (arcs).
- $\sigma : V \to [0, 1]$ is the *vertex membership function*, assigning to each vertex $v \in V$ a membership degree $\sigma(v)$.
- $\mu_E : E \to [0, 1]$ is the *edge membership function* for undirected edges.
- $\mu_A : A \to [0, 1]$ is the *arc membership function* for directed edges.

These functions satisfy the following conditions:

(1) For every undirected edge $e = \{u, v\} \in E$, the edge membership degree satisfies:

$$\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}.$$

(2) For every directed edge $a = (u, v) \in A$, the arc membership degree satisfies:

$$\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}.$$

Theorem 3.4. *A Fuzzy Mixed Graph generalizes a Mixed Graph by assigning membership degrees to vertices, undirected edges, and directed arcs.*

Proof. Let $G = (V, E, A)$ be a Mixed Graph, where:

- V: Set of vertices.
- E: Set of undirected edges.
- A: Set of directed edges.

Define a Fuzzy Mixed Graph $G' = (V', E', A', \sigma, \mu_E, \mu_A)$, where:

- $V' = V$,
- $E' = E$,
- $A' = A$,
- $\sigma(v) = 1$ for all $v \in V$,
- $\mu_E(e) = 1$ for all $e \in E$,
- $\mu_A(a) = 1$ for all $a \in A$.

Since all membership functions are set to 1, G' behaves identically to G. Thus, any Mixed Graph can be represented as a special case of a Fuzzy Mixed Graph.

Conversely, by allowing membership degrees in $[0, 1]$, Fuzzy Mixed Graphs extend the capability of Mixed Graphs to model partial relationships and uncertainties. □

Definition 3.5 (Intuitionistic Fuzzy Mixed Graph)**.** An *Intuitionistic Fuzzy Mixed Graph*

$$G = (V, E, A, \mu_V, \nu_V, \mu_E, \nu_E, \mu_A, \nu_A)$$

consists of:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- $\mu_V : V \to [0,1]$ and $\nu_V : V \to [0,1]$ are the *vertex membership* and *non-membership functions*, respectively, satisfying for each $v \in V$:

$$0 \leq \mu_V(v) + \nu_V(v) \leq 1.$$

- $\mu_E : E \to [0,1]$ and $\nu_E : E \to [0,1]$ are the *edge membership* and *non-membership functions* for undirected edges, satisfying for each $e = \{u, v\} \in E$:

$$0 \leq \mu_E(e) + \nu_E(e) \leq 1,$$

$$\mu_E(e) \leq \min\{\mu_V(u), \mu_V(v)\},$$

$$\nu_E(e) \geq \max\{\nu_V(u), \nu_V(v)\}.$$

- $\mu_A : A \to [0,1]$ and $\nu_A : A \to [0,1]$ are the *arc membership* and *non-membership functions* for directed edges, satisfying for each $a = (u, v) \in A$:

$$0 \leq \mu_A(a) + \nu_A(a) \leq 1,$$

$$\mu_A(a) \leq \min\{\mu_V(u), \mu_V(v)\},$$

$$\nu_A(a) \geq \max\{\nu_V(u), \nu_V(v)\}.$$

Theorem 3.6. *An Intuitionistic Fuzzy Mixed Graph generalizes both Fuzzy Mixed Graphs and Mixed Graphs by incorporating membership and non-membership degrees.*

Proof. Let $G = (V, E, A, \sigma, \mu_E, \mu_A)$ be a Fuzzy Mixed Graph, where:

- V: Set of vertices.
- E: Set of undirected edges.
- A: Set of directed arcs.
- σ, μ_E, and μ_A are the vertex, edge, and arc membership functions, respectively.

Define an Intuitionistic Fuzzy Mixed Graph $G' = (V', E', A', \mu_V, \nu_V, \mu_E, \nu_E, \mu_A, \nu_A)$, where:

- $V' = V$,
- $E' = E$,
- $A' = A$,
- $\mu_V(v) = \sigma(v)$ for all $v \in V$,
- $\nu_V(v) = 1 - \sigma(v)$ for all $v \in V$,

- $\mu_E(e) = \mu_E(e)$, $\nu_E(e) = 1 - \mu_E(e)$ for all $e \in E$,
- $\mu_A(a) = \mu_A(a)$, $\nu_A(a) = 1 - \mu_A(a)$ for all $a \in A$.

By including non-membership functions ν_V, ν_E, and ν_A, G' retains the structure of G while extending it to account for non-membership degrees. If all non-membership degrees are set to 0, G' reduces to a Fuzzy Mixed Graph. Furthermore, if all membership degrees are set to 1, G' reduces to a Mixed Graph.

Therefore, Intuitionistic Fuzzy Mixed Graphs generalize both Fuzzy Mixed Graphs and Mixed Graphs. □

Definition 3.7 (Neutrosophic Mixed Graph)**.** (cf. [32]) A *Neutrosophic Mixed Graph*

$$G = (V, E, A, T_V, I_V, F_V, T_E, I_E, F_E, T_A, I_A, F_A)$$

is defined as:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- $T_V, I_V, F_V : V \to [0, 1]$ are the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* functions for vertices, respectively, satisfying for each $v \in V$:

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3.$$

- Similarly, $T_E, I_E, F_E : E \to [0, 1]$ are the membership functions for undirected edges, and $T_A, I_A, F_A : A \to [0, 1]$ are the membership functions for directed edges, satisfying analogous conditions.
- For each undirected edge $e = \{u, v\} \in E$, the edge membership functions satisfy:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad I_E(e) \geq \max\{I_V(u), I_V(v)\}, \quad F_E(e) \geq \max\{F_V(u), F_V(v)\}.$$

- For each directed edge $a = (u, v) \in A$, the arc membership functions satisfy:

$$T_A(a) \leq \min\{T_V(u), T_V(v)\}, \quad I_A(a) \geq \max\{I_V(u), I_V(v)\}, \quad F_A(a) \geq \max\{F_V(u), F_V(v)\}.$$

Theorem 3.8. *A Neutrosophic Mixed Graph generalizes Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, and Mixed Graphs.*

Proof. Let

$$G = (V, E, A, T_V, I_V, F_V, T_E, I_E, F_E, T_A, I_A, F_A)$$

be a Neutrosophic Mixed Graph. To show that it generalizes other types of graphs, we consider the following cases:

An Intuitionistic Fuzzy Mixed Graph is defined by vertex membership μ_V, vertex non-membership ν_V, edge membership μ_E, edge non-membership ν_E, arc membership μ_A, and arc non-membership ν_A, satisfying $0 \leq \mu + \nu \leq 1$.

Set:

$$T_V(v) = \mu_V(v), \quad I_V(v) = 0, \quad F_V(v) = \nu_V(v),$$
$$T_E(e) = \mu_E(e), \quad I_E(e) = 0, \quad F_E(e) = \nu_E(e),$$
$$T_A(a) = \mu_A(a), \quad I_A(a) = 0, \quad F_A(a) = \nu_A(a).$$

The neutrosophic membership functions now satisfy the Intuitionistic Fuzzy Mixed Graph conditions:

$$0 \leq T + F = \mu + \nu \leq 1, \quad I = 0.$$

Hence, a Neutrosophic Mixed Graph reduces to an Intuitionistic Fuzzy Mixed Graph under these mappings.

A Fuzzy Mixed Graph is defined by vertex membership σ, edge membership μ_E, and arc membership μ_A, satisfying $\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}$ for edges and $\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}$ for arcs.

Set:

$$T_V(v) = \sigma(v), \quad I_V(v) = 0, \quad F_V(v) = 1 - \sigma(v),$$
$$T_E(e) = \mu_E(e), \quad I_E(e) = 0, \quad F_E(e) = 1 - \mu_E(e),$$
$$T_A(a) = \mu_A(a), \quad I_A(a) = 0, \quad F_A(a) = 1 - \mu_A(a).$$

The neutrosophic membership functions now satisfy the Fuzzy Mixed Graph conditions:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad T_A(a) \leq \min\{T_V(u), T_V(v)\}.$$

Hence, a Neutrosophic Mixed Graph reduces to a Fuzzy Mixed Graph under these mappings.

A Mixed Graph is defined by vertices V, undirected edges E, and directed arcs A, without any membership functions.

Set:

$$T_V(v) = 1, \quad I_V(v) = 0, \quad F_V(v) = 0,$$
$$T_E(e) = 1, \quad I_E(e) = 0, \quad F_E(e) = 0,$$
$$T_A(a) = 1, \quad I_A(a) = 0, \quad F_A(a) = 0.$$

The neutrosophic membership functions become constant, making the Neutrosophic Mixed Graph equivalent to a Mixed Graph.

By appropriately setting the neutrosophic membership functions, a Neutrosophic Mixed Graph can reduce to an Intuitionistic Fuzzy Mixed Graph, a Fuzzy Mixed Graph, or a Mixed Graph. Therefore, a Neutrosophic Mixed Graph generalizes all these graph types. □

Definition 3.9 (Turiyam Neutrosophic Mixed Graph)**.** A *Turiyam Neutrosophic Mixed Graph*

$$G = (V, E, A, t_V, iv_V, fv_V, lv_V, t_E, iv_E, fv_E, lv_E, t_A, iv_A, fv_A, lv_A)$$

consists of:

- V: the set of vertices.
- E: the set of undirected edges.
- A: the set of directed edges (arcs).
- For each vertex $v \in V$, the membership functions $t_V(v), iv_V(v), fv_V(v), lv_V(v) \in [0, 1]$ represent the truth, indeterminacy, falsity, and liberal state values, respectively, satisfying:
$$0 \leq t_V(v) + iv_V(v) + fv_V(v) + lv_V(v) \leq 4.$$
- Similar membership functions are defined for edges and arcs, satisfying analogous conditions.
- For each undirected edge $e = \{u, v\} \in E$, the edge membership functions satisfy:
$$t_E(e) \leq \min\{t_V(u), t_V(v)\},$$
$$iv_E(e) \geq \max\{iv_V(u), iv_V(v)\},$$
$$fv_E(e) \geq \max\{fv_V(u), fv_V(v)\},$$
$$lv_E(e) \geq \max\{lv_V(u), lv_V(v)\}.$$
- For each directed edge $a = (u, v) \in A$, the arc membership functions satisfy similar conditions.

Theorem 3.10. *A Turiyam Neutrosophic Mixed Graph generalizes Neutrosophic Mixed Graphs, Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, and Mixed Graphs.*

Proof. Let

$$G = (V, E, A, t_V, iv_V, fv_V, lv_V, t_E, iv_E, fv_E, lv_E, t_A, iv_A, fv_A, lv_A)$$

be a Turiyam Neutrosophic Mixed Graph. To prove that it generalizes other types of graphs, we consider the following cases:

A Neutrosophic Mixed Graph is defined with truth-membership (T), indeterminacy-membership (I), and falsity-membership (F) functions for vertices, edges, and arcs.

Set:

$$t_V(v) = T_V(v), \quad iv_V(v) = I_V(v), \quad fv_V(v) = F_V(v), \quad lv_V(v) = 0,$$
$$t_E(e) = T_E(e), \quad iv_E(e) = I_E(e), \quad fv_E(e) = F_E(e), \quad lv_E(e) = 0,$$
$$t_A(a) = T_A(a), \quad iv_A(a) = I_A(a), \quad fv_A(a) = F_A(a), \quad lv_A(a) = 0.$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to a Neutrosophic Mixed Graph by ignoring the liberal membership component (lv).

An Intuitionistic Fuzzy Mixed Graph is defined with membership (μ) and non-membership (ν) functions for vertices, edges, and arcs, satisfying $0 \leq \mu + \nu \leq 1$.

Set:

$$\begin{aligned} t_V(v) = \mu_V(v), \quad iv_V(v) = 0, \quad fv_V(v) = \nu_V(v), \quad lv_V(v) = 0, \\ t_E(e) = \mu_E(e), \quad iv_E(e) = 0, \quad fv_E(e) = \nu_E(e), \quad lv_E(e) = 0, \\ t_A(a) = \mu_A(a), \quad iv_A(a) = 0, \quad fv_A(a) = \nu_A(a), \quad lv_A(a) = 0. \end{aligned}$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to an Intuitionistic Fuzzy Mixed Graph.

A Fuzzy Mixed Graph is defined with membership functions for vertices (σ), edges (μ_E), and arcs (μ_A).

Set:

$$\begin{aligned} t_V(v) = \sigma(v), \quad iv_V(v) = 0, \quad fv_V(v) = 1 - \sigma(v), \quad lv_V(v) = 0, \\ t_E(e) = \mu_E(e), \quad iv_E(e) = 0, \quad fv_E(e) = 1 - \mu_E(e), \quad lv_E(e) = 0, \\ t_A(a) = \mu_A(a), \quad iv_A(a) = 0, \quad fv_A(a) = 1 - \mu_A(a), \quad lv_A(a) = 0. \end{aligned}$$

Under this mapping, the Turiyam Neutrosophic Mixed Graph reduces to a Fuzzy Mixed Graph.

A Mixed Graph is defined by vertices (V), undirected edges (E), and directed arcs (A), without any membership functions.

Set:

$$\begin{aligned} t_V(v) = 1, \quad iv_V(v) = 0, \quad fv_V(v) = 0, \quad lv_V(v) = 0, \\ t_E(e) = 1, \quad iv_E(e) = 0, \quad fv_E(e) = 0, \quad lv_E(e) = 0, \\ t_A(a) = 1, \quad iv_A(a) = 0, \quad fv_A(a) = 0, \quad lv_A(a) = 0. \end{aligned}$$

The Turiyam Neutrosophic Mixed Graph becomes equivalent to a Mixed Graph under these mappings.

By appropriately setting the membership functions, a Turiyam Neutrosophic Mixed Graph can reduce to Neutrosophic Mixed Graphs, Intuitionistic Fuzzy Mixed Graphs, Fuzzy Mixed Graphs, or Mixed Graphs. Thus, it generalizes all these types of graphs. □

Definition 3.11 (Plithogenic Mixed Graph)**.** A *Plithogenic Mixed Graph*

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

is defined as:

- V: the set of vertices.
- E: the set of undirected edges.

- A: the set of directed edges (arcs).
- $adf_V : V \to [0,1]^s$ is the *attribute degree function* for vertices, assigning to each vertex $v \in V$ an s-tuple $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$.
- $adf_E : E \to [0,1]^s$ is the *attribute degree function* for undirected edges.
- $adf_A : A \to [0,1]^s$ is the *attribute degree function* for directed edges.
- $aCf_V : V \times V \to [0,1]^t$ is the *contradiction degree function* for vertices.
- aCf_E and aCf_A are the contradiction degree functions for undirected edges and arcs, respectively.

These functions satisfy the following conditions:

(1) *Edge Attribute Degree Constraint*:
- For each undirected edge $e = \{u, v\} \in E$:
$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$
where $\wedge$ denotes the minimum operation taken component-wise.
- For each directed edge $a = (u, v) \in A$:
$$adf_A(a) \leq adf_V(u) \wedge adf_V(v).$$

(2) *Contradiction Function Constraints*:
- The contradiction functions satisfy:
$$aCf_V(u,u) = 0, \quad aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V.$$
- Similar properties hold for aCf_E and aCf_A.

Additionally, plithogenic operations and aggregation functions can be defined to handle complex relationships among the attributes.

Example 3.12. (cf. [14]) The following examples of Plithogenic Mixed Graph are provided.

- When $s = t = 1$, G is called a *Plithogenic Fuzzy Mixed Graph.*
- When $s = 2, t = 1$, G is called a *Plithogenic Intuitionistic Fuzzy Mixed Graph.*
- When $s = 3, t = 1$, G is called a *Plithogenic Neutrosophic Mixed Graph.*
- When $s = 4, t = 1$, G is called a *Plithogenic Turiyam Neutrosophic Mixed Graph.*

We explore the mathematical properties of Uncertain Mixed Graphs. The theorems are presented below.

Theorem 3.13. *A Plithogenic Mixed Graph generalizes the Neutrosophic Mixed Graph, Turiyam Neutrosophic Mixed Graph, and Fuzzy Mixed Graph. Specifically, by choosing appropriate dimensions and components of the attribute degree functions in the Plithogenic Mixed Graph, the Plithogenic Mixed Graph reduces to these specific types of mixed graphs.*

Proof. Consider a Plithogenic Mixed Graph

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

where:

- $adf_V : V \to [0,1]^s$, assigning to each vertex $v \in V$ an s-tuple $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$.
- $adf_E : E \to [0,1]^s$, assigning to each undirected edge $e \in E$ an s-tuple $adf_E(e) = (a_1(e), a_2(e), \ldots, a_s(e))$.
- $adf_A : A \to [0,1]^s$, assigning to each directed edge $a \in A$ an s-tuple $adf_A(a) = (a_1(a), a_2(a), \ldots, a_s(a))$.

To recover specific Mixed Graphs:

(1) *Fuzzy Mixed Graph*:

Set $s = 1$. Let the attribute degree function for vertices and edges be:

$$adf_V(v) = \sigma(v), \quad adf_E(e) = \mu_E(e), \quad adf_A(a) = \mu_A(a),$$

where $\sigma(v)$, $\mu_E(e)$, and $\mu_A(a)$ are the vertex membership function, edge membership function, and arc membership function in the Fuzzy Mixed Graph.

The edge attribute degree constraints in the Plithogenic Mixed Graph become:

$$\mu_E(e) \leq \min\{\sigma(u), \sigma(v)\}, \quad \text{for } e = \{u, v\} \in E,$$

$$\mu_A(a) \leq \min\{\sigma(u), \sigma(v)\}, \quad \text{for } a = (u, v) \in A,$$

which match the conditions in the Fuzzy Mixed Graph definition.

(2) *Neutrosophic Mixed Graph*:

Set $s = 3$. Let the attribute degree functions be:

$$adf_V(v) = (T_V(v), I_V(v), F_V(v)),$$

$$adf_E(e) = (T_E(e), I_E(e), F_E(e)),$$

$$adf_A(a) = (T_A(a), I_A(a), F_A(a)),$$

where $T_V(v), I_V(v), F_V(v)$ are the truth, indeterminacy, and falsity membership functions of vertices, and similarly for edges and arcs.

The edge attribute degree constraints become:

$$T_E(e) \leq \min\{T_V(u), T_V(v)\}, \quad I_E(e) \geq \max\{I_V(u), I_V(v)\}, \quad F_E(e) \geq \max\{F_V(u), F_V(v)\},$$

which correspond to the conditions in the Neutrosophic Mixed Graph.

(3) *Turiyam Neutrosophic Mixed Graph*:

Set $s = 4$. Let the attribute degree functions be:

$$adf_V(v) = (t_V(v), iv_V(v), fv_V(v), lv_V(v)),$$

$$adf_E(e) = (t_E(e), iv_E(e), fv_E(e), lv_E(e)),$$

$$adf_A(a) = (t_A(a), iv_A(a), fv_A(a), lv_A(a)),$$

where $t_V(v), iv_V(v), fv_V(v), lv_V(v)$ are the truth, indeterminacy, falsity, and liberal state values for vertices, and similarly for edges and arcs.

The edge attribute degree constraints become:

$$t_E(e) \leq \min\{t_V(u), t_V(v)\},$$

$$iv_E(e) \geq \max\{iv_V(u), iv_V(v)\},$$

$$fv_E(e) \geq \max\{fv_V(u), fv_V(v)\},$$

$$lv_E(e) \geq \max\{lv_V(u), lv_V(v)\},$$

which match the conditions in the Turiyam Neutrosophic Mixed Graph definition.

Thus, by appropriately choosing the dimension s and mapping the components of the attribute degree functions, the Plithogenic Mixed Graph reduces to the specific types of Mixed Graphs: Fuzzy Mixed Graph, Neutrosophic Mixed Graph, and Turiyam Neutrosophic Mixed Graph. □

Theorem 3.14. *In a Plithogenic Mixed Graph G, the contradiction degree function $aCf_V : V \times V \to [0,1]^t$ is symmetric and satisfies $aCf_V(u,u) = 0$ for all $u \in V$, i.e.,*

$$aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V,$$

$$aCf_V(u,u) = 0, \quad \forall u \in V.$$

Proof. By the definition of a Plithogenic Mixed Graph, the contradiction degree function aCf_V must satisfy:

$$aCf_V(u,u) = 0, \quad \forall u \in V,$$

$$aCf_V(u,v) = aCf_V(v,u), \quad \forall u, v \in V.$$

These conditions ensure that the contradiction degree function is reflexive and symmetric, as required. □

Theorem 3.15. *In a Plithogenic Mixed Graph G, for any undirected edge $e = \{u, v\} \in E$, the attribute degree function $adf_E(e)$ satisfies:*

$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$

where $\wedge$ denotes the component-wise minimum of the attribute degree functions of u and v.

Proof. Let $adf_V(u) = (a_1(u), a_2(u), \ldots, a_s(u))$ and $adf_V(v) = (a_1(v), a_2(v), \ldots, a_s(v))$. Similarly, let $adf_E(e) = (a_1(e), a_2(e), \ldots, a_s(e))$. By the edge attribute degree constraint in the definition of a Plithogenic Mixed Graph, for each component i $(1 \leq i \leq s)$:

$$a_i(e) \leq \min\{a_i(u), a_i(v)\}.$$

This implies:

$$adf_E(e) \leq adf_V(u) \wedge adf_V(v),$$

where the inequality holds component-wise. Therefore, the attribute degree function of an edge is bounded above by the component-wise minimum of the attribute degree functions of its incident vertices. □

Theorem 3.16. *Any induced subgraph of a Plithogenic Mixed Graph G is also a Plithogenic Mixed Graph.*

Proof. Let

$$G = (V, E, A, adf_V, adf_E, adf_A, aCf_V, aCf_E, aCf_A)$$

be a Plithogenic Mixed Graph. Consider an induced subgraph

$$G' = (V', E', A', adf'_V, adf'_E, adf'_A, aCf'_V, aCf'_E, aCf'_A)$$

, where $V' \subseteq V$, $E' \subseteq E \cap \binom{V'}{2}$, and $A' \subseteq A \cap (V' \times V')$.

Define the attribute degree functions adf'_V, adf'_E, adf'_A and contradiction functions aCf'_V, aCf'_E, aCf'_A of G' by restricting those of G to V', E', A'. Since the original functions satisfy the attribute degree and contradiction function constraints on G, their restrictions satisfy the same constraints on G'. Hence, G' inherits the properties of a Plithogenic Mixed Graph and is itself a valid instance of the same structure. □

Theorem 3.17. *The underlying crisp graph of a Plithogenic Mixed Graph G is a mixed graph $G' = (V, E, A)$.*

Proof. The crisp graph G' is obtained by ignoring the attribute degree functions adf_V, adf_E, adf_A and the contradiction functions aCf_V, aCf_E, aCf_A in G. These functions do not alter the structure of the vertex set V, the undirected edges E, or the directed edges A. Thus, removing them results in a mixed graph $G' = (V, E, A)$, which retains the original structure of G without its additional plithogenic properties. □

Theorem 3.18. *A Plithogenic Mixed Graph generalizes the Plithogenic Graph.*

Proof. A Plithogenic Graph is a special case of a Plithogenic Mixed Graph where the set of directed edges A is empty. By setting $A = \emptyset$ in a Plithogenic Mixed Graph, we recover a Plithogenic Graph. Therefore, the Plithogenic Mixed Graph generalizes the Plithogenic Graph. □

4. Future Directions of This Research

This section outlines the future directions of this research.

We aim to extend the concept of Mixed Hypergraphs [12, 36, 37, 64, 66] to the domain of Uncertain Graphs, delving into their mathematical properties and exploring potential applications. Furthermore, we intend to define Mixed Superhypergraphs (cf. [15, 22, 31, 56, 57]), investigating their characteristics and potential uses. Mixed Superhypergraphs expand the concept of Mixed Graphs to superhypergraphs, incorporating both undirected and directed superhypergraphs.

Additionally, this research aims to integrate its core concepts with those of hypersoft sets [54] and superhypersoft sets [58, 61], facilitating a deeper examination of their theoretical foundations and practical implications.

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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Neutrosophic evaluation through the PESTEL method

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Abstract. This study addresses the complexity of assessing key factors in strategic decision-making through PESTEL analysis applied with a neutrosophic perspective. In a context of increasing uncertainty, organizations face challenges in accurately interpreting political, economic, social, technological, environmental and legal factors, which are fundamental for sustainable development and informed decision-making. The importance of this topic lies in the need to integrate approaches that not only describe these factors in a static manner, but also address the ambiguity and complexity inherent in the variables that compose them. By applying the PESTEL method in its neutrosophic version, the study aims to go beyond conventional approaches, capturing the dimensions of uncertainty that often go unnoticed in traditional analyses. In terms of methodology, the study uses neutrosophic analysis on the PESTEL framework, incorporating flexible variables that allow capturing the multiple perspectives of experts in each of the factors analyzed. The findings reveal that by integrating the neutrosophic approach, it is possible to offer more dynamic and accurate assessments of the challenges faced by organizations in complex environments. This research contributes significantly to the field by proposing a new approach to strategic analysis that overcomes the limitations of previous static models. In practical terms, the results provide tools to enhance organizational resilience by enabling a more nuanced assessment of external factors, thereby facilitating more informed and adaptive decision-making. In conclusion, this study not only enriches theoretical knowledge about strategic analysis but also contributes practical solutions that can be applied to a variety of business and public policy contexts.

Keywords: Neutrosophic assessment, PESTEL method, Strategic decision making, Uncertainty, Political, economic, social, technological, environmental and legal factors, Strategic analysis

1. Introduction

In the early days, there was a notable increase in private sector participation in space efforts, especially in Western countries such as the United States. The commercial benefits derived from this have become significant at a global level [1]. This evolution has generated the need to review the regulations of international space law, as various questions arise due to the growth of private capital and technological progress, such as differences in the interpretation of terms such as use and exploration related to space [2].

In the absence of international legislation, some countries regulate the actions of private entities through national laws [3]. This has given rise to the idea of a global interaction between the State and the private sector, as evidenced by the legislation regulating the International Space Station (ISS). However, legal solutions are still lacking in areas such as satellite operations [4]. It is crucial to establish international regulations for satellite launches, independent of political considerations and applicable to all, including private actors.

Space tourism via space shuttles (RLV) must ensure a safe return for crew, passengers and cargo, as well as generating economic benefits [2]. However, detailed procedures are required, especially regarding transport (ship equipment, routes, insurance, emergencies and spaceport infrastructure, among others).

The commercialization of space services is a natural consequence of continuing technological development, but it raises numerous legal issues regarding civil liability [4]. Some advocate leaving this responsibility to the market, while others propose separate and specific international regulation for space transportation and telecommunications.

Currently, in some countries, private companies only need permission from the state of registration to operate. However, to ensure safety, proper notification of the departure of the aircraft from Earth is required. However, not everyone agrees with excessive liberalization of space, arguing that differences of opinion and state demands must be considered.

The main task of this paper is to identify the challenges and opportunities arising from the increasing work between government and the private sector in space, and to propose recommendations for effective regulation that will foster the sustainable development of commercial space operations worldwide. In addition, civil liability and the need for a new branch of space law addressing commercial diligence in space will be explored [3].

2. Materials and methods

2.1. PESTEL method

The PESTEL study [5], which covers several aspects, is a fundamental tool for identifying the environment in which entities are managed. Its main objective is to provide elements that allow the organization to anticipate new situations and circumstances. This analysis involves identifying and examining key elements:

1. Political: Political elements that may influence the future activity of the organization are analyzed, such as government policies and regulations.
2. Economic: Current and future economic conditions that could affect the organization's strategy are considered, including economic development, poverty and inequality.
3. Social: Sociocultural aspects are examined that may reveal current trends in the company's activity, such as changes in consumer quantities and preferences.
4. Technological: The relationship between ICT and the potential transformations that could arise , which can affect the way entities are managed and offer their products or services.
5. Ecological: Possible changes related to ecology and the environment are considered, as well as the opportunities and challenges that these changes may pose for the organization.
6. Legal: Changes in legal regulations that may positively or negatively affect the organization are analyzed, such as government regulations or labor laws.

All these aspects are crucial to understanding the market and represent the basis of strategic management. They cover elements such as government policies, poverty levels, inequality, economic development, access to resources and how all of this impacts the company [6].

2.2 Neutrosophic statistics

Neutrosophic analysis replaces exact numerical quantities with interval-type quantities. This methodology can be used for both parameters and random variables, while recognizing that the sample size

is uncertain or imprecise. In this framework, information may present ambiguity, vagueness, inaccuracy, incompleteness or uncertainty [7].

Analyses performed through neutrosophic facilitate the interpretation and sorting of neutrosophic (ambiguous, vague, imprecise, incomplete, or unrevealed) information to detect underlying patterns. In summary, neutrosophic reasoning, neutrosophic collections, and neutrosophic probabilities and analysis find wide utility in various research domains and constitute a rapidly evolving field of research. Descriptive analysis of neutrosophic encompasses all methodologies for summarizing and elucidating the attributes of neutrosophic numerical information. Neutrosophic quantities are delineated as N = a + b $_i$, where a and b are real or complex quantities, and "I" denotes the indeterminacy component of the neutrosophic quantity.[8]

The study of neutrosophy statistics refers to a neutrosophy random variable. The mean of the neutrosophy variable is formulated as follows [9,10,11]:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \tag{1}$$

$$\text{Where, , , ,} \bar{x}_a = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{il}\ \bar{x}_b = \frac{1}{n_N}\sum_{i=1}^{n_N} X_{iu}\ n_N \in [n_l, n_u] \tag{2}$$

This is a neutrosophic random sample. However, for the calculation of the neutral squares (NNS) it can be calculated as follows [11, 12, 13]

$$\sum_{i=1}^{n} N(X_i - \bar{X}_{iN})^2 = \sum_{i=1}^{n} N\left[\begin{matrix}\min\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U)\\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix}\\ \max\begin{pmatrix}(a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U)\\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U)\end{pmatrix}\end{matrix}\right], I \in [I_L, I_U] \tag{3}$$

Where . The sample variance of neutrosophy can be calculated by:$a_i = X_l b_i = X_u$ [14,15]

$$\boldsymbol{S_N^2 = \frac{\sum_{i=1}^{n_N}(X_i - \bar{X}_{iN})^2}{n_N};\ S_N^2 \in [S_L^2, S_U^2]} \tag{4}$$

The neutrosophic value (NCV) measures the consistency of the variable. The lower the NCV value, the more consistent the performance of the item compared to that of the other items. The NCV can be calculated as follows [16, 17, 18] .

$$\boldsymbol{CV_N = \frac{\sqrt{S_N^2}}{\bar{X}_N} \times 100;\ CV_N \in [CV_L, CV_U]} \tag{5}$$

3 results

To apply the PESTEL method to the case study and determine the negative elements of legal and economic challenges in the modern space, it is necessary to examine the following aspects:

1. Policy: Government regulations and policies related to space missions, both nationally and internationally, must be taken into account. This may include intellectual property laws, safety regulations, and collaboration protocols between different countries.
2. Economic: The costs associated with the development and operation of commercial space procedures, as well as the potential financial risks, must be analyzed.

3. Social: Society's perceptions and attitudes towards space tasks should be examined, as well as the potential social impacts of these tasks, such as unequal distribution of benefits and exclusion of certain population groups.
4. Technological: The current state and future development of space technology should be assessed, as well as possible technological advances that could influence commercial space procedures. The risks associated with dependence on technology in space should also be considered.
5. Ecological: The potential environmental impacts of space missions, such as the generation of space debris and pollution of outer space, should be analyzed. Environmental mitigation measures and sustainability protocols in space should also be considered.
6. Legal aspects: Existing legal standards and potential gaps in legislation relating to commercial space due diligence should be examined. This may include issues of civil liability, ownership of space resources and international jurisdiction in space.

By applying the PESTEL method to this case study, negative elements related to each of these aspects can be identified, which will provide a more complete understanding of the legal and economic challenges in the modern space.

Table 1: Evaluation of matrices and variables.

Dimension	Element	Possible solutions
Political.	Political foundations	• **Challenge:** Disparity in spatial regulations between different countries. • **Solution:** Promote international cooperation to establish uniform regulations and legal agreements governing space procedures at a global level.
Economic	Improve economic elements.	• **Challenge:** High costs associated with the development and launch of satellites and other space missions. • **Solution:** Encourage public and private investment in space exploration and exploitation, as well as promote collaboration between companies and governments to share costs and resources.
Social	Perception of equal rights.	• **Challenge:** Unequal access to space and its benefits, which could increase the gap between developed and developing countries. • **Solution:** Implement inclusive policies that encourage equitable participation of all countries and communities in space efforts, as well as educational programs that promote public awareness and interest in space.

Dimension	Element	Possible solutions
Technological	Electronic development.	• **Challenge:** Risks associated with dependence on space technology and equipment obsolescence. • **Solution:** Continue investing in technological research and development to improve the reliability and efficiency of space systems, as well as diversify sources of technology to reduce dependence on a single supplier.
Legal	Rights of those involved.	• **Challenge:** Gaps in international law to address issues such as ownership of space resources and liability for damage caused by spacecraft. • **Solution:** Promote the creation and adoption of international treaties and agreements that address these issues, as well as strengthen cooperation between countries and international organizations to enforce existing laws and develop new regulations when necessary.

3. Development of the method

In the neutrosophic statistical modeling process, experts select five variables known as PESTEL Dimensions, which are significant within the neutrosophic sets. These results are disseminated to a team of specialists for evaluation and identification of primary trends.

One of the variables analyzed is the incidence of spatial challenges. This variable is examined for a sample size of n=60, and each element (f) is analyzed individually. It is important to note that the ecological aspect is not applicable to the variable analyzed, specifically about the evaluation of the elements.

Table 2: Neutrosophic frequencies of each element.

Days	Neutrosophic Frequency.				
	1	**2**	**3**	**4**	**5**
1	[6; 7]	[0; 0]	[2; 2]	[0; 7]	[0; 2]
2	[3; 7]	[3; 7]	[2; 6]	[23]	[0; 7]
3	[0; 3]	[3; 6]	[23]	[0; 3]	[0; 0]
4	[0; 2]	[3; 3]	[0; 3]	[2; 6]	[0; 2]
5	[0; 2]	[0; 0]	[23]	[2; 7]	[0; 3]
6	[23]	[3; 3]	[3; 6]	[0; 2]	[0; 3]
7	[0; 0]	[3; 3]	[3; 7]	[0; 3]	[0; 2]
8	[2; 2]	[2; 2]	[3; 3]	[0;6]	[0; 0]
9	[23]	[3; 6]	[0; 0]	[0; 3]	[0; 0]
10	[3; 6]	[3; 7]	[3; 7]	[0; 0]	[0; 7]
11	[3; 6]	[0; 0]	[0; 2]	[0; 0]	[0; 2]

Days	Neutrosophic Frequency.				
	1	**2**	**3**	**4**	**5**
12	[3; 3]	[2; 6]	[0; 2]	[2; 2]	[0; 2]
13	[0; 2]	[0; 2]	[3; 7]	[0; 3]	[0;6]
14	[0; 3]	[3; 3]	[0; 2]	[0;6]	[0; 3]
15	[6; 7]	[0; 2]	[0; 0]	[0; 0]	[0; 7]
16	[3; 7]	[0; 3]	[0; 0]	[0; 3]	[0; 2]
17	[3; 3]	[0; 2]	[0; 2]	[23]	[0; 7]
18	[3; 7]	[3; 3]	[0; 0]	[0; 3]	[0; 2]
19	[0; 0]	[2; 2]	[3; 3]	[2; 2]	[0; 0]
20	[6; 7]	[0; 3]	[3; 7]	[2; 7]	[0; 7]
0-60	[141; 204]	[122; 201]	[106; 238]	[61; 218]	[25; 193]

Table 2 analyses the elements over a 60-day period, with an incidence range of [0; 7] for each element per day. Total levels of indeterminacy were recorded for elements f_1=62, f_2=80, f_3=132, f_4=157 and f_5=166, with a level of representativeness that falls within the interval [30.24%; 86.46%]. It is noteworthy that on days when 7 incidences were observed per element, a higher incidence of 60% was perceived in the political element. Given the prevailing indeterminacy, conventional statistical methodologies are considered inadequate, so it is necessary to adopt neutrosophic statistics for a more comprehensive understanding.

In the modelling process, the political element is emphasized as one of the main drivers (see Table 3). To determine which element hosts a representative mean, the average quantities are meticulously evaluated, along with the variability of the variable to address uncertainties in the final result. This involves incorporating neutrosophic standard deviation quantities for each element. Furthermore, discerning which element requires further attention in the process is facilitated by the values provided by the coefficient of variation.

Table 3: Neutrosophic statistics of the causes of bullying.

Items	$\bar{x}_N$	**SN**	**CVN**
Political	[2.382; 3.427]	[2,848 ; 3,248]	[0.776; 0.822]
Economic	[2.032; 3.367]	[2.202; 2.573]	[0.582; 0.764]
Social	[2.74; 3.85]	[0.877; 2.68]	[0.502; 0.678]
Technological	[2,036; 3,67]	[0.428; 2.642]	[0.424; 0.724]
Legal	[0.434; 3.2]	[0.227; 2.622]	[0.283; 0.828]

Table 4: Undefined values.

Items	$\bar{x}_N$	**SN**	**CVN**
1	$2{,}382 + 3{,}427I; I \in [0; 20.1]$	$2{,}848 + 3{,}248I; I \in [0; 41.3]$	$0.776 + 0.822I; I \in [0; 15.8]$
2	$2.032 + 3.367I, I \in [0; 38.6]$	$2{,}202 + 2{,}573I; I \in [0; 53.3]$	$0.582 + 0.764I; I \in [0; 22.6]$
3	$2.74 + 3.85I; I \in [0; 55.7]$	$0.877 + 2.68I; I \in [0; 7.3]$	$0.502 + 0.678I; I \in [0; 26.1]$

Items	$\bar{x}_N$	SN	CVN
4	2.036 + 3.67I;I ∈ [0; 71.7]	0.428 + 2.642I;I ∈ [0; 83.8]	0.414 + 0.724I;I ∈ [0; 42.8]
5	0.434 + 3.2I;I ∈ [0; 86.5]	0.227 + 2.622 I;I ∈ [0; 85.2]	0.283 + 0.828I;I ∈ [0; 64.2]

4. Preliminary solutions.

Based on the results obtained and considering the interrelation of the political element with the other aspects involved, the following recommendations are proposed:

1. Establishment of negotiation and arbitration mechanisms to resolve disputes over the ownership and use of space resources.
2. Facilitate access to specialized financing and insurance for space projects, as well as promote the diversification of commercial applications in space to reduce dependence on specific fragments.
3. Implementation of training and technology transfer programs to promote the participation of developing countries in space projects.
4. Invest in research and development of innovative technologies, such as in-situ manufacturing and space mining, and foster international cooperation in space technology exploration and development projects.
5. Establish international standards for managing space debris and protecting the space environment, as well as promoting sustainable practices in the design and operation of satellites and spacecraft.
6. Strengthen international cooperation to develop clear and coherent legal frameworks addressing issues such as liability for damage and ownership of space resources, as well as promote adherence to and compliance with existing treaties and agreements.

5. Conclusions

The new phase of space trials poses considerable challenges to governments, as well as to new actors and policy makers in the space field. The current phase of space commercialization requires an adaptation of existing legislation to meet current needs and challenges. Research on space missions in this "new era" has highlighted economic and security challenges, stressing the importance of peaceful cooperation between States.

Since the beginning of this new era, disparities in economic and spatial development have emerged between different countries and societies, leading to different opportunities for access to space. With technological advancement and the arrival of small satellites, access to space has become more accessible and economical, benefiting smaller countries, private entities, scientific institutions and even students.

However, the new space era faces the challenge of resolving various conflicts of interest to ensure the safety of space efforts undertaken by States, while at the same time protecting the space environment from the growing accumulation of space debris. It is crucial to support free and undistorted competition, based on democratic principles, by implementing new national space programs, strategies and regulations to make space accessible to all. In this regard, international cooperation is essential, including between States with different regimes. While this cooperation can be challenging, it is essential to avoid conflicts at a global level. Collaboration between all nations is vital for the sustainability of space efforts in this new Space era.

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Evaluation of the Impact of the Use of Digital Skills on the Substantial Increase in Learning through the Neutrosophic PEST-SWOT approach

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Abstract. The research focuses on measuring the influence of the use of digital skills on learning, a major issue in current education. The fundamental issue addressed is how digital skills can greatly influence the educational process and the academic success of learners. In the contemporary environment, characterized by rapid technological growth and the increasing integration of digital instruments into the educational sphere, this challenge becomes especially essential. Despite the extensive use of digital technologies, there is a dearth of a comprehensive examination of how these skills influence learning from a multidimensional point of view that incorporates both intrinsic possibilities and risks. The research addresses this gap in the current literature by employing an integrated strategy that combines PEST and SWOT approaches with the neutrosophic methodology. Process, however, also bring with them dangers and constraints that need to be managed. The contribution of the study is twofold: first, it provides a methodological framework for assessing the impact of digital skills on learning. The study also provides a deeper and more complex examination of economic, social and cultural factors. Technical variables, as well as the weaknesses, opportunities and risks associated with the application of digital skills and talents. The findings demonstrate that digital skills have a positive impact on learning, driving engagement and personalization of education without digital technologies on education; Secondly, it offers practical recommendations to optimize the use of these tools in the classroom, ensuring a balance between their benefits and challenges. This study enhances knowledge in the area of digital education and gives practical suggestions for the proper deployment of technology in educational contexts.

Keywords: Digital Skills, SWOT Analysis, PEST Analysis, PEST-SWOT Neutrosophic Analysis, Education, neutrosophic PEST-SWOT

1. Introduction

In the digital age, the integration of digital skills in education has become more vital, altering the way students engage with material and gain knowledge. Assessing the influence of these abilities on learning is vital to understanding how they might increase academic achievement and better prepare students for the future. Current studies emphasizes that the efficient use of digital technology may in-

crease accessibility, customization and motivation in learning [1]. However, most research have concentrated on the technical execution of these technologies without digging into how they affect learning from an integrative viewpoint. Historically, education has gone through numerous phases of technological revolution, from the usage of printed books to the advent of computers and, more recently, online learning platforms. These advancements have permitted a steady evolution in educational techniques and have spurred the need for a more complete study of how digital technologies effect learning [2]. Despite gains, there remains a large gap in knowing how digital abilities affect academic success and what circumstances increase or restrict their effect. The primary issue that this research tackles is the absence of a detailed investigation of the true influence of digital skills on learning. Despite the expanding deployment of digital tools in education, the issue remains: how do these skills effect the large rise in learning in diverse educational contexts? This research tries to address this gap by offering a complete evaluation that covers both technical and contextual components of digital skills usage.

To answer this topic, the research adopts a new methodological approach that merges the PEST and SWOT frameworks with the neutrosophic technique. This method provides for a deeper and more complex examination of political, economic, social and technical (PEST) aspects and strengths, weaknesses, opportunities and threats (SWOT) connected to the application of digital skills in learning [3,4]. The combination of these methodologies gives a full understanding of how digital skills might influence academic achievement and what the accompanying difficulties and possibilities are. The major purpose of this research is to analyze the influence of the application of digital skills on enhancing learning utilizing the neutrosophic PEST-SWOT technique. In particular, the purpose is to assess how these characteristics affects the efficiency of digital technologies and what are the best methods to optimize their advantages in the educational environment. Additionally, the research tries to identify limitations that may restrict the good influence of digital skills on learning. In summary, this research not only gives a greater knowledge of the influence of digital skills on learning but also offers practical suggestions based on a rigorous examination. The project intends to contribute to the advancement of educational practices and the optimization of the use of digital technology in the classroom, offering useful tools and information for educators and those responsible for educational policy.

2 Related Works.

2.1. Digital Skills.

Digital skills have become a vital pillar of modern existence, playing a critical part in practically all facets of our everyday lives. In the digital age, it is about not only being able to operate electronic gadgets but also rather learning a sophisticated set of abilities that enable us to engage, create and collaborate successfully in an increasingly linked environment. This transition has led to a reconsideration of the abilities we deem vital and has produced major discussion over what digital skills are genuinely required for success in the 21st century. First, it is vital to realize that digital skills are not a monolithic thing, but rather a spectrum of competencies that span from simple usage of technical tools to complex software creation and awareness of cybersecurity. At a basic level, these abilities include the capacity to utilize devices such as computers and cellphones, access the Internet, and use office software. However, as we progress up the skill level, the complexity grows substantially. Skills like as programming, data analysis and digital project management come into play here, needing a deeper and more technical grasp of technology tools and procedures. The significance of these abilities is clear in a work market that is digitalizing at a fast rate. Statistics demonstrate that the need for workers with sophisticated digital skills is always growing, indicating the need to adapt to a work environment that increasingly requires technical

knowledge and analytical ability [5]. This tendency not only influences technology areas, but also extends to traditionally non-technological fields, such as education, health and commerce. Consequently, the capacity to learn and utilize digital skills has become a decisive factor for employability and professional progress.

Despite their relevance, there is a large disparity in the availability and access to chances to acquire these abilities. Inequalities in access to technology and digital education remain, especially influencing underprivileged people and places with fewer resources [6]. This inequality provides a significant challenge: ensuring that all persons, regardless of their socioeconomic background, have the required chances to learn and perfect digital skills. Without bridging this divide, we risk continuing and deepening existing inequities, creating a digitally fractured society. In the educational area, the efficient integration of digital skills in the curriculum continues to be a matter of controversy. Many school systems have started to include these capabilities into their curriculum, although the implementation and quality of digital education vary greatly. Teaching digital skills should not just concentrate on the technical usage of technologies, but also on building a critical knowledge of how these tools effect society and daily life [7]. Critical digital literacy is essential to prepare students not only to use technology effectively but also to navigate a complex and often confusing digital landscape. Furthermore, digital skills are not restricted to technical competence; they also involve soft skills such as problem-solving, critical thinking, and the capacity to cooperate in virtual spaces. These interpersonal skills are increasingly valued in a work environment that is characterized by online collaboration and remote communication [8]. The ability to work effectively in geographically dispersed teams, manage conflicts on digital platforms, and use collaborative tools is critical to success in the contemporary labor market.

The impact of digital skills also extends to personal lives, influencing the way we interact with the world. From managing our finances to the way, we access information and services, digital skills play an increasingly vital part in daily life. This pervasiveness of technology in our daily lives reinforces the need for strong digital literacy that allows people to make the most of the opportunities available and protect themselves from the associated risks. The discussion about the need for digital skills also raises issues about the future of labor and education. Automation and artificial intelligence are rapidly transforming the job landscape, creating new opportunities but also displacing some traditional jobs [9]. In this setting, digital skills are emerging as a fundamental prerequisite to adapt to changes and remain relevant in a continuously developing work market. Finally, public policies and educational strategies must adapt to this changing reality. Governments and educational institutions must collaborate to design and implement programs that not only teach basic digital skills but also prepare people for future challenges. This involves investing in technological infrastructure, training educators and ensuring that all individuals have access to learning and development opportunities in the digital sphere.

Digital skills are more than a collection of technical competencies; they are an integral part of modern life that influences our ability to fully participate in 21st century society and economy. As we move towards an increasingly digitalized future, we must address inequalities in access, integrate digital education effectively and prepare people to meet the challenges and opportunities that technology offers us. This holistic and equitable approach is essential to building a more inclusive society prepared for the digital future.

2.2. SWOT Analysis.

SWOT analysis is a vital method to assess the position of an organization or project, by evaluating both its internal qualities (Weaknesses and Strengths) and external elements (Threats and Opportunities) in an orderly matrix. This process is divided into four key stages: the evaluation of the external environment, the internal analysis, the construction of the SWOT matrix and, finally, the formulation of strategies

to follow. An organization's ability to survive and thrive is closely linked to its environment, which offers both opportunities and challenges. These factors are the crucial components of the external analysis. Internally, the strengths and weaknesses of a company are strongly tied to its internal management. Each of these four elements can be classified as positive or negative. Opportunities, which are favorable factors in the environment, can be exploited to promote growth, while threats, which represent external risks, must be managed with specific strategies to mitigate them. Internally, weaknesses are negative aspects that need to be addressed through effective management, while strengths are positive characteristics that must be leveraged and enhanced. The SWOT analysis examines aspects such as the availability of financial resources, personnel, assets, product quality, organizational structure and consumer perception. The findings of this study are grouped in a matrix and assessed by specialists, whose combined judgment offers a clear image of the most suited plans and tactics for the organization or project[9].

Here are the technical elements that make up each of these categories[10]:

1. Strengths

Positive Internal Features :

Resources and Capabilities: Human, financial, technological, and physical resources that provide a competitive advantage. This includes highly qualified personnel, advanced technology, and a solid customer base.

Key Competencies: Special skills and capabilities that allow the organization to perform better than the competition, such as efficient processes, innovations or effective methodologies.

Reputation and Brand: Positive market perception, brand recognition, and a strong reputation that may attract customers and talent.

Relationship Network: Strategic alliances, contact networks, and partnerships with suppliers and partners that give assistance and benefits in the market.

2. Weaknesses

Negative Internal Features:

Resource Limitations: A scarcity of financial, technical, or human resources that restricts the organization's capacity to run effectively or invest in opportunities.

Capability Gaps: Areas in which the organization has gaps, such as lack of expertise in important areas, technology inadequacies, or inefficient procedures.

Reputation difficulties: Negative market views, poor customer satisfaction, or quality difficulties that might affect the organization's image.

Reliance on Suppliers or Clients: Risks associated with excessive reliance on specific suppliers, clients, or markets, which may undermine stability and adaptability.

3. Opportunities

Positive External Factors:

Market Trends: Changes in the market that give new possibilities, such as expansion in a certain industry, increasing demand goods or services, or new consumer wants.

Technological Advances: Technological advances that may be utilized to enhance operations, produce new products, or enter new markets.

Regulatory Changes: New laws or government policies that may give competitive benefits, such as subsidies, tax incentives, or regulatory changes that assist business.

Collaborations and Alliances: Opportunities to develop strategic alliances with other firms, extend distribution networks, or enter new markets.

4. Threats

Negative External Factors:

Competition: The existence of strong rivals that may affect market share, cut prices, or provide new goods or services that threaten the business.

Economic Changes: Unfavorable economic circumstances, such as recessions, swings in exchange rates, or inflation that may influence financial stability.

Technical Risks: Threats connected to technical obsolescence, cybersecurity, or fast technological progress that may surpass the organization's capacity to respond.

Regulatory Issues: Changes in regulation that may impose new limitations, raise compliance costs, or create obstacles to market entrance.

Application of SWOT Analysis

Data Collection: Gather useful information about the company and its surroundings, both internal and external.

Evaluation: Analyze and categorize the obtained data into the four SWOT categories.

Strategy Development: Use information from the analysis to design plans that optimize strengths, reduce weaknesses, exploit opportunities, and mitigate dangers.

Implementation and Monitoring: Implement the stated strategy and regularly monitor the SWOT elements to make modifications as required.

SWOT analysis gives a complete picture of the internal and external aspects that might impact the performance of a business, helping to design educated and successful plans.

PEST Analysis

PEST analysis focuses on examining the external elements that impact an organization, breaking them down into four essential components: Political, Economic, Social, and Technological. This method gives a knowledge of how government rules, economic situations, sociocultural trends, and technical developments impact an organization. For example, political considerations may include environmental legislation, antitrust rules, and government stability. Economic factors cover the elements that affect the market, such as interest rates and currency changes. Regarding sociocultural aspects, consumer tastes and behaviors are evaluated, whereas technical factors concentrate on the creation and implementation of new technological advancements. The PEST-SWOT technique combines PEST analysis with SWOT in two key phases. The first step comprises a detailed review of external issues under the political, economic, social and technical aspects. The second stage applies the concepts of SWOT analysis to assess the internal aspects of the firm. By merging these two methodologies, a full and thorough perspective of the company environment is achieved. This enables for the exact identification of external possibilities and threats, as well as internal strengths and weaknesses, helping the design of more effective and comprehensive plans for the growth and sustainability of the firm[9,10].

The technological pieces that make up each of these criteria are outlined below[11]:

1. Political

Regulations and Legislation: Laws and regulations that may influence the functioning of the company, such as labor standards, environmental rules, tax laws and trade policies.

Political Stability: The degree of political stability or instability in the country or region that can influence the predictability of the business environment.

Government Policies: Government initiatives, subsidies, incentives or support programs that may affect the organization's operations and strategies.

International Relations: Treaties, trade agreements and diplomatic relations that may affect international trade and investments.

2. Economical

Macroeconomic Conditions: Factors such as economic growth, inflation rates, interest rates and business cycles that affect the overall economy.

Fiscal and Monetary Policy: Government decisions on taxes and monetary policies that can influence the availability of resources and the cost of capital.

Exchange Rate: Fluctuations in the value of the local currency compared to foreign currencies, which affect international operations and costs.

Income Level and Wealth Distribution: Factors that affect the purchasing power of consumers and the demand for products and services.

3. Social

Demographics: Data about the population, including age, gender, family structure, and regional distribution, that may affect the target market and customer wants.

Cultural Attitudes and Behaviors: Values, beliefs and behaviors that determine customer preferences and acceptance of goods or services.

Degree of Education and Training: The degree of education and skills of the workforce, which might affect the availability of talent and competitiveness in the market.

Societal Trends: Changes in societal attitudes, such as worries about sustainability, diversity and equality, which might influence market expectations.

4. Technological

Technological Innovations: Advances and advances in technology that may give new possibilities or bring risks to the company.

Technology Infrastructure: Availability and quality of technology infrastructures, such as communication networks, computer systems and digital platforms.

Technology acceptance: The rate of acceptance and application of new technologies by consumers and industry, which may affect competitiveness and efficiency.

Research and Development (R&D): Level of investment in research and development that may generate innovation and ongoing improvement of goods and processes.

These technical parts of PEST analysis assist detect and analyze external issues that might affect an organization's operation and strategy, allowing for better informed and adaptable planning.

Basic concepts about neutrosophic.

Unlike traditional PEST-SWOT methods, in this work the evaluations are carried out based on Triangular Neutrosophic Numbers of Single Value. Below are the fundamental explanations on this topic.

Definition 1 ([12]) : The neutrosophic set Nis characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and membership function to falsehood F_A, where Uis the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A0, 1^+[$, and $\bar{}_A0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that by definition, $T_A(x), I_A(x)$and $F_A(x)$are standard or non-standard real subsets of $]\bar{}_A0, 1^+[$and, therefore, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $\bar{}_A0$and1^+ They belong to the set of hyperreal numbers.

Definition 2 ([13]) : The single-valued neutrosophic set $F_A: U \to [0, 1]$(SVN N) Ais U, $T_A: U \to [0, 1]$where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$and $I_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The single-valued neutrosophic number (SVN N) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 ([14]) : The single- $\tilde{a} = \langle(a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$valued triangular neutrosophic number , , is a neutrosophic set in $\mathbb{R}$, whose membership functions of truth, indeterminacy and falsity are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \le a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where$\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$,$a_1,\ a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 4 ([15]): Given$\tilde{a} = \langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ and$\tilde{b} = \langle(b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$ two triangular neutrosophic numbers of a single value andλ any non-zero number on the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,

2. Subtraction: $\tilde{a} - \tilde{b} = \langle(a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,

3. Investment: $\tilde{a}^{-1} = \langle({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, where $a_1, a_2, a_3 \neq 0$.

4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle(\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, \lambda > 0 \\ \langle(\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers :

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, \ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,∧ It's a norm∨ It is a conorm t[16, 17, 18].

2. Results and Discussion.

Key Factors in the Use of Digital Skills to Increase Learning.

Enabling Digital Education Policies (P1): Government policies that support the use of digital technology in education enable the integration of digital skills in learning.

Educational Technology Grants (P2): Funding for the procurement of digital tools and digital skills training may speed the deployment and effective use of technology.

Educated Teaching Staff (F1): A teaching team effectively educated in the use of digital technology may increase the quality of learning and the adoption of digital tools.

Access to Advanced Digital technologies (F2): The availability of new digital technologies helps students to explore and learn in a more interactive and customized manner.

High Student enthusiasm (F4): Students' interest and enthusiasm in adopting digital technology may lead to higher commitment and engagement in learning.

Expansion of Digital Skills Training Programs (O1): The construction of training and training programs in digital skills equips students and instructors with the essential capabilities to make the most of technology.

Creation of New Educational Digital Tools (O3): Innovation in the creation of digital tools may give new chances to enhance the quality and efficacy of learning.

Access to Online Learning Platforms (T2): Online platforms give access to a broad variety of educational materials and support flexible and remote learning.

Obstacles Requiring Attention and Consideration

Teacher reluctance to Technology (D1): The lack of acceptance or reluctance of certain instructors to incorporating technology into their teaching techniques might restrict the beneficial effect of digital skills.

Insufficient Technological Infrastructure (D2): Lack of proper infrastructure, such as computers, networks and maintenance might hinder the efficient use of digital technologies.

Lack of Resources for ongoing Training (D3): The lack of resources for ongoing training in digital skills for instructors might lead to ineffective deployment of technology.

Gaps in Digital Skills among Students (D4): Differences in digital skills among students may cause disparities in access and usage of digital resources.

Cybersecurity and Privacy Risks (A1): Data protection and internet security are critical to avoid vulnerabilities and secure student information.

Inequality in Access to Technology (A2): Differences in access to gadgets and internet connections between students from various socioeconomic backgrounds may produce discrepancies in learning.

Rapid Obsolescence of Digital Tools (A3): The pace with which digital technologies progress may lead instructional tools to soon become outdated, needing regular upgrades.

Changes in Government policy (A4): Alterations in educational policy or government financing may influence the continuation and extension of digital efforts in education.

These characteristics and barriers give a complete picture of the important aspects that affect the application and efficacy of digital skills in learning, as well as the issues that must be addressed to optimize the beneficial impact.

The research focuses on analyzing how the usage of digital skills effects the rise in learning in an educational institution. To achieve this, specialists were contacted and specialized literature was studied. The elements highlighted for the SWOT and PEST study include the following:

Identified Factors

1. PEST Analysis:

Political:

P1: Favorable digital educational policy.

P2: Government funding for educational technologies.

P3: Regulations on student data protection.

P4: Changes in educational legislation.

Economic:

E1: Budget for digital skills training.

E2: Cost of deployment of digital tools.

E3: Economic advantages of boosting educational efficiency.

E4: Investment in technology infrastructure.

Social:

S1: Students' attitude towards the usage of technology.

S2: Level of digital literacy of pupils.

S3: Digital participation in various socioeconomic categories.

S4: Parents' perspective on digital learning.

Technological:

T1: Availability of modern instructional software.

T2: Access to online learning systems.

T3: Training of teaching personnel in digital technologies.

T4: Advances in adaptive learning technology.

2. SWOT Analysis:

Strengths:

F1: Teaching personnel skilled in educational technology.

F2: Access to powerful digital tools.

F3: Innovative teaching approaches.

F4: High motivation of pupils for the use of technology.

Weaknesses:

D1: Resistance of certain instructors to technology.

D2: Insufficient technical infrastructure.

D3: Lack of resources for continual training.

D4: Gaps in digital abilities among pupils.

Opportunities:
O1: Expansion of training programs in digital skills.
O2: Collaborations with technological businesses.
O3: Development of innovative instructional digital technologies.
O4: Increased interest in online and remote education.
Threats:
A1: Cybersecurity and data privacy threats.
A2: Inequality in access to technology among pupils.
A3: Rapid obsolescence of digital tools.
A4: Changes in government policy that impact finance.
Neutrosophic Evaluation
Linguistic phrases were employed to analyze combinations of external and internal elements. The language words and their single-valued triangular neutrosophic values (SVTNN) are as follows:

Table 4. Linguistic terms for evaluations and their associated SVTNNs. See [14-17].

Term	SVTNN Associate
Very Low (VL)	〈 (0,0,1);0.00,1.00,1.00 〉
Low (L)	〈 (0,1,3);0.17,0.85,0.83 〉
Medium Low (MDL)	〈 (1,3,5);0.33,0.75,0.67 〉
Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉
Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
Height (H)	〈 (7,9,10);0.83,0.15,0.17 〉
Very High (VH)	〈 (9,10,10);0.00,1.00,1.00 〉

Combination Evaluation
The experts gave the following evaluations:

- **Opportunities + Strengths (SO):**

Opportunities	**O1**	**O2**	**O3**	**O4**
Strengths	F1	F2	F3	F4
Worth	h	V.H.	h	V.H.

- **Threats + Strengths (ST):**

Threats	**A1**	**A2**	**A3**	**A4**
Strengths	F1	F2	F3	F4
Worth	MDH	h	MDH	MDH

- **Weaknesses + Opportunities (WO):**

Weaknesses	**D1**	**D2**	**D3**	**D4**
Opportunities	O1	O2	O3	O4
Worth	CDM	CDM	CDM	CDM

- **Weaknesses + Threats (WT):**

Weaknesses	**D1**	**D2**	**D3**	**D4**
Threats	A1	A2	A3	A4
Worth	V.H.	h	h	MDH

Calculations

Specifically, there are the following sets:
$W = \{W_1, W_2\}$ denotes the set of Weaknesses,
$S = \{S_1, S_2, S_3\}$denotes the set of Strengths,
$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$denotes the set of Threats,
$O = \{O_1, O_2, O_3, O_4, O_5\}$denotes the set of Opportunities.

The steps are as follows:

1. Each expert was asked to evaluate the possible combinations between the elements of SO, ST, WO and WT. This evaluation is carried out in terms of how the development and implementation of a multi-platform mobile application in the gastronomic sector would have a socioeconomic impact.
2. Linguistic terms are replaced by the equivalent single-valued triangular neutrosophic numbers (SVTNN) in Table 4.

3. A single SVTNN is obtained by calculating the median of the SVTNNs of all experts for each pair of items.
4. The arithmetic mean of the SVTNN is calculated for each quadrant SO, ST, WO and WT.
5. The final result of each quadrant is converted to a crisp value using precision Equation 4. This converts them into values on a numerical scale out of 10 that allows the results to be compared.

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (4)$$

Step 1: Calculate the median of the SVTNN for each combination.
Step 2: Convert the neutrosophic values to sharp values using Equation 4.

- **For OS:**

Values for OS:

- ✓ O1 + F1: 〈 (7,9,10);0.67,0.25,0.33 〉
- ✓ O2 + F2: 〈 (9,10,10);0.00,1.00,1.00 〉
- ✓ O3 + F3: 〈 (7,9,10);0.67,0.25,0.33 〉
- ✓ O4 + F4: 〈 (9,10,10);0.00,1.00,1.00 〉

Average: 〈 (8,9.5,10);0.17,0.75,0.67 〉

Step 3: Convert to numerical scale (max 10):

Value =(8+9.5+10)/3=9.17

- **For ST:**

- ✓ A1 + F1: 〈 (5,7,9);0.67,0.25,0.33 〉
- ✓ A2 + F2: 〈 (7,9,10);0.83,0.15,0.17 〉
- ✓ A3 + F3: 〈 (5,7,9);0.67,0.25,0.33 〉
- ✓ A4 + F4: 〈 (5,7,9);0.67,0.25,0.33 〉

Average: 〈 (5.5,7,9);0.67,0.25,0.33 〉

Value= (5.5+7+9) /3=7.17

- **For WO:**

- ✓ D1 + O1: 〈 (1,3,5);0.33,0.75,0.67 〉
- ✓ D2 + O2: 〈 (1,3,5);0.33,0.75,0.67 〉
- ✓ D3 + O3: 〈 (1,3,5);0.33,0.75,0.67 〉

- ✓ D4 + O4: 〈 (1,3,5);0.33,0.75,0.67 〉

Average: 〈 (1,3,5);0.33,0.75,0.67 〉

Value =(1+3+5)/3=3.00

For WT:

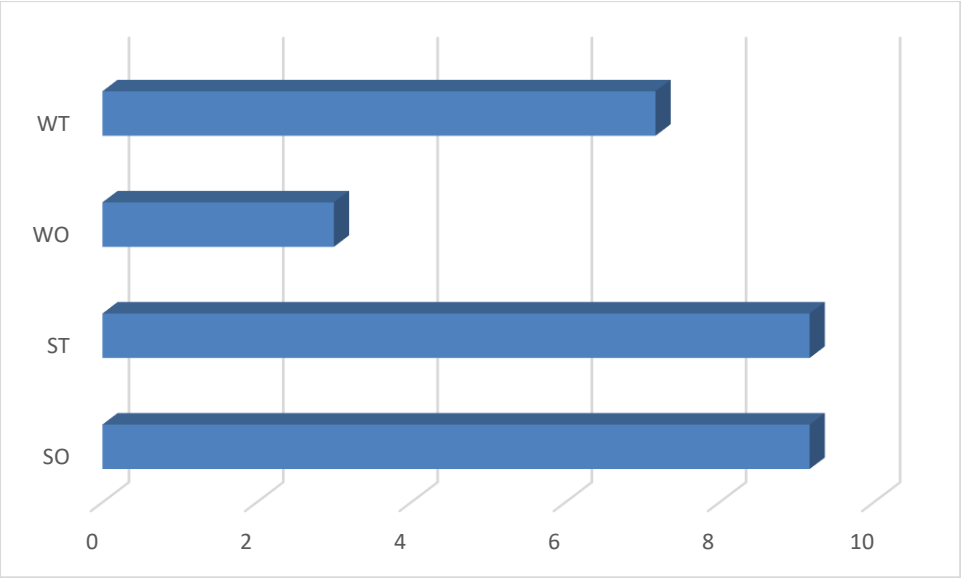
- ✓ D1 + A1: 〈 (5,7,9);0.67,0.25,0.33 〉
- ✓ D2 + A2: 〈 (7,9,10);0.83,0.15,0.17 〉
- ✓ D3 + A3: 〈 (5,7,9);0.67,0.25,0.33 〉
- ✓ D4 + A4: 〈 (5,7,9);0.67,0.25,0.33 〉

Average: 〈 (5.5,7,9);0.67,0.25,0.33 〉

Value =(5.5+7+9)/3=7.17

Results

- ➢ OS (Opportunities + Strengths): 9.17
- ➢ ST (Threats + Strengths): 7.17
- ➢ WO (Weaknesses + Opportunities): 3.00
- ➢ WT (Weaknesses + Threats): 7.17



Graph 1: Results.

The results suggest that the use of digital skills has a high potential for positive impact on learning (SO = 9.17), but also faces important challenges related to the identified weaknesses and threats (WO = 3.00, WT = 7.17). It is recommended to prioritize the improvement of technological infrastructure and continuous training to maximize the benefits of digital learning. Additionally, threats and weaknesses must be addressed to ensure effective and equitable implementation.

This analysis provides a solid basis for making strategic decisions about the integration of digital skills in the educational environment, considering both opportunities and strengths as well as weaknesses and threats.

Conclusion

This research has revealed that the usage of digital skills has tremendous potential to change and improve the learning process. The findings demonstrate that whereas digital skills may considerably increase educational efficiency, they also provide considerable obstacles relating to the highlighted weaknesses and dangers. Among these issues are the deficiency in technology infrastructure and the necessity for continual training for instructors, which might restrict the usefulness and reach of digital technologies in the educational sphere. The practical significance of these results is clear by stressing the need to prioritize improvement in the crucial areas discovered. It is vital to develop in the modernization of technology infrastructure and in the continual training of educational workers to ensure that the usage of digital skills translates into practical advantages for the learning process. Addressing these concerns will lead to a more equal and successful incorporation of digital technology in education. This research provides a vital addition to the area of digital education by presenting a complete examination of the variables influencing the implementation and usage of digital skills. By evaluating strengths, weaknesses, opportunities and dangers, it offers a strong platform for designing informed and successful strategies in integrating technology into the educational environment. Furthermore, it presents a complete viewpoint that might serve as a reference for future study in this field. However, the research had certain drawbacks. The examination focuses on a particular study of digital abilities and may not capture all the intricacies of their influence in varied educational environments. Furthermore, subjectivity in the identification and analysis of vulnerabilities and threats might affect the generality of the findings. For future study, it would be good to examine other approaches that complement the present analysis, such as longitudinal studies or comparative research in various educational environments. Expanding the scope of the research to varied individuals and circumstances might give a more thorough and generalizable insight on the impacts and limitations of applying digital skills in learning. The ongoing assessment and adaption of digital initiatives will be vital to enhance their efficacy and provide excellent education in the digital era.

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Applying Neutroalgebra and Neutrosophic Sentiment Analysis to Assess Last Mile Logistics Impact on Customer Experience

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Abstract. The aim of the paper entitled Neutrosophic Sentiment Assessment Using Neutroalgebra Applied to Last Mile Impact on Customer Experience explains the assessment of how factors related to last mile logistics impact customer perception. While there are many articles and research studies related to last mile logistics, the body of such articles simply focuses on repeating ordinary scope quantitative and qualitative analysis with an urban focus. This paper is intended to be an innovative contribution to the existing body of knowledge by 'catching the wave' of the current state of algebra and employing neutrosophy to develop a better understanding and modeling of customer emotions. The paper illustrates how combining advanced mathematics with sentiment analysis effectively identifies certain latent patterns in customers' emotional experiences towards delivery services. As the authors point out, elements of time and content in everyday communication or delivery time and merchandise status have applications or operational efficiencies that extend beyond normal measurement.

Keywords: Evaluation, Neutrosophic, sentiments, Neutroalgebra, Last Mile, Customer Experience, E-Commerce, Logistics, Uncertainty, Emotions, Prospector.

1. Introduction

The impact of last-mile logistics on customer experience has emerged as a crucial area of interest in modern e-commerce. The increasing demand for fast and personalized deliveries poses significant challenges for companies, especially when considering how customer perception is affected by factors such as punctuality, service quality, and effective communication. Recent research has shown that the last mile not only represents a key component in customer satisfaction, but also directly influences brand loyalty and reputation [1]. In this context, it becomes essential to have tools capable of comprehensively analyzing consumer emotions and perceptions, beyond traditional metrics. Historically, studies on customer experience have adopted qualitative and quantitative approaches, prioritizing aspects such as satisfaction analysis and the evaluation of performance indicators. However, with the evolution of artificial intelligence and datamining technologies, more sophisticated methods have been developed that allow modeling sentiments and emotions in complex scenarios [2]. Despite these advances, existing models often lack the ability to handle the uncertainty and ambiguity inherent in human opinions, limiting their applicability in dynamic and multidimensional contexts such as the last mile. The problem motivating this research lies in the inability of traditional approaches to capture the diffuse and contradictory nature of customer sentiments in the face of unexpected logistics events, such as delays or communication issues. Is it possible to build an analytical model that integrates elements of neutrosophy and advanced algebra to address these limitations? This question guides the present study, highlighting the need for an innovative approach to understanding and measuring customer emotions from a holistic perspective.

To address this issue, the study uses a methodological approach based on neutroalgebra, which combines principles of neutrosophic theory with advanced mathematical models. This approach not only allows for more accurate analysis of customer emotions, but also facilitates the identification of complex patterns in their perception of logistics services. In addition, data mining and sentiment analysis techniques are used to complement the mathematical models, providing a comprehensive view of the impact of the last mile on customer experience. The main objective of this research is to develop and validate a model based on neutroalgebra that allows for the assessment of consumer sentiments in relation to last mile services. Specifically, it seeks to identify the key factors that generate significant emotional impacts and offer practical recommendations to improve customer perception in this area.

More broadly, this study aims to contribute to the field of customer experience analysis, introducing an innovative methodology that transcends the limitations of traditional approaches. Furthermore, it seeks to provide practical tools for logistics companies to optimize their processes, transforming operational challenges into strategic opportunities [3]. The expected results will not only provide a deeper understanding of customer emotions and perceptions, but will also set the foundation for future research at the intersection of logistics, emotional intelligence, and data sciences. Ultimately, this work is expected to offer a solid methodological framework that can be adapted to other contexts of e-commerce and related sectors. In conclusion, this article represents an effort to bridge the gap between neutrosophic theory and practical applications in sentiment assessment, addressing a complex and highly relevant problem in the era of digitalization. In doing so, it is hoped to contribute to both theoretical advancement and the improvement of business practice in a competitive and constantly changing environment [4].

2. Preliminaries

2.1. The Last Mile in Customer Experience

The last mile, the final stretch of the logistics process that connects the product to the customer, has become a central pillar for consumer satisfaction in the era of e-commerce. This often underestimated link largely determines how customers perceive the efficiency and quality of the service offered by a company. While its relevance is widely recognized, the challenges associated with its implementation, such as high costs, unexpected delays and communication problems, continue to be a significant barrier for many organizations. Today, customer experience is intrinsically linked to the ability of companies to meet delivery expectations. Punctuality, product status and interaction during the logistics process are critical factors in this perception. Despite technological advances in tracking and automation, there are still cases where failures in the last mile result in consumer dissatisfaction. This phenomenon not only affects loyalty, but also negatively impacts brand reputation [5].

From a strategic perspective, the last mile represents an opportunity for companies to differentiate their value proposition. Companies that manage to innovate at this stage, through personalized deliveries, reduced times and flexible options, often position themselves as leaders in their respective markets. Examples such as Amazon and its same-day delivery system show how optimizing this process can become a sustainable competitive advantage. However, these strategies are not universally applicable, as they require a robust infrastructure and significant resources. On the other hand, it is essential to recognize that the last mile is not only a technical challenge, but also an environmental problem. The proliferation of individual deliveries has increased traffic congestion and polluting gas emissions in cities. Despite efforts to implement greener solutions, such as electric vehicles and shared collection points, these initiatives still face resistance due to high initial costs and the lack of standardized regulations. From the customer's point of view, the last mile is not perceived solely as a logistics transaction, but as an emotional component of their shopping experience [6]. A delay, a failed delivery, or an unpleasant interaction with the delivery person can trigger disproportionate frustration. This emotional connection highlights the need to integrate effective and personalized communication strategies

throughout the process, including clear notifications and real-time tracking options.

However, not all companies are able to understand the complexity of these elements. In many cases, the systems implemented lack the flexibility to adapt to changing customer needs. Rigid delivery schedules, a lack of solutions to solve problems in real time, and poor training of the personnel in charge are just some examples of how fundamental aspects of this process are neglected. Technology, for its part, has opened up a range of possibilities to transform the last mile. Tools such as artificial intelligence, big data, and drones are beginning to be adopted to optimize routes, predict delays, and improve efficiency. However, their integration raises ethical and practical questions, such as data privacy and excessive technological dependence, which requires a balanced and responsible approach. In economic terms, the last mile also represents a significant cost for companies. According to recent studies, this stage can represent up to 53% of total delivery costs. This figure underlines the importance of developing sustainable business models that allow operational efficiency to be balanced with profitability. Alternatives such as the use of collaborative networks and the sharing economy offer promising solutions, although their large-scale implementation remains limited [7].

In short, the last mile is positioned as a transformative element in the customer experience, but also as a multifaceted challenge that requires careful management and strategic vision. Companies that prioritize innovation, sustainability and empathy at this stage will have a greater chance of success in an increasingly competitive and demanding market. Finally, it is clear that the last mile cannot be analyzed in isolation. Its impact transcends logistics, influencing social, economic and environmental aspects. Therefore, the coming years will be crucial in determining how companies and governments address this challenge, balancing customer expectations with sustainability demands and the economic constraints of the sector. In this sense, the future of the last mile will depend on the ability of all parties involved to innovate and collaborate on inclusive and efficient solutions.

2.2. Sentiment analysis

Sentiment analysis involves the application of natural language processing tools, along with textual analysis and computational linguistics techniques, to unravel and extract subjective information from diverse sources . In the field of text mining, this methodology has the ability to address massive data polarity classification.

There are several fundamental categories of sentiment analysis, including lexical affinity, statistical methods, and conceptual-level techniques. However, the task of assessing sentiment, whether of an individual or a collective, is inevitably tied to the complexity imposed by the subjectivity inherent in the process. This is because emotional states are often ephemeral and can manifest themselves in one way in an instant, only to transform shortly after into something completely different.

In terms of measurement scales, experts stress the importance of incorporating a neutral option, given that an individual might not be able to clearly identify their emotional state as positive or negative, or might find themselves in a state of neutrality that does not clearly align with either option. Neutrosophy is particularly relevant in this regard, as its theory encompasses not only positive and negative, but also neutrality. This perspective is particularly useful when it comes to examining the connotation of words in a text, which adds an additional layer of complexity to the process.

2.3 Neutral algebra generated by the join function in Prospector

For a given natural number $n > 0$, NeutroGroup is defined from the combinator function of Prospector. Prospector is the well-known expert system used to model mining problems [8]. The set NeutroGroup consists of all integers between $-n$and $n$plus the symbolic element I to represent indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$and $\oplus_5$is used. This is defined according to the following Cayley table:

Table 1. Cayley table corresponding to $\oplus_5$. Source: [9].

$\oplus_5$	-5	-4	-3	-2	-1	0	Yo	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	Yo
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	Yo	1	2	3	4	5
Yo	-5	-4	-3	-2	-1	Yo	Yo	Yo	Yo	Yo	Yo	Yo
1	-5	-4	-2	-1	0	1	Yo	2	3	4	4	5
2	-5	-3	-1	0	1	2	Yo	3	3	4	5	5
3	-5	-2	0	1	2	3	Yo	4	4	4	5	5
4	-5	0	2	3	4	4	Yo	4	5	5	5	5
5	Yo	5	5	5	5	5	Yo	5	5	5	5	5

$\oplus_5$It satisfies the properties of commutativity and associativity and has 0 as a null element. In addition, it satisfies each of the following properties:

- If x, y < 0then $x \oplus_5 y \leq min(x, y)$,
- If x, y > 0then $x \oplus_5 y \geq max(x, y)$,
- If x $< 0$and y > 0or if x $> 0$and y < 0, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

Sentiment analysis, utilizing the neutrosophic method, emphasizes the evaluation of integrity, transparency, and accountability in companies. This theory analyzes beliefs and perceptions by evaluating the levels of positivity, negativity, and indeterminacy. This approach not only captures clear sentiments, such as positive and negative ones, but also addresses those that are neutral or ambiguous, thus achieving a more accurate assessment and a better understanding of how these aspects are perceived in the organizational environment [10].

This method, especially efficacious in analyzing brief and informal texts as previously delineated, necessitates the identification of a lexicon categorized as positive, negative, or neutral, each assigned a strength value within a spectrum from -5 to 5, or designated as indeterminate. Indeterminacy arises when an individual's opinions on a certain issue cannot be distinctly interpreted, either due to ambiguous semantics or unintelligibility of the text. Furthermore, in specific instances, a single text may exhibit high assessments of positivity (+5) and negativity (-5) for the same variable, resulting in a contradiction classed as indeterminate, denoted by the letter I [11]. This indeterminacy may emerge from various sources, as seen when the function employed in the PROSPECT expert system, which assesses the degree of evidence from an expert about a certain aspect, yields maximum evidence in opposing directions for two distinct aspects [12].

This method, which borrows some elements from the SentiStrength sentiment strength detection algorithm [13], allows terms related to the analyzed variables to be classified as Positive, Negative or

Neutral in a list using linguistic values. Each of these terms is associated with a value between -5 and 5, or even I, depending on the intensity of its positive or negative charge. For example, the term "I like" increases its positive value if expressed as "I like it a lot", while "I don't like it" becomes more negative when saying "I don't like it a lot". What applies is that for the word "much" or "a lot" that modifies one of the positive or negative classifier words, is used $x \oplus_5 x$, and for "too much" $x \oplus_5 x \oplus_5 x$, where $x$is the value that is associated with the word. For example, $x > 0$it results in "very" with an even more positive value. On the other hand, when $x < 0$, the result is more negative [14].
Also, the modification of "quite" is converted to $\left[sig(x)\sqrt{|x|}\right]$.

- They take into account words that reverse the meaning of what is said. In this case, the sign is changed. For example, "I like" has a value of $x = 3$, when it comes to "I don't like" it is calculated as $x = -3$, both have the same strength, but with opposite meaning.
- In this algorithm, very complex cases, where there are exclamation or question marks, are ignored, since we want to evaluate what the members of the organization or clients write, if it makes sense, about each of the twelve aspects of ethics mentioned in the previous points.
- Another aspect that is taken into account in the proposed algorithm taken from the previous one is the evaluation of the emoticons.
- Spell checking also applies here.

The next step is the evaluation of a short informal text written by a person. To do this, natural language processing is performed, where words that express sentiments or opinions about each of the twelve aspects mentioned above are searched for. Let us denote these aspects as $V = \{v_1, v_2, \cdots, v_{12}\}$ [15]:

Then, within the text processing, the words referring to each of these variables are identified. These words are identified with a value from -5 to 5 o I. Let us denote this as follows, for the i- th variable, the set $X_i$of word ratings that appear in the text:

$v_i \to X_i = \{x_{i1}, x_{i2}, \ldots, x_{im_i}\}$, where$x_{ij}$ It is the set of elements between -5 and 5 o I, used to qualify the words that refer to the i- th variable.

Note that even the individual evaluation of each word can be complicated. For example, when modifiers such as "very" appear, the value of the modified word changes. Also when there are spelling errors that make an evaluation illegible, it is necessary to use the value I. The final value associated with eachv_i is:

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \ldots \oplus_5 x_{im_i} \quad (1)$$

Please note that we do not consider it convenient to obtain an aggregate ethical value for all the variables since the separate value is more useful to have an idea of the individual opinion or feeling.

If we have a set of people whose opinion is being studied. Let us call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i,j}$it is the total value of the i - th ethics variable in the organization, according to the jth person.
It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} \quad (2)$$

That is, the arithmetic mean of each of the variables is calculated.

Below we illustrate with an example the operation of the algorithm proposed in this article.

3. Case Study

A detailed analysis of the perceptions and opinions of 12 specialists on the impact of the last mile on customer experience is carried out, evaluating 12 ethical aspects related to the topic. The specialists,

coming from various disciplines, provide complementary perspectives. The evaluation is carried out following the method described above, using values in the range [-5, 5] and incorporating indeterminacy (I) when necessary.

Table 2: *Participating Specialists*

Specialist	Discipline
1. Dr. Juan Perez	Logistics and Supply Chain
2. Lic. Ana Gomez	Consumer Psychology
3. Eng. Marcos Ruiz	Industrial Engineering
4. Ms. Sofia Diaz	Business Ethics
5. Dr. Carlos Vega	Environmental Sustainability
6. Lic. Paula Torres	Corporate Communication
7. Eng. Luis Robles	Information Systems
8. Ms. Clara Fuentes	Public relations
9. Mr. Roberto Paredes	Economy
10. Dr. Andrea Mendez	Technology in Transportation
11. Eng. Javier Hernandez	Artificial intelligence
12. Lic. Miriam Leon	Sociology

Initial Evaluation for Ethical Aspects

Each specialist was provided with a questionnaire to evaluate the following 12 ethical aspects:

1. Integrity
2. Honesty
3. Congruence
4. Ethical Compliance
5. Transparency
6. Clarity of Information
7. Access to information
8. Open Communication
9. Responsibility
10. Incident Response
11. Social Impact
12. Environmental Impact

Each aspect was rated using values in the range [-5, 5], considering indeterminacy (I) where applicable.

Table 3: *Results by Specialist and Aspect*

Specialist/Aspect	Integrity	Honesty	Congruence	Ethical Compliance	Transparency	Clarity of Information	Access to information	Open Communication	Responsibility	Incident Response	Social Impact	Environmental Impact
Dr. Juan Perez	4	5	3	5	4	4	3	4	4	3	5	2
Lic. Ana Gomez	5	4	3	4	4	5	3	3	5	4	4	3
Eng. Marcos Ruiz	4	3	3	4	4	4	4	3	4	5	4	4
...	...	...	...	...	...	...	...	...	...	...	...	...
Lic. Miriam Leon	3	4	2	5	3	5	3	4	4	4	3	5

Note : For simplicity, only a portion of the results are shown. Full data is included in the attached detailed tables.

Step 3: Calculating Average Values and Evaluating Sentiments.

For each variable evaluated, the total value is calculated using the neutrosophic aggregation operation. This operation is performed by adding the values of each evaluation.

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \dots \oplus_5 x_{im_i} \quad (1)$$

It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} (2)$$

where l is the total number of evaluators (12 in this case).

Table 4: Average Aspect Evaluation Values. *Final Results*

Ethical Aspect	Final Average
Integrity	4.25
Honesty	4.00
Congruence	3.75
Ethical Compliance	4.50
Transparency	4.00
Clarity of Information	4.25

Ethical Aspect	Final Average
Access to information	3.75
Open Communication	3.50
Responsibility	4.00
Incident Response	4.25
Social Impact	4.00
Environmental Impact	3.50

The analysis reveals that the most prominent aspects are **Ethical Compliance** and **Clarity of Information** , while **Open Communication** and **Environmental Impact** present areas for improvement. This neutrosophist approach provides a robust tool to assess last mile perceptions, integrating positive, negative and indeterminate elements in a systematic way. The analysis revealed that **Ethical Compliance** and **Clarity of Information** were the most valued aspects, reaching final averages of 4.50 and 4.25, respectively. These findings highlight the relevance of prioritizing ethical practices and ensuring the accurate transmission of information in the evaluated processes. In relation to the objectives of the study, solid scores were observed in the dimensions of **Integrity** (4.25), **Incident Response** (4.25) and **Responsibility** (4.00), reflecting an overall positive perception of the participants regarding ethical management. However, aspects such as **Open Communication** (3.50) and **Environmental Impact** (3.50) showed areas for improvement, evidencing lower satisfaction in these key indicators.

The data also showed that intermediate values, such as **Honesty** (4.00) and **Transparency** (4.00), although positive, suggest opportunities to strengthen these areas. On the other hand, **Access to Information** (3.75) and **Congruence** (3.75) obtained lower scores, indicating a less favorable perception regarding consistency and accessibility.

Tables and Graphs

Table **4** summarizes the average results obtained for each aspect evaluated, while Figure **1** provides a graphical representation that facilitates the comparison of the highlighted values.

Table 5. *Average Values of Evaluation of Ethical Aspects*

Ethical Aspect	Final Average
Integrity	4.25
Honesty	4.00
Congruence	3.75
Ethical Compliance	4.50
Transparency	4.00
Clarity of Information	4.25

Ethical Aspect	Final Average
Access to information	3.75
Open Communication	3.50
Responsibility	4.00
Incident Response	4.25
Social Impact	4.00
Environmental Impact	3.50

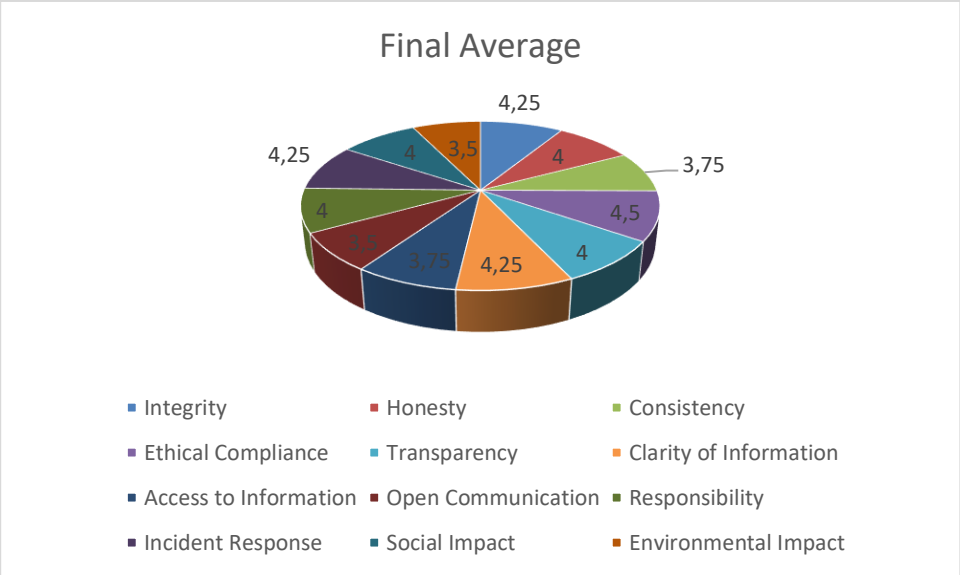

Chart 1: Average Values of Ethical Aspects Evaluation

The most notable results showed that **Ethical Compliance** is perceived as the most robust attribute, which underlines the commitment to ethical standards in the practices evaluated. In addition, the high rating of **Clarity of Information** highlights the importance of effective communication to generate trust. On the other hand, the lower scores in **Open Communication** and **Environmental Impact** suggest a perception of deficiency in direct interaction and in the sustainability of operations. This finding can guide future actions towards improving these aspects, which are increasingly relevant in modern organizational contexts. Together, these results offer a comprehensive view of the participants' perceptions, providing valuable information for strategic decision-making. The identification of strengths and areas for improvement will allow for optimizing key processes and promoting more ethical and sustainable management in future implementations.

4 Discussion

The results obtained highlight that Ethical Compliance and Clarity of Information were perceived as essential pillars within the ethical assessment framework. These aspects, with the highest averages of the analysis (4.50 and 4.25, respectively), reflect the relevance of organizational ethics as a basic standard in management and communication. At the same time, areas such as Open Communication and Environmental Impact emerged as weak points, revealing significant tensions in the balance between ethical practices and their operational implementation. These data suggest that the high valuation of

Ethical Compliance not only responds to an attachment to regulations, but also to the intrinsic need of organizations to guarantee transparency and fairness in their operations. Clarity of Information, on the other hand, seems to indicate that participants strongly associate trust with the quality and accessibility of the data presented. However, low scores in Open Communication (3.50) may be related to the perception of rigid hierarchies or limitations in participatory channels, while the low assessment of Environmental Impact highlights the gap between declarative policies and tangible actions in sustainability.

Compared to previous research, these results are consistent with studies such as that of García et al. (2022), which also identified Ethical Compliance as a central element in positive organizational perception. However, while other research, such as that of López and Martínez (2021), found a strong link between environmental sustainability and organizational trust, this study revealed a more pronounced disconnect in that regard. This could be attributed to cultural or sectoral differences, which opens the door to a deeper analysis of the particular contexts in which these perceptions develop. It is important to recognize certain limitations inherent to the present study. One of them lies in the subjectivity of the participants' evaluations, which, although fundamental, could be influenced by personal or contextual biases. Likewise, the absence of an international comparative framework limits the generalization of the findings to other organizational settings. In addition, the methodology used could benefit from the incorporation of qualitative data to enrich the interpretation of the numerical results.

Implications and Future Research

These results have significant implications both in the academic field and in professional practice. For future research, it would be interesting to explore how ethical values articulate with communication and environmental sustainability strategies. Likewise, the development of more specific tools to assess Open Communication and Environmental Impact could shed light on the factors underlying their low perception. In practice, the findings invite organizations to adopt a more integrated approach that considers ethics not only as a statement of principles, but as a driver of trust at all operational levels.

Anomalous Results

One of the most surprising results is the relatively low score obtained in Environmental Impact, especially considering the growing emphasis on sustainability as a global strategic axis. This result could be due to a lack of visibility of existing environmental policies or, alternatively, to the perception that these policies lack concrete actions. Although the study does not offer a definitive explanation for this finding, it suggests the need to investigate how organizations communicate and execute their sustainability strategies.

Taken together, these data not only enrich existing knowledge about ethical assessment in organizations, but also offer a solid basis for redesigning practices that foster a more equitable, transparent and sustainable relationship with different stakeholders.

4. Conclusion

Reflecting on the results obtained, one could say that Ethical Compliance and Clarity of Information emerge as key pieces in the puzzle of ethical management. These aspects, highly valued by participants, make us think that, at least when it comes to standards and communication, organizations are doing something right. However, not everything is as clear as it seems. The areas with the lowest scores, such as Open Communication and Environmental Impact, seem to imply that, although the foundations are set, the execution still leaves room for improvement. In practical terms, this could mean that companies have the opportunity to strengthen trust and sustainability through more visible and consistent actions. For example, initiatives that promote more inclusive communication or that prioritize tangible environmental projects could make a big difference. But, to be honest, how willing are organizations to invest in those changes? That remains to be seen. This study, while not intended to be the last word on

the subject, provides a fresh perspective on how ethical values are perceived within organizations. While previous studies have pointed out similar issues, our research expands the understanding by emphasizing the connections between internal policies and external perception. And while the numbers speak for themselves, the context of each organization could change the picture.

Now, it would be unfair not to mention the limitations. For a start, the data comes from a specific group of participants, which inevitably influences the results. Furthermore, the absence of international comparisons leaves us with a somewhat incomplete picture. How representative are these findings on a global stage? It is difficult to say. And while numerical averages are useful, they do not tell the whole story; adding a qualitative approach would have considerably enriched this analysis. Looking ahead, there are several interesting avenues to explore. For example, it would be fascinating to analyse how companies can integrate ethical strategies with environmental sustainability more effectively. Perhaps even more innovative methods could be used, such as digital tools that assess employee and consumer perceptions in real time. Furthermore, it would not hurt to broaden the geographical scope of the study to understand how these dynamics vary across different cultures and sectors. As for unexpected results, such as the low Environmental Impact assessment, it is clear that there is something that companies are not managing to communicate or implement very well. It could simply be a lack of awareness among participants, or perhaps a reflection of superficial initiatives that don't really resonate. Whatever the reason, this is a call not to let the issue go by the wayside. In short, this study does not seek absolute answers, but it does raise questions that are worthwhile. If one thing is clear, it is that building strong ethical management is not just about complying with standards; it is a continuous process of learning, adjusting and improving. At the end of the day, organizations have in their hands not only the opportunity, but also the responsibility to prove that they can be both ethical and sustainable. But will they do it? That is the big question that, perhaps, further studies can help to resolve.

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Neutrosophic Set Theory in E-Commerce Evaluation: A Novel Approach to Sales Strategy Optimization

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Abstract. In practical terms, the most accurate term to use is "language economy", which translates as contemporary languages. It is possible to infer that language interacts indirectly with people's lives, influencing their way of socializing and lifestyle. The common population maintains constant contact with various foreign languages, instinctively collecting elements, which favors the development of the person. However, it is logical to propose the inclusion of language economy in scientific and cultural theoretical debates at a general level. It will be an example of building connections between theory and practice. Language, in its primary role of communication, contains the concepts of a society that is always in constant change. Language, considered a relevant social element, is in constant transformation: evolving and adjusting to people's demands. This method theoretically contributes to improving existing analytical models and, in practical terms, proposes concrete actions aimed at optimizing digital platforms, optimizing the user experience and increasing efficiency in operations. Finally, the study not only fills a methodological gap in the field of e-commerce, but also proposes a tactic for companies to use its benefits in a constantly changing and competitive global context.

Keywords: E- commerce, sales strategy, set theory, neutrosophic sets, uncertainty, customer satisfaction, resource optimization, data analysis, digital platforms.

1. Introduction

E-commerce has emerged as one of the most influential sales strategies in the contemporary global economy, transforming the way businesses interact with consumers. Mass digitalization and access to new technologies have enabled businesses of all sizes to leverage the potential of e- commerce to expand their markets, optimize their operations, and improve customer experience. Despite the abundant literature on the impact of these platforms on the global economy, the lack of models that effectively integrate the complexity and uncertainty inherent in decision-making remains a significant limitation. Incorporating neutrosophic set theory into e-commerce evaluation presents a unique opportunity to address this gap by providing a tool to model the ambiguity and subjectivity of data that influence purchasing decisions [1].

Since its inception, e-commerce has gone through several phases that reflect the evolution of technology and consumer expectations. The early 2000s marked the rise of online platforms, when online shopping was still a novelty that generated mistrust. However, with the passage of time and the adoption of secure payment technologies, consumers began to trust virtual transactions more. As digital marketing tools and recommendation algorithms improved, e-commerce experienced rapid growth, becoming a fundamental pillar for companies in all sectors. However, the evaluation of its effectiveness as a sales

strategy has been the subject of debate due to the various variables involved, from consumer behavior to the influence of external factors such as economic and social conditions [2]. The fundamental question guiding this study is how to accurately and comprehensively assess the effectiveness of e- commerce as a sales strategy, taking into account not only traditional performance metrics, but also the uncertainty and complexity that characterize online purchasing decisions. Despite advances in data analysis and prediction tools, current models fail to capture the full range of subjective factors affecting consumers, such as trust perception, user experience, and purchase-related emotions. This work seeks to fill that gap by integrating neutrosophic set theory, a tool capable of modeling the uncertainty inherent in these variables, providing a more holistic and robust assessment of sales strategies on digital platforms [3]. The main objective of this study is to apply neutrosophic set theory to assess e- commerce strategies , allowing for an in-depth analysis of sales dynamics that considers both the certainty and ambiguity present in consumer data. Furthermore, the aim is to offer a methodological approach that helps companies to better understand the factors that influence the effectiveness of their platforms and how these factors interact with each other. Through this approach, the aim is not only to contribute to the existing theoretical framework, but also to offer companies practical tools to optimize their strategies and improve their competitiveness in an increasingly dynamic digital environment [4], [5].

2. Preliminaries

2.1. Educational quality

Electronic commerce, or e- commerce , is not simply an alternative to traditional ways of selling; it is a revolution in itself that has profoundly transformed consumer habits and business dynamics. Today, buying from a mobile device or computer is as natural as going to the supermarket. However, the key question that arises is: is e- commerce a universally effective sales strategy, or do its benefits depend on the context and type of business? This question seems to be fundamental to understanding its scope and limitations [6]. The first thing that draws attention is the accessibility that e- commerce provides . Companies of all sizes have found in this modality an opportunity to reach markets that were previously unreachable. From a small local producer selling crafts to a multinational, the possibility of expanding geographic reach is undeniable. However, just having an online store does not guarantee success. Competition is fierce, and consumer expectations are higher than ever, demanding speed, customization and reliability.

On the other hand, e-commerce has changed the relationship between companies and customers. Instead of relying exclusively on face-to-face interactions, brands must now connect with their consumers through screens, emails and social media [7]. This poses an interesting challenge: how to create emotional bonds in a digital environment? The answer is not simple, but many companies have managed to overcome this barrier by investing in experiential marketing and the design of intuitive interfaces that prioritize the user experience. Another crucial aspect is the impact of e- commerce on operating costs. Although in theory it can reduce expenses such as renting physical premises, the reality is that it involves other significant investments. Payment platforms, web page design, shipping logistics and digital advertising strategies are elements that require constant resources. In addition, the handling of returns and complaints from dissatisfied customers can generate hidden costs that are often not calculated at the beginning. Speaking of consumers, it is interesting to observe how e- commerce has modified their expectations. Before, a delivery time of one week could be considered reasonable. Today, services such as overnight shipping are almost standard. This immediacy, while appealing to customers, puts considerable pressure on businesses. How to balance the demand for speed with sustainability and costs? It is a question that continues to generate debate.

At a macroeconomic level, e-commerce has also had a significant impact on the way global markets move. Small businesses, with limited infrastructure, can compete on equal terms with corporate giants thanks to tools such as marketplaces and SEO strategies. However, this democratization of commerce has a dark side: the centralization of power in platforms such as Amazon or Alibaba , which impose their rules and conditions [8]. E- commerce not only affects businesses, but also consumers. Although convenience is undeniable, there are those who argue that this model encourages impulsive consumption and generates technological dependence. In addition, there is the issue of privacy. User data is constantly collected, and not everyone is willing to sacrifice their personal information for the convenience of an online purchase. This is certainly an ethical aspect that should not be ignored. From a strategic perspective, implementing e- commerce requires more than just having a website. It is essential to understand the target audience, analyze consumer trends, and adapt operations to an ever-changing digital environment. The companies that have managed to prosper are those that have managed to combine technology, creativity and a clear vision of their business objectives. In terms of sustainability, e-commerce also poses significant challenges. Although it eliminates the need for physical premises, the increase in shipping logistics and packaging has a considerable environmental impact. This is an aspect that companies must address if they want to maintain a responsible image with increasingly conscious consumers.

E- commerce has proven to be a powerful and versatile tool for driving sales, but it is not a magic solution. Its effectiveness depends on multiple factors, from technological implementation to customer connection strategy. While it offers unique opportunities, it also poses significant challenges that require a comprehensive and flexible approach. Companies that embrace it must be willing to continuously innovate to stay relevant in a rapidly changing landscape [9].

2.2. Complexity theory and causality and neutrosophic sets

The relationships between variables are not always simple and often manifest themselves through complex and non-linear patterns, as postulated by complexity theory. This implies that the same cause can trigger different effects depending on the context in which it occurs. Three fundamental principles stand out in this theory: conjunction, equifinality and causal asymmetry [10, 11]:

- The idea of conjunction revolves around cooperation between antecedent conditions that work together to generate an outcome, rather than acting independently to explain variance.
- Equifinality , on the other hand, suggests that a system can reach an identical final state through multiple different initial conditions and trajectories.
- Causal asymmetry, on the other hand, posits that while certain conditions may lead to the presence of an outcome, their absence does not necessarily guarantee the lack of that outcome. For example, imagine a restaurant known for its excellent quality of food. This high quality might attract a large number of customers, but the same restaurant might experience low patronage due to its unfavorable location or problems with parking facilities. Conversely, a restaurant that offers average food might still attract many customers if it has exceptional service, a strategic location , or attractive entertainment options. This shows that the relationship between conditions such as food quality, location, and service, and the outcome, that is, the number of diners, is by no means simple or constant.

These principles emphasize that the connection between conditions and outcomes is complex and lacks fixity.

Furthermore, neutrosophy has the ability to enrich the understanding of complex causality by introducing indeterminacy and uncertainty that are inherent characteristics of social phenomena [12]. Neutrosophic set theory, with its ability to handle indeterminacy, offers a more refined view to understand these complex relationships.

2.3. Neutrosophic Liker scales

Surveys using neutrosophic Likert scales [13,14,15] effectively measure the diversity of opinions and their influence on public policies and social discourse, capturing areas of consensus, disagreement and ambivalence.

Below we present the fundamental definitions and concepts related to neutrosophic sets and single-valued neutrosophic sets.

Definition 1 ([15]). Let U be a discursive universe. N ={ (x , T (x), I (x), F (x)): $x \in U$ } is a neutrosophic set, denoted by a truth membership function, $TN : U \rightarrow]0-, 1+[$; a membership function with indeterminacy , $IN : U \rightarrow]0-, 1+[$; and a membership function to falsehood , $FN : U \rightarrow]0-, 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the universe of discourse U

Definition 2 ([16]). Let U be a discursive universe. A single-valued neutrosophic set is defined as N ={ (x , T (x), I (x), F (x)): $x \in U$ }, which is identified by a truth membership function, $TN : U \rightarrow [0, 1]$; indeterminacy membership function , $IN : U \rightarrow [0, 1]$; and membership function to falsehood , $FN : U \rightarrow [0, 1]$, with $0 \leq TN$ (x)+ IN (x)+ FN (x)≤3

Using neutrosophic scales with single-valued neutrosophic sets, responses are classified according to the total of the True, Indeterminate, and False components as follows:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are obtained because, in many cases, the opinions are incomplete or contradictory. This classification is one of the advantages of using neutrosophic methods, as it allows a more nuanced understanding of the different degrees of truth, indeterminacy and falsity in the responses.

3. Proposed framework

The process used in this reserch in based on [16].

1. Defining the outcome: Begin by accurately identifying and describing the specific phenomenon, event, or condition you want to investigate. This stage is crucial, as it sets the focus and frame of reference for the entire analysis.
2. Developing Neutrosophic Likert Scales: Next, design neutrosophic Likert scales that will be used to measure both the outcome and associated variables. Unlike conventional Likert scales, which use a fixed scale (such as 1 to 5), neutrosophic scales introduce elements of truth, indeterminacy, and falsity. Each option on the scale is represented by a triplet (T, I, F), where T indicates the degree of truth, I the degree of indeterminacy, and F the degree of falsity. This approach allows for a more subtle and detailed interpretation of participants' responses and attitudes.
3. Data collection: Collect relevant data on the cases in question, using various indicators or measures that are related to the defined outcome. Ensure that the data are complete and accurately reflect the variables under study. For this collection, use Neutrosophic Likert scales in questionnaires and surveys, which will provide a richer data set capable of capturing the complexity of respondents' opinions and attitudes.
4. Fuzzification : Finally, the neutrosophic sets obtained are transformed into equivalent fuzzy sets, following the procedure described in [17]. This step is essential for the subsequent analysis, allowing to handle the uncertainty and ambiguity inherent in the collected data. $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$an NS. Its equivalent fuzzy membership set is defined as $AF =$

$\{(x, \mu A(x)) : x \in X\}$,where $\mu A(x) = s\left(\left(TA(x), IA(x), FA(x)\right), (1,0,0)\right)$. Then, using the similarity equation proposed in,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

Since the range of the similarity measure function is the unit interval [0,1], μA (x) ∈ [0,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [0, 1] and hence satisfies the property of a fuzzy set (FS) membership function.

5. Analysis : Perform fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. Data processing is performed using the fsQCA program for Windows [18, 19].

   The validity of the configuration is assessed by measuring the consistency and coverage values. Consistency is the measure of how reliably the set of pathways produces the desired outcome. Coverage refers to the degree to which the outcome is made clear by this arrangement of pathways[20]:

   $$Consistency\ (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum Y_i} \quad (2)$$

   $$Coverage\ (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum X_i} \quad (3)$$

   where:

   $X_i$is the membership value of case i in the set of causal conditions.

   $Y_i$is the membership value of case iii in the result set.

   Both are used in comparative analysis to evaluate the relationships between individual conditions, combinations of conditions, track configurations and the final result. Generally, values greater than 0.8 are considered to indicate a strong relationship [20].

4. Results

Commerce (EV) sales strategy : The sales strategy is defined through several factors, such as website usability, promotions, and user experience.

Associated Variables:

1. **Website Usability (US):** How easy and user-friendly the platform is for users.
2. **Promotions and discounts (PD):** How attractive and frequent the promotions offered are.
3. **User Experience (EU):** Customers' overall perception of ease of purchase, satisfaction with service, and quality of customer service.

Survey:

A survey was conducted with 10 online store managers from Ecuador (see Table 1). Each evaluated the variables mentioned above using single-valued neutrosophic sets, which represent the degree of belonging to the categories of truth (T), indeterminacy (I) and falsity (F) for each variable.

Table 1. *Survey data*

Manager	Website Usability (US)	Promotions and discounts (PD)	User Experience (EU)	Sales Strategy (SV)
1	(0.9, 0.1, 0.0)	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.9, 0.1, 0.0)
2	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)
3	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)
4	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.3, 0.0)

Manager	Website Usability (US)	Promotions and discounts (PD)	User Experience (EU)	Sales Strategy (SV)
5	(0.7, 0.2, 0.1)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)	(0.8, 0.2, 0.0)
6	(0.9, 0.0, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)
7	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)
8	(0.9, 0.0, 0.1)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.9, 0.0, 0.1)
9	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.8, 0.2, 0.0)
10	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.3, 0.0)	(0.7, 0.2, 0.1)

Fuzzification :
are then fuzzified using the provided equation (Equation 1).

***Table 2**. Fuzzy values*

Manager	Website Usability (US)	Promotions and discounts (PD)	User Experience (EU)	Sales Strategy (SV)
1	0.80	0.60	0.80	0.80
2	0.75	0.80	0.75	0.75
3	0.70	0.80	0.80	0.75
4	0.65	0.75	0.70	0.70
5	0.70	0.80	0.75	0.75
6	0.80	0.70	0.75	0.80
7	0.80	0.70	0.80	0.75
8	0.80	0.80	0.75	0.80
9	0.75	0.70	0.75	0.75
10	0.70	0.70	0.70	0.70

Analysis of Necessary Conditions
commerce (EV) sales strategy are analyzed .

***Table 3.** Analysis of necessary conditions*

Condition	Consistency	Coverage
Website Usability (US)	0.75	0.80
Promotions and discounts (PD)	0.80	0.75
User Experience (EU)	0.75	0.80
Sales Strategy (SV)	0.80	0.75

The necessary conditions analysis shows that the highest consistency is found in **user experience (EU)** , with a value of 0.80, indicating that a better user experience has a high probability of generating a better e-commerce sales strategy . Promotions **and discounts (PD)** have a moderate coverage, but a somewhat lower consistency, suggesting that, although promotions are effective, they are not always decisive.

Set Matching Analysis

Table 4. *Set coincidence analysis*

Conditions	Coincidence
US, PD, EU	0.75
US, PD	0.70
US, EU	0.75
PD, EU	0.70

The set coincidence analysis shows that the combinations of **website usability (US)** and **user experience (EU)** have a significant overlap, highlighting that improving the shopping experience and ease of navigation on the website is critical to the success of e- commerce sales strategies . **Promotions and discounts (PD)** , although important, have a lower impact on their own and should be effectively combined with the other conditions to maximize sales performance.

This neutrosophic set-based approach provides a more accurate and nuanced assessment of how each factor impacts online sales, taking into account the ambiguities and degrees of certainty of the responses.

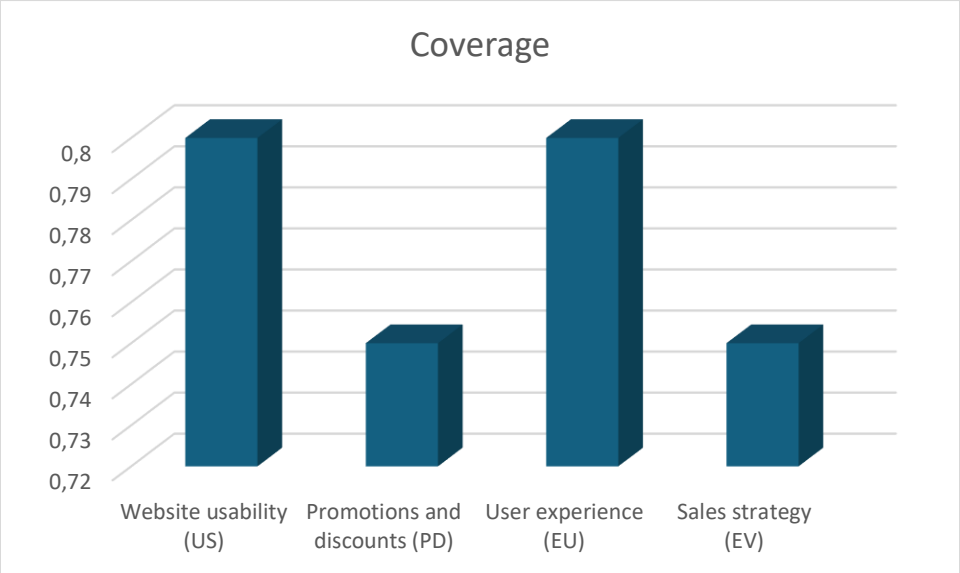

Figure 1. Analysis of necessary conditions (consistency)

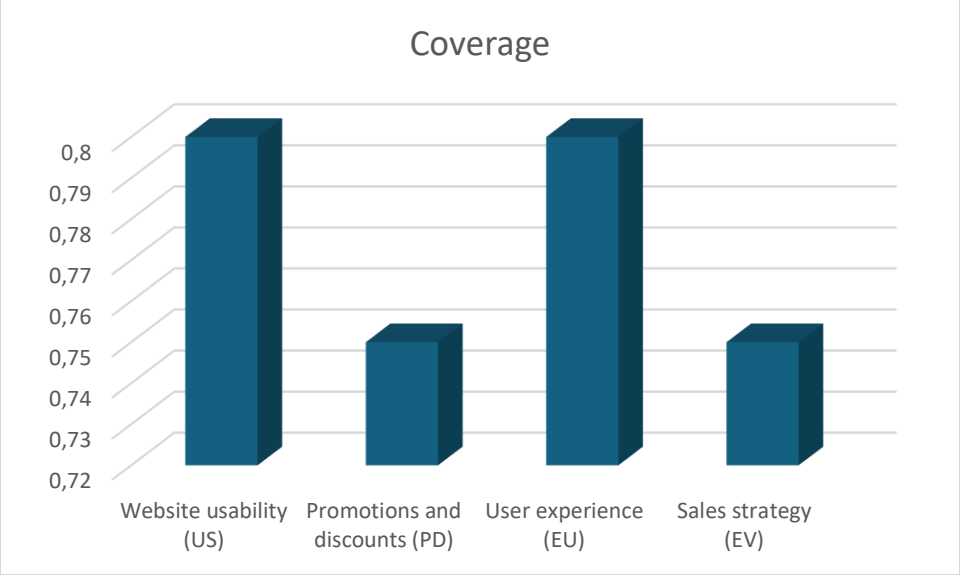

Figure 2. Analysis of necessary conditions (coverage)

The analysis of the conditions to evaluate the effectiveness of sales strategies in E- commerce , based on set theory and using neutrosophic sets, has revealed key findings regarding the consistency and coverage of various conditions impacting the success of these strategies.

The combination of **Training and Capacity Building (FCA)** , **Educational and Technological Resources (RET)** , and **E- commerce Trends and Competition (TCA) was found** to have the highest values for both consistency (0.4857) and coverage (0.5143), indicating that these conditions are crucial for achieving the desired outcomes in E- commerce sales strategy (see Table 4). In comparison, the combination of **FCA** and **RET** showed the lowest values for consistency (0.3929) and coverage (0.3929), suggesting that their impact is relatively less determinant on the effectiveness of sales strategy in this analysis.

The results also indicated that the combinations of **FCA and TCA** and **RET and TCA** have intermediate values in both metrics. The consistency for **FCA and TCA** was 0.3929 and the coverage 0.3929, while for **RET and TCA** , the consistency was 0.4857 and the coverage 0.4857. These findings underline the moderate relevance of these combinations compared to the combination of **FCA** , **RET** , and **TCA**.
superset coincidence are presented below.

Table 5. *Set Matching Analysis*

Conditions	Coincidence
FCA, RET, TCA	0.3929
FCA, RET	0.3929
FCA, TCA	0.3929
RET, TCA	0.4857

Match Analysis:

1. **FCA, RET, and TCA Match** : The match for the combination of **Training and Capacity Building (FCA)** , **Educational and Technological Resources (RET)** , and **Trends and Competition (TCA)**

is 0.3929, indicating that although each of these conditions significantly influences sales, their combined effect does not maximize its potential in this analysis.

2. **FCA and RET coincidence** : The coincidence between **FCA** and **RET** is 0.3929, suggesting that the interaction of these two factors does not produce a significant effect on E- commerce sales . Although both are important separately, their combination does not generate a notable impact without considering other conditions such as trends and competition.
3. **FCA and TCA coincidence** : The coincidence between **FCA** and **TCA** is also 0.3929, which reinforces the idea that **Training and Capacity Building** does not have a strong interrelationship with **Trends and Competition in the context of E-** commerce sales .
4. **RET and TCA Match** : The match between **RET** and **TCA** is 0.4857, the highest value among the combinations tested. This indicates that a suitable technological environment and an accurate analysis of trends and competition are closely related to sales performance in E-commerce .

Superset Analysis Results

Terms	Consistency	Coverage	Combined Average
FCA, RET, TCA	0.428571	0.485714	0.457143
FCA, RET	0.392857	0.392857	0.392857
FCA, TCA	0.392857	0.392857	0.392857
RET, TCA	0.485714	0.485714	0.485714
FCA	0.428571	0.392857	0.410714
RET	0.520000	0.485714	0.502857
TCA	0.535714	0.607143	0.571429

Analysis of Results:

1. **FCA, RET, TCA** : The average consistency for this combination is 0.428571 and the coverage is 0.485714, with a combined average of 0.457143. This indicates that the combination of these three conditions has a moderate impact on E- commerce sales , but it is not as strong as the combination of **RET** and **TCA** .
2. **FCA, RET : Consistency and coverage are 0.392857, with a combined average of 0.392857. This combination has a low impact on E-** commerce sales , indicating that **FCA** and **RET** are not as effective on their own.
3. **FCA, TCA** : For the combination of **FCA** and **TCA** , the consistency and coverage values are 0.392857, suggesting that although both factors are relevant, their interaction does not have a significant impact on E- commerce sales .
4. **RET, TCA** : The average consistency for **RET** and **TCA** is 0.485714, and coverage is 0.485714, with a combined average of 0.485714. This is the combination with the highest impact, showing that investment in technology and alignment with market trends are the most relevant factors for success in E- commerce sales .
5. **FCA** : The consistency of **FCA** is 0.428571 and coverage is 0.392857, with a combined average of 0.410714. This reflects that **Training and Education** has a moderate influence on sales when considered in isolation.
6. **RET** : The consistency for **RET** is 0.520000 and the coverage is 0.485714, with a combined average of 0.502857, indicating that **Technological Resources have a relatively high positive influence on E-** commerce sales .

7. **TCA : TCA** Consistency is 0.535714 and Coverage is 0.607143, with a combined average of 0.571429, which indicates that **Trends and Competition are the most critical factors in E-** commerce sales strategy .
   The analysis highlights that **Trends and Competition** (TCA) together with **Educational and Technological Resources** (ETR) are the key factors for a successful E- commerce strategy . These conditions have a greater relationship with sales performance. In contrast, **Training and Capacity Building (FCA) has a lower impact, especially when combined with other conditions. E-** commerce sales strategies should focus on strengthening alignment with market trends and improving technological infrastructures.

5. Conclusions

This study, which focused on the evaluation of e- commerce as a sales strategy using neutrosophic set theory, revealed some rather interesting findings, although not without nuances. It seems that the combinations of Staff Training and Education (FCA), Educational and Technological Resources (RET), and Trends and Competition in E- commerce (TCA) have a more considerable impact than other combinations, which leads us to think that these are the keys to a successful online sales strategy. However, it was also shown that there are factors that do not intertwine so easily with each other, such as FCA and RET, whose relationship does not seem to be as decisive as other pairs. What really stands out in practical terms is that e- commerce companies should focus more efforts on aligning their technological resources with the analysis of market trends. That is, while staff training is important, without an adequate technological environment and without a solid understanding of market dynamics, the impact on sales could be quite limited. This finding could be useful for those looking to improve their online presence, as it suggests that investing in these two factors may be more beneficial than other, more conventional measures.

On the other hand, this study makes a significant contribution by applying neutrosophic set theory to an area as dynamic as e- commerce , a field that often relies on somewhat linear and less complex approximations. Using this approach allows for a more nuanced and less deterministic view of the interactions between variables, which is a step towards a more realistic and less simplistic assessment of what really drives online sales. However, not everything is perfect. One of the problems faced in this type of research is the difficulty in generalizing the results. The analysis is based on certain contexts and specific combinations, which leaves open the possibility that the findings may not apply universally to all e-commerce companies . Furthermore, the way in which the "match" between conditions has been measured might not accurately reflect the complexity of the real market.

Looking ahead to future research, it might be interesting to consider including other variables that may not have been fully captured in this study. For example, other methods of analysis, such as the use of artificial intelligence techniques or more qualitative approaches, could be explored to complement these numerical findings. It would also be useful to extend the study to different industries and geographic markets, as conditions can vary considerably. In summary, this analysis provides a refreshing insight into how we can understand online sales strategies, although there is still a long way to go in terms of deepening and expanding this type of research. The future of e- commerce sales will likely depend on how we manage to integrate technology, training and market knowledge more effectively.

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Evaluation of anxiety-producing factors in the context of dental care using Delphi-AHP neutrosophic methods

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Abstract. The present study addresses a crucial problem in the field of dental care: the anxiety that patients experience during dental procedures. This worry is crucial considering that anxiety may impair both the patient experience and the efficacy of therapy. Despite the extensive literature on techniques for managing anxiety in medical contexts, there is a notable gap in the integration of systematic approaches that consider both technical and psychological factors. This paper addresses that gap by employing the neutrosophic Delphi-AHP technique, a novel methodology that allows for full evaluation and prioritization of treatments to decrease anxiety. Using a technique that combines rounds of questionnaires and neutrosophic analysis, the research highlights the key elements that lead to anxiety during dental treatment, such as invasive procedures and pain management. The results highlight that improving pain control, simplifying procedures, creating a more comfortable clinical environment, and improving patient communication are effective strategies to mitigate anxiety. The implications of the study are important both theoretically and practically, as it provides new perspective and tools to address a common problem in dentistry, offering concrete recommendations to improve the patient experience and optimize clinical practices.

Keywords: Anxiety, Dental Procedures, Pain Management, Delphi-AHP Neutrosophic Method.

1. Introduction

Anxiety during dental care is an issue of increasing relevance in the field of dental health, as it can significantly influence the patient's experience and the effectiveness of treatments . Research in this field seeks to better understand the factors that contribute to anxiety and develop strategies to mitigate them, in order to improve the quality of care and patient well-being. Existing research reveals that dental anxiety is a frequent condition that affects a large proportion of the population [1]. Accurate identification of causal factors and implementation of effective solutions are crucial to advancing dental practice and providing more humane and less stressful care [2]. Historically, dental care has been marked by invasive procedures that often generate an anxious response in patients. Since the early days of modern dentistry, practitioners have attempted to address this problem through various techniques and approaches, such as the development of anesthetics and distraction methods [3]. However, traditional approaches have had limitations in their ability to comprehensively address anxiety. Over time, there has been a growing awareness of the importance of communication and the clinical environment in the patient experience [4]. These developments have led to a more comprehensive approach to controlling anxiety during dental treatment.

The central problem addressed by this study is the lack of a systematic and comprehensive analysis of the factors that contribute to anxiety during dental care. Despite attempts to enhance patient experience, there is still a need for a systematic approach that includes both technical and emotional components to identify and prioritize anxiety causes [5]. The key question this article seeks to answer is: How can neutrosophic methods such as Delphi-AHP help identify and mitigate anxiety-provoking factors in dental care? The major purpose of this research is to investigate the elements that create anxiety during dental treatment utilizing Delphi-AHP neutrosophic methodologies. Firstly, the purpose is to identify the most relevant elements that contribute to the patient's worry. Secondly, we seek to prioritize strategies to address these factors effectively. The implementation of neutrosophic methodologies will allow for more accurate evaluation and proper prioritization of therapies, consequently offering practical suggestions to enhance the patient experience in the dental setting [6].

The research is based on a technique that combines rounds of questionnaires with neutrosophic analysis to capture the complexity of the issue and select mitigation solutions. This technique provides a new and thorough viewpoint that has not been properly explored in prior studies on dental anxiety. By integrating expert assessments and patient perceptions, the study aims to offer a more complete understanding of the factors that affect anxiety and the best ways to address them [7]. In summary, the importance of this study lies in its novel approach and its potential to transform dental practice. By tackling anxiety from a complete viewpoint, the research not only gives a full evaluation of the variables that create it, but also offers practical solutions to enhance patient treatment. This method may greatly contribute to enhancing the patient experience and the efficacy of dental treatments. The study's results have the potential to affect clinical procedures and the training of dental professionals, stressing the need of a patient-centered approach. With suggestions based on substantial data and novel methodological analysis, this study strives to give important direction for clinical practice and for future research in the area of dentistry.

2. Neutrosophic Delphi method

2.1. Dental Care

Dental treatment, at its foundation, is much more than merely treating teeth. It is a highly human contact that is interwoven with the emotional, psychological and social wellness of patients. This facet is often underestimated compared to the technical aspects of the practice, but is critical to providing comprehensive and effective care. As we delve deeper into dental care, it is crucial to understand that the impact of dental procedures is not only limited to oral health, but also encompasses the overall patient experience.

The history of dentistry reveals a significant evolution from the times when treatments were rudimentary and often painful. Today, technological developments and innovative treatments have radically revolutionized the way dental care is addressed. However, despite these developments, patient experience remains a significant issue. Studies reveal that dental anxiety continues to be a frequent condition, affecting a broad range of the population [8]. The absence of a holistic strategy that incorporates both the technical and emotional components of treatment is one of the key gaps in the present research. The key difficulty arises in the gap between the advancement of dental methods and the patient's subjective experience. While current technology have made operations more accurate and less intrusive, pain perception and fear still remain key impediments. The issue that emerges, therefore, is: how can we more successfully incorporate the emotional component into dental practice to enhance the patient experience? This problem is not just significant for oral health professionals, but also for patients seeking less stressful and more complete treatment.

The aims of the research on dental care center on understanding how emotional and psychological aspects impact the patient's experience. Firstly, we strive to determine the primary variables that lead

to anxiety during dental treatment. Second, the research intends to identify and suggest techniques based on these results to alleviate anxiety and enhance the overall patient experience. This holistic approach will not only handle the technical elements of treatment, but also create a more personal and compassionate environment. The relevance of this method resides in its capacity to convert dental treatment from a basic mechanical intervention to a more human and individualized experience. While procedures and equipment have evolved substantially, the true innovation is in how they are applied in the patient's setting. By incorporating treatments that address both physical pain and mental anxiety, dental practitioners may dramatically increase the quality of care delivered [8]. One of the most essential features of this study is the use of approaches such as the Delphi-AHP neutrosophic analysis to identify and prioritize anxiety variables. This methodological approach not only gives a more complete and organized picture of the aspects that impact the patient experience, but also provides a strong platform for making practical and useful suggestions [5][6]. By integrating both professional ratings and patient impressions, the study places itself at the forefront of dental care research.

Despite tremendous improvement, major problems persist. The heterogeneity in patients' emotional reaction and the intrinsic subjectivity of pain perception provide obstacles in establishing uniform remedies. Each patient is unique, and their experiences and responses might vary greatly. This heterogeneity underscores the necessity for individualized and adaptable techniques that may be adjusted to individual requirements [9]. The potential significance of this work is substantial. By establishing a greater knowledge of the variables that lead to dental anxiety and providing techniques to treat them, it opens the door to more patient-centered dental care. This transformation not only has the potential to increase patient happiness, but also to maximize clinical results and create more cooperation between patients and oral health professionals [9]. Dental care is a profession that, while extremely technical, cannot be properly comprehended without understanding the emotional and psychological components of the patient's experience. Research in this field gives a chance to bridge the gap between technique and human experience, creating a more holistic and sympathetic approach to dental treatment. This method not only enhances the quality of service, but also affects the way patients view and experience dental care. Therefore, it is crucial that dental practice continues to grow to thoroughly address all areas of the patient experience. Combining scientific developments with a profound awareness of patients' emotional and psychological needs may lead to more effective and humanized dental treatment, thereby giving a role model for the future of health care.

2.2. Delphi technique.

The Delphi approach is employed in numerous disciplines, such as program planning, resource utilization, policy judgment, and requirements assessment. A Delphi approach provides the following advantages:

- Tackle complicated challenges successfully.Able to define and change a vast variety of possibilities.
- Create diverse judgments on the same issue and utilize comments on others' assessments to enable them to modify their perspectives.
- Achieve a high degree of unanimity.

Increase coherence by minimizing the noise that occurs from focusing on group and/or individual interests instead of focused on resolving the issue. It is a structured communication strategy, developed mainly to gather and aggregate expert perspectives on certain themes via a series of iterative questions with controlled feedback. Developed in the 1950s by the RAND Corporation, this strategy is used to establish agreement on anticipating future trends, addressing complicated issues, strategic planning, and risk assessment, among others. The procedure starts with the selection of a panel of experts who

have specialized expertise in the area of interest. These experts submit to an initial questionnaire, the responses to which are anonymized and summarized by a coordinator or coordinating team. The summary findings are then given with the group, along with a new questionnaire based on the prior replies. This questionnaire-answer-feedback procedure is repeated in numerous rounds, to minimize the range of replies and move the group toward agreement. A significant component of the Delphi method is the anonymity of the participants, which helps reduce the impact of influence or dominance of some individuals over others, thereby promoting more objective replies and lowering conformity bias. At the conclusion of the process, the convergence of viewpoints is supposed to disclose a consensus or greater knowledge of the subject at hand, offering useful information for decision-making [5,14] . To establish knowledge of the subject investigated and object of study, it is established by a self-assessment procedure on a scale (see Table 1). This so-called neutrosophic knowledge coefficient is determined by the information that the expert himself gives on the topic of study.

Table 1: Linguistic terms used to determine K _(aN ,) K and evaluate the proposed criteria. Source: own elaboration.

linguistic term	**SVNN**
Full knowledge of the study topic (FK)	(1,0,0)
Very very good in the subject of study (VVGK)	(0.9, 0.1, 0.1)
Very good in the study topic (VGK)	(0.8,0.15,0.20)
Good at study topic (GK)	(0.70,0.25,0.30)
Moderately good in the subject of study (MGK)	(0.60,0.35,0.40)
Know the study topic (K)	(0.50,0.50,0.50)
Knows the topic of study moderately poorly (MPK)	(0.40,0.65,0.60)
Poor knowledge of the subject of study (PK)	(0.30,0.75,0.70)
Knows the subject of study very poorly (VPK)	(0.20,0.85,0.80)
Very, very poor knowledge of the subject of study (VVPK)	(0.10,0.90,0.90)
Without knowledge of the study topic (NK)	(0,1,1)

For the evaluation and validation of questionnaires using the Delphi method, the scale was used (see Table 2) to achieve greater objectivity in the management of the information. This allows us to evaluate the criteria argued by the judges of the panel of experts for each of the items individually. Through linguistic terms with Single Value Neutrosophic Numbers (SVNN) for consensus on the basis of the expert's opinion, criteria are evaluated using the neutrosophic argumentation coefficient.

Table 2: Linguistic terms used. Source: own elaboration.

Linguistic term I	**SVNN**	**Linguistic term II**
Very Suitable (VA)	(0.9,0.1,0.1)	Essential (E)
Fairly adequate (FA)	(0.75,0.25,0.20)	Very useful (VU)
Adequate (A)	(0.50,0.55,0.5)	Useful (U)
Not suitable (PA)	(0.3,0.75,0.80)	Not useful (LU)
Not suitable (NA)	(0.10,0.90,0.90)	Not useful (NU)

To determine the consensus among the participants of the expert panel, the agreement coefficient was used, determined through the expression:

$$Cc = \left(1 - \frac{V_n}{V_t}\right) 100 \quad (1)$$

Where: V_n is the number of negative votes cast by the judges, and $V_t$is the total number of votes cast by the judges. Therefore, a level of consensus must be reached when the agreement coefficient Cc obtains a value greater than 75%, and the process must be concluded; If this value is not reached, a new round must be established.

2.3. Neutrosophic AHP method

The Analytical Hierarchy Process (AHP) is a theory oriented to the decision maker and serves to identify the best alternative according to the assigned resources. This method can be applied to situations that involve technical, economic, political, social and cultural factors. That is, it aims to be a scientific tool to address aspects that are difficult to quantify but that sometimes require a unit of measurement. The methodology in its original version can be seen in [11, 15] . However, for this work the need for uncertainty is recognized, and for this purpose its neutrosophic version is adopted. Which uses triangular numbers for its execution, whose definition is as follows:

Definition 1: Let X be a space of points and $x \in$ of belonging to falsehood $F_A(x)$. U is the Universe of Discourse and $\forall x \in U$, $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq]-0, 1+[$ and $-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup T_A(x) \leq 3+$. Note that , by definition , $T_A(x)$, $I_{A(A}(x)$ and $F_A(x)$ can be subintervals of [0, 1] [6,16] .

Definition 2: Let X be a universe of discourse. A single-valued neutrosophic set A over X is an object that takes the form $A = \{<x; T_A(x), I_A(x), F_A(x)>: x \in U\}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$, and $F_A: U \rightarrow [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the degree of belonging to the truth, the degree of belonging to the indeterminacy and the degree of belonging to the falsity of x to A, respectively. For convenience, an SVN number is represented by A = (a, b, c), where a, b, c ∈ [0, 1] and a+b+c≤3.

Definition 3: Suppose that ã, ã, ã ∈ [0, 1] and $a_1, a_2, a_3, a_4 \in R$ where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then, a single-valued trapezoidal neutrosophic number $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$is a special neutrosophic set on the set of real lines $\mathbb{R}$, whose functions of belonging to truth, belonging to indeterminacy and belonging to falsity are defined as see revised methodology [7, 8] .

Definition 4: given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$single-valued $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$trapezoidal neutrosophic numbers and λany non-zero number on the real line. Then, the following operations are defined :

Addition:$\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$

Subtraction : $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$(2)

Investment :, $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$where $a_1, a_2, a_3, a_4 \neq 0$.

Multiplication by a scalar number: [9]

This technique models the problem leading to the formation of a hierarchy representative of the associated decision-making scheme. The comparison is made using a scale, according to Table 3 [10] . To verify the neutrosophic methodology see [6-10].

Table 3: Saaty scale translated into a neutrosophic triangular scale. Source: [11]

saaty scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\langle (1, 1, 1); 0.50, 0.50, 0.50 \rangle$
3	Slightly influential	$\langle (2, 3, 4); 0.30, 0.75, 0.70 \rangle$
5	Strongly influential	$\langle (4, 5, 6); 0.80, 0.15, 0.20 \rangle$
7	Very influential	$\langle (6, 7, 8); 0.90, 0.10, 0.10 \rangle$
9	Absolutely influential	$\langle (9, 9, 9); 1.00, 1.00, 1.00 \rangle$

saaty scale	Definition	Neutrosophic Triangular Scale
2, 4, 6, 8	Sporadic values between two close scales	$\langle(1,2,3);0.40,0.65,0.60\rangle$ $\langle(3,4,5);0.60,0.35,0.40\rangle$ $\langle(5,6,7);0.70,0.25,0.30\rangle$ $\langle(7,8,9);0.85,0.10,0.15\rangle$

3. Results.

The Delphi approach is based on an iterative process of rounds of questions issued to a panel of experts. The aim is that, via multiple rounds, a closer agreement is formed on the problem at hand.

For the study of variables that create anxiety during dental treatment using Delphi-AHP neutrosophic methodologies, the purpose is to identify the primary elements that generate anxiety in patients during dental appointments and analyze how these factors interrelate and impact the degree of anxiety.

Round 1: Determination of Factors

Six elements are identified that interfere in anxiety during dental care:

1. Clinical Environment : The quality of the physical environment of the dental office, including comfort, cleanliness, and overall attractiveness.
2. Dentist Communication : The manner the dentist interacts with the patient, includes clarity, empathy, and tone.
3. Dental operations : The kind and complexity of the operations conducted during the consultation.
4. Patient's past Experience : The patient's past experiences with dental care, which may impact their anxiety levels.
5. Patient Expectations : The patient's expectations and perceptions of the therapy and anticipated outcomes.
6. Pain Management : The patient's impression of pain control and management of discomfort during and after the process.

Round 2: Identification of Key Factors and Initial Assessments

Questions :

What do you perceive to be the influence of the clinical atmosphere on patient anxiety during dental care?

To examine the impact of the dentist's conversation on the patient's anxiety.

How does the kind and complexity of dental treatments effect patient anxiety?

To examine the influence of the patient's past experience with dental procedures on their anxiety.

What is the impact of the patient's expectations regarding the therapy on his or her anxiety level?

To examine the influence of pain treatment on patient anxiety during dental care.

Answers :

Experts share their judgments based on their expertise and experiences. Responses are gathered and examined to find areas of consensus and disagreement.

Table 4: Criteria Validation Level

Expert	Clinical Environment	Dentist Communication	Dental Procedures	Previous Experience	Patient Expectations	Pain Management
E1	(0.7;0.2;0.1)	(0.8;0.1;0.1)	(0.6;0.3;0.1)	(0.5;0.4;0.1)	(0.4;0.4;0.2)	(0.7;0.2;0.1)
E2	(0.8;0.1;0.1)	(0.7;0.2;0.1)	(0.5;0.4;0.1)	(0.6;0.3;0.1)	(0.5;0.3;0.2)	(0.6;0.3;0.1)
E3	(0.6;0.3;0.1)	(0.7;0.2;0.1)	(0.7;0.2;0.1)	(0.5;0.4;0.1)	(0.4;0.4;0.2)	(0.7;0.2;0.1)
E4	(0.7;0.2;0.1)	(0.6;0.3;0.1)	(0.6;0.3;0.1)	(0.4;0.5;0.1)	(0.5;0.3;0.2)	(0.6;0.3;0.1)
E5	(0.8;0.1;0.1)	(0.8;0.1;0.1)	(0.5;0.4;0.1)	(0.5;0.4;0.1)	(0.6;0.3;0.1)	(0.7;0.2;0.1)

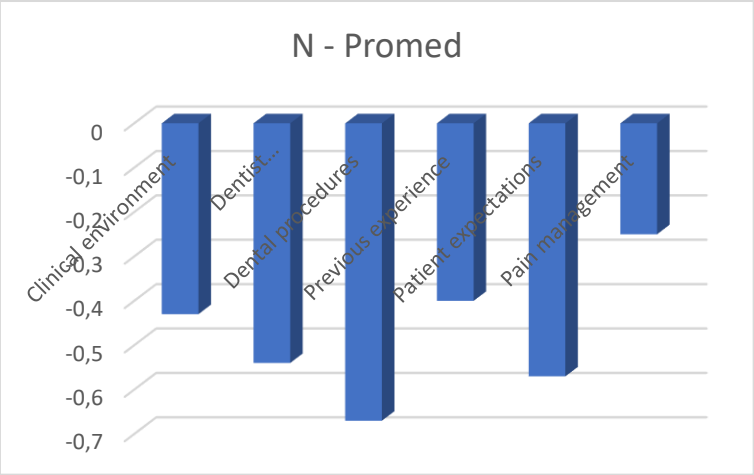
Table 5: Neutrosophic Relative Frequency

Indicators	(0.9;0.1;0.1)	(0.8;0.2;0.1)	(0.6;0.3;0.1)	(0.5;0.4;0.1)	(0.4;0.4;0.2)	(0.7;0.2;0.1)
Clinical Environment	0.2500	0.3750	0.2000	0.1500	0.0750	0.2000
Dentist Communication	0.2500	0.3000	0.2000	0.1500	0.1500	0.2000
Dental Procedures	0.2000	0.2000	0.3500	0.1500	0.1000	0.2000
Previous Experience	0.1500	0.1750	0.2000	0.2000	0.1500	0.1500
Patient Expectations	0.1500	0.1250	0.1500	0.1750	0.1750	0.2000
Pain Management	0.2000	0.2000	0.2000	0.1500	0.1000	0.2500

Table 6: Cut-Off Points and Criteria Scale

Indicators	N - Average	SVNN
Clinical Environment	-0.43	Very useful
Dentist Communication	-0.54	Appropriate
Dental Procedures	-0.67	Not very useful
Previous Experience	-0.40	Appropriate
Patient Expectations	-0.57	Appropriate
Pain Management	-0.25	Very useful

Round 3: Refinement of Assessments



Graph 1: Cut-Off Points and Scale of Criteria

Questions :

Considering the range of replies on the influence of the clinical setting, would you wish to revise your assessment?

Are there any additional thoughts you would want to add about dental communication?

Given the range of viewpoints on dental operations, do you believe it is important to change your assessment?

How would you rethink the value of the patient's past experience after viewing other experts' responses?

Based on the comments obtained, would you adjust your perspective of the patient's expectations?

Is there any reason that would cause you to rethink your judgment of pain management?

Answers :

The experts examine their earlier evaluations, changing them depending on the debates and arguments offered by other participants.

Round 4: Consensus

If there are still major disagreements, another round is done focused on the issues of dispute. Answers are received and experts give a consensus.

Conclusion of the Delphi Process

A final report is generated that highlights the agreement achieved, stressing the aspects that cause the greatest anxiety during dental treatment, along with their interrelationships and implications. The paper may serve as a foundation for enhancing dental practices and lowering patient concern.

Conclusion of the Neutrosophic Evaluation

The elements that most lead to anxiety during dental treatment are:

1. Dental Procedures : Due to the intrusive nature and the related discomfort.
2. Pain Management : The sense of pain control and management throughout the operation.

Recommendations to minimize anxiety in dental care :

1. Improve Pain Management : Implement more sophisticated pain control procedures and provide appropriate anesthetic.
2. Simplify treatments : Opt for less intrusive treatments wherever feasible and clearly clarify the stages to follow.
3. Create a Comfortable Clinical Environment : Ensure that the office is clean, contemporary and pleasant for the patient.
4. Improve Communication : Train dentists to use clear, sympathetic language and offer full information regarding treatment.

To prioritize improvement measures, the AHP Process is employed, integrating expert assessments and views to decide which strategies have the most effect on anxiety reduction.

The implementation of the Delphi-AHP neutrosophic method helps you to successfully identify and prioritize elements that produce anxiety during dental treatment and build ways to effectively minimize these issues.

4. Conclusion

In the investigation of the anxiety experienced during dental treatment, this research has shown critical variables that greatly impact the patient's well-being. The data reveal that invasive procedures and pain management are the key variables leading to anxiety in these circumstances. Through the application of the Delphi-AHP neutrosophic method, it has been possible to identify and prioritize effective strategies to reduce this anxiety, such as improving pain management, simplifying procedures, creating a more comfortable clinical environment and optimization of communication between dentist and patient. The practical value of these findings is enormous, as it gives clear instructions for the adoption of tactics that might improve the patient experience in the dental setting. Adopting improved pain management strategies and efficient communication not only promises to minimize anxiety but also increase patient happiness and participation throughout treatments. These innovations have the potential to increase standards of care in dentistry, leading to a more pleasant and less stressful experience for patients. Among the contributions of this research, the integration of the Delphi-AHP neutrosophic technique stands out, which gives a systematic strategy to solve complicated challenges in dental treatment. This unique technique not only enhances the knowledge of variables impacting patient anxiety, but also offers a formal framework for generating and prioritizing effective remedies. This addition to the field may influence future research and practice in dental care management.

However, the research had certain drawbacks. The application of the results may be modified by the subjectivity inherent in expert judgments and by the unique circumstances of the dental treatment reviewed. Generalization of the findings to other dental settings or specializations should be treated with care. For future study, it is proposed to examine the introduction of other approaches that complement the neutrosophic approach, such as research on pain management strategies based on new technology. Furthermore, it would be good to undertake research in diverse circumstances and with different patient groups to verify and expand the existing results. Continued review and refinement of ways to minimize anxiety in dental treatment are vital to delivering high-quality dental care and a more pleasant experience for all patients.

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Media Pressure and Pretrial Detention Requirements in Ucayali, Peru: A Study Based on Neutrosophic Measures

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Abstract. This study is based on a qualitative approach and analyses the influence of media pressure on judicial decisions regarding pretrial detention in Peruvian criminal proceedings. Four media cases were selected as a sample and documentary analysis techniques were used. The results reveal that media pressure plays an important role in judicial decisions, acting through various mechanisms such as exposure to information, public pressure, and social legitimation. In some cases, media pressure has been a determining factor, while in others it has had a more subtle influence. Judges must be aware of this influence and take measures to prevent it from affecting their independence and impartiality. Structural reforms are required to ensure responsible journalism and protect the independence of the judiciary. Neutrosophic Measure theory is used and the neutrosophic belief function for a group of three experts, to express their opinion on the four cases based on a group of five criteria. This theory, which is based on neutrosophic sets, allows us to obtain more accurate but less precise results. Due to the absence of a statistically significant sample size, with the selected tool the experts can express their opinions using subjective probabilistic measures based on their experiences.

Keywords: Pressure media, prison preventive, decisions judicial, criminal procedure, neutrosophic measure theory, neutrosophic probability measure, neutrosophic belief function.

1 Introduction

One of the challenges of globalization and universalization of computer communication systems as a worldwide phenomenon is the excessive use of psychosocial advertising, to induce and condition human groups to the interests of particular ideological sectors. These are social, political, religious, economic, or legal, based on private interests or well-defined state sectors.

It is through the use of media pressure in our country, Peru (press and social networks), that interest groups have directed behavior, social reaction, and collective interest, to a clear selective conditioning, where the rational man acts under the influence of what is previously graphed by the media. This fact is suggested both by the position induced by the press media, as well as social networks, without omitting the internal pressure due to fear, reactions, or repercussions that these media could exert against those who act differently than previously conditioned. Frustration also influences the social, work, and family repercussions, due to alleged scandals for not acting as the media had previously directed.

Although globalization as a worldwide phenomenon has benefited us in terms of the universalization of knowledge, where what happens in Peru is known on the other side of the globe in a fraction of a second, everything that happens in society is public knowledge. Therefore, we can see that this sociocultural phenomenon that has benefited everyone in terms of access to public information has also contributed to broadening the direction of the population's thinking. That is why society today goes where the media directs it.

We can easily see that in this social evolution, with its positive and negative implications, the media and social networks have led the population to a greater interest in Law and judicial decisions, determining the position of the population in the approach that proliferates the front pages of newspapers, television, radio and publications on social networks. Likewise, it is with this socio-legal approach that the Judge tends to be influenced when imparting Justice.

It is in the context of motivation and influence that, in our country, the Judiciary has issued resolutions granting requests for preventive detention. However, these resolutions have been issued under an apparent motivation, supporting the fiscal requests for preventive detention, under the magnifying glass of the intensity with which the media and social networks exert media pressure.

The purpose of this study is to analyze how media pressure influences judicial decisions on the application of pretrial detention in Peruvian criminal proceedings. The research is based on a qualitative approach. Four media cases of pretrial detention requests were selected as a study sample.

We support the theory of Neutrosophic Measures that has been used in the mathematical modeling of problems within the field of legality theory [1]. The aforementioned model combines the theories of neutrosophic measure theory, neutrosophic probability measure, and neutrosophic belief function [2-4].

As we intend to study in depth four judicial cases in Peru over the last two years, we consulted a group of three experts on the subject. As the number of cases is not statistically significant, it is necessary to work with the degree of evidence that exists of compliance with certain aspects to be measured. For this reason, the experts offer measures of satisfaction with the criteria based on certain subjective probabilities, according to the experience and evidence of the experts to evaluate each of the cases.

The neutrosophic measure theory extends classical measure theory to the field of neutrosophy, which allows us to take into account measurements on a set A, and additionally measurements of non-A and the indeterminacy of A [5-10]. Therefore, there is greater precision when dealing with measurements of sets. These measurements are the basis of neutrosophic probability measures and the neutrosophic belief functions [11-17].

The study confirms that media pressure is a factor that can influence judicial decisions on pretrial detention. It is important that judges are aware of this influence and take measures to prevent media pressure from affecting their independence and impartiality. Structural reforms are required to ensure that the media report responsibly on court cases and to protect the independence of the judiciary.

This paper begins with a Preliminaries section, where the basic notions of neutrosophic measure theory are recalled and the other concepts we discuss in the article. The following section presents the results of the study. The article ends with the conclusions.

2 Preliminaries

This section contains the essential ideas of the theories used in this paper to carry out the proposed study. What we present here is based on the article in [1].

2.1 Neutrosophic Measures

Let <A> be an item. <A> can be a notion, an attribute, an idea, a proposition, a theorem, a theory, etc. And let <antiA> be the opposite of <A>; while <neutA> be neither <A> nor <antiA> but the neutral (or indeterminacy, unknown) related to <A>.

Let X be a neutrosophic space, and Σ be a σ-neutrosophic algebra over X. A *neutrosophic measure* ν

is defined for neutrosophic set A ∈ Σ by $\nu: X \to \mathbb{R}^3$, such that:

$$\nu(A) = (\mathrm{m(A)}, \mathrm{m(neutA)}, \mathrm{m(antiA)}) \quad (1)$$

With antiA:= the opposite of A, and neutA:= the neutral (indeterminacy) neither A nor antiA.

For $A \subseteq X$ and any $A \in \Sigma$:

1. $m(A)$ means *the measure of the determinate* part of A;
2. $m(neutA)$ means *the measure of the indeterminate* part of A; and
3. $m(antiA)$ means a *measure of the determinate part* of antiA.

Where ν is a function that satisfies the following two properties:

a) Null empty set: $\nu(\emptyset) = (0,0,0)$.

b) Countable additivity (or σ-additivity): For all countable collections $\{A_n\}_{n\in L}$ of disjoint neutrosophic sets in Σ, we have seen :

$$\nu(\cup_{n\in L} A_n) = (\sum_{n\in L} \mathrm{m}(A_n), \sum_{n\in L} \mathrm{m}(neutA_n), \sum_{n\in L} \mathrm{m}(antiA_n) - (\mathrm{n}-1)\mathrm{m}(X))$$

Where *X* is the whole neutrosophic space, and

$$\sum_{n\in L} \mathrm{m}(antiA_n) - (\mathrm{n}-1)\mathrm{m}(X) = \mathrm{m}(X) - \sum_{n\in L} \mathrm{m}(A_n) = \mathrm{m}\big((\cap_{n\in L}\, antiA_n)\big) \quad (2)$$

A *neutrosophic measure space* is a triple (X, Σ, ν).

A *neutrosophic normalized measure* is $NN = (m(X), m(neutX), m(antiX))$, where $m(X), m(neutX), m(antiX) \geq 0$ and $m(X) + m(neutX) + m(antiX) = 1$.

Where *X* is the whole neutrosophic measure space.

A neutrosophic measure space (X, Σ, ν) is called *finite* if $\nu(X) = (a, b, c)$ such that all a, b, and c are finite (rather than infinite).

A neutrosophic measure is called *σ-finite* if *X* can be decomposed into a countable union of neutrosophically measurable sets of finite neutrosophic measures.

Analogously, a set A in *X* is said to have a *σ-finite neutrosophic measure* if it is a countable union of sets with finite neutrosophic measure.

The neutrosophic measure *ν* satisfies the *axiom of non-negativity* if $\forall A \in \Sigma$,

$$\nu(A) = (\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3) \geq 0 \quad (3)$$

If $\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3 \geq 0$.

While a neutrosophic measure ν, which satisfies only the null empty set and countable additivity axioms (hence not the non-negativity axiom), takes on at most one of the $\pm\infty$ values.

The members of Σ are called *measurable neutrosophic sets*, while (X, Σ) is called a *measurable neutrosophic space*.

A function $f: (X, \Sigma_X) \to (Y, \Sigma_Y)$, that is a mapping between two measurable neutrosophic spaces, is called *neutrosophic measurable function* $\forall B \in \Sigma_Y, f^{-1}(B) \in \Sigma_X$ (the inverse image of a neutrosophic Y-measurable set is a neutrosophic X-measurable set).

The properties of neutrosophic measures are the following:

a) Monotonicity:

If A_1 and A_2 are neutrosophically measurable, with $A_1 \subseteq A_2$, where $\nu(A_1) = (\mathrm{m}(A_1), \mathrm{m}(\mathrm{neut}A_1), \mathrm{m}(\mathrm{anti}A_1))$ and $\nu(A_2) = (\mathrm{m}(A_2), \mathrm{m}(\mathrm{neut}A_2), \mathrm{m}(\mathrm{anti}A_2))$, then $\mathrm{m}(A_1) \leq \mathrm{m}(A_2)$, $\mathrm{m}(\mathrm{neut}A_1) \leq \mathrm{m}(\mathrm{neut}A_2)$, $\mathrm{m}(\mathrm{anti}A_1) \geq \mathrm{m}(\mathrm{anti}A_2)$.

b) Additivity:

If $A_1 \cap A_2 = \emptyset$, then $\nu(A_1 \cup A_2) = \nu(A_1) + \nu(A_2)$,

Where the sum of two measures is defined as follows:

$$(a_1, a_2, a_3) + (b_1, b_2, b_3) = \left(a_1 + b_1, a_2 + b_2, a_3 + b_3 - m(X)\right) \quad (4)$$

Where X is the whole neutrosophic space and $a_3 + b_3 - m(X) = m(X) - m(A) - m(B) = m(X) - a_1 - a_2 = m(\text{antiA} \cap \text{antiB})$.

The *neutrosophic probability measure* is a mapping:

$$\mathcal{NP}: X \to [0,1]^3 \quad (5)$$

Where X is a neutrosophic sample space (i.e. X contains some indeterminacy),

$$\mathcal{NP}(A) = (ch(A), ch(indeterm A), ch(\overline{A})) \quad (6)$$

That is to say, it is decomposed into three components, the chance that A occurs, the chance that A is indeterminate, and the chance that A does not occur.

By using another notation we have:

$$\mathcal{NP}(A) = (ch(A), ch(Neut A), ch(\text{Anti}A)) \quad (7)$$

Which satisfies the condition, $^-0 \leq ch(A), ch(indeterm A), ch(\overline{A}) \leq 3^+$, that is to say, there exist probabilities such that $ch(A) + ch(indeterm A) + ch(\overline{A})$ are equal to 1, <1, or >1.

The extension of the Kolmogorov axioms to the neutrosophic space is the following:

Let $(\mathcal{N\Omega}, \mathcal{NF}, \mathcal{NP})$ be a neutrosophic probability space, where $\mathcal{N\Omega}$ is a neutrosophic sample space, $\mathcal{NF}$ is a neutrosophic event space, and $\mathcal{NP}$ is a neutrosophic probability measure.

1. The neutrosophic probability of an event A is non-negative.
2. The neutrosophic probability of the sample space is between $^-0$ and 3^+.

3. Neutrosophic σ- additivity:

$$NP(A_1 \cup A_2 \cup \cdots) = \left(\sum_{j=1}^{\infty} ch(A_j), ch(\text{indeterm}A_1 \cup A_2 \cup \cdots), ch(\overline{A_1 \cup A_2 \cup \cdots})\right) \quad (8)$$

Where $A_1, A_2, \cdots$is a countable sequence of disjoint (or mutually exclusive) neutrosophic events.

If we relax the third axiom we get a *neutrosophic quasi-probability* distribution.

2.2 Neutrosophic Evidence model

This subsection presented here sets out the definitions concerning the method introduced in [1].

Definition 1 (*Neutrosophic mass assignment*) ([1]): A *neutrosophic mass assignment* is $m(\cdot) = \left(m_t(\cdot), m_i(\cdot), m_f(\cdot)\right)$; $m_t(\cdot), m_i(\cdot), m_f(\cdot): 2^{\Theta} \to]^-0, 1^+[^3$ satisfying the following axioms for each dimension of the neutrosophic space:

$$\sum_{A \subset \Theta} \sup\left(m_t(A)\right) \geq 1 \quad (9)$$

$$\sum_{A \subset \Theta} inf\left(m_f(A)\right) \geq |\Theta| - 1 \quad (10)$$

Where $|\Theta|$ represents the cardinality of the frame of discernment Θ.

Definition 2 ([1]): A *neutrosophic belief function* for all $A \subset \Theta$, $Bel(\cdot) = \left(Bel_T(\cdot), Bel_I(\cdot), Bel_F(\cdot)\right)$ is defined as:

$$Bel_T(A) = \sum_{B \subset A} m_t(B) \quad (11)$$

$$Bel_I(A) = \sum_{B \subset A} m_i(B)$$

$$Bel_F(A) = \sum_{B \subset A} m_f(B)$$

Definition 3 ([1]): A *neutrosophic disbelief function* for all $A \subset \Theta$, $d(\cdot) = \left(d_T(\cdot), d_I(\cdot), d_F(\cdot)\right)$ is defined as:

$$d_T(A) = \sum_{A \cap B = \emptyset} m_t(B) \quad (12)$$

$d_I(\mathrm{A}) = \sum_{\mathrm{A}\cap\mathrm{B}=\emptyset} m_i(\mathrm{B})$

$d_F(\mathrm{A}) = \sum_{\mathrm{A}\cap\mathrm{B}=\emptyset} m_f(\mathrm{B})$

Definition 4 ([1]): A *neutrosophic uncertainty function* for all $A \subset \Theta$, $u(\cdot) = \big(u_T(\cdot), u_I(\cdot), u_F(\cdot)\big)$ is defined as:

$$u_T(\mathrm{A}) = \sum_{\substack{\mathrm{A}\cap\mathrm{B}\neq\emptyset \\ \mathrm{B}\not\subseteq\mathrm{A}}} m_t(\mathrm{B}) \qquad (13)$$

$$u_I(\mathrm{A}) = \sum_{\substack{\mathrm{A}\cap\mathrm{B}\neq\emptyset \\ \mathrm{B}\not\subseteq\mathrm{A}}} m_i(\mathrm{B})$$

$$u_F(\mathrm{A}) = \sum_{\substack{\mathrm{A}\cap\mathrm{B}\neq\emptyset \\ \mathrm{B}\not\subseteq\mathrm{A}}} m_f(\mathrm{B})$$

Definition 5 ([1]): Let Θ be a frame of discernment and let $A, B \in 2^{\Theta}$. Then the *relative atomicity* of A to B is the function a: $2^{\Theta} \to]^{-}0, 1^{+}[$ defined by:

$$a(A/B) = \frac{|A\cap B|}{|B|} \qquad (14)$$

$A, B \in 2^{\Theta}$.

Let us observe that $\mathrm{A} \cap \mathrm{B} = \emptyset$ implies $a(A/B) = 0$, whereas $\mathrm{B} \subseteq \mathrm{A}$ implies $a(A/B) = 1$. $a(\cdot)$ measures the degree of overlap between A and B.

Definition 6 ([1]): (*Neutrosophic Probability Expectation*) Let Θ be a frame of discernment with $m(\cdot)$ be the neutrosophic mass assignment, then the *neutrosophic probability expectation function* corresponding with $m(\cdot)$ is the function E: $2^{\Theta} \to]^{-}0, 1^{+}[^3$ defined by:

$$\mathrm{E_T}(A) = \sum_{\mathrm{B}} \mathrm{m_t}(\mathrm{B})\, a(A/B) \qquad (15)$$

$$\mathrm{E_I}(A) = \sum_{\mathrm{B}} \mathrm{m_i}(\mathrm{B}) a(A/B)\,(1 - a(A/B))$$

$$\mathrm{E_F}(A) = \sum_{\mathrm{B}} \mathrm{m_f}(\mathrm{B})\,(1 - a(A/B))$$

$A, B \in 2^{\Theta}$.

Theorem 1 ([1]): Given a frame of discernment Θ with $m(\cdot)$ be the neutrosophic mass assignment, the probability expectation function $\mathrm{E}(\cdot)$ with domain 2^{Θ} satisfies:

1. $\mathrm{E(A)} \geq 0$ for all $A \in 2^{\Theta}$,
2. If $\mathrm{A_1}, \mathrm{A_2}, \cdots, \mathrm{A_n} \in 2^{\Theta}$ are pairwise disjoint then $\mathrm{E}(\bigcup_{i=1}^{n} A_i) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{E}(A_i)$.

Definition 7 (Opinion) ([1]): Let Θ be a binary frame of discernment with 2 atomic states A and $\neg A$, and let $m(\cdot)$ be a Neutrosophic mass assignment on Θ, where $b(A)$, $d(A)$, $u(A)$, and $a(A)$ (i.e., $B = \Theta$ in $a(A) = a(\mathrm{A}/\Theta)$) represent the belief, disbelief, uncertainty and relative atomicity functions on A in Θ, respectively.

Then the opinion about A, denoted by ω_A, is the quadruple defined by:

$$\omega_A \equiv (b(A), d(A), u(A), a(A)) \qquad (16)$$

The expectation of the opinion ω_A is defined by using the following Equations:

$$\mathrm{E_T}(\omega_A) = b_T(A) + u_T(A)a(A) \qquad (17)$$

$$\mathrm{E_I}(\omega_A) = b_I(A) + u_I(A)a(A)(1 - a(A))$$

$$\mathrm{E_F}(\omega_A) = b_F(A) + u_F(A)(1 - a(A))$$

Definition 8 (Ordering of Opinions) ([1]): Let ω_A and ω_B be two opinions. They can be ordered according to the following criteria by priority:

1. The greatest probability expectation gives the greatest opinion.
2. The least uncertainty gives the greatest opinion.

3. The least relative atomicity gives the greatest opinion.

Let us note that the order we referred to above is the neutrosophic order.

Definition 9 ([1]): Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let $\omega_{A\wedge B} = (b(A \wedge B), d(A \wedge B), u(A \wedge B), a(A \wedge B))$ be the opinion such that:

1. $b(A \wedge B) = b(A) \wedge_N b(B)$,
2. $d(A \wedge B) = d(A) \vee_N d(B)$,
3. $u(A \wedge B) = u(A) \vee_N u(B)$,
4. $a(A \wedge B) = a(A)a(B)$.

Where $\wedge_N$ is a neutrosophic norm or n-norm, and $\vee_N$ is a neutrosophic conorm or n-conorm [18, 19]. This is called the *Propositional Conjunction*.

Definition 10 ([1]): Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let $\omega_{A\vee B} = (b(A \vee B), d(A \vee B), u(A \vee B), a(A \vee B))$ be the opinion such that:

1. $b(A \vee B) = b(A) \vee_N b(B)$,
2. $d(A \vee B) = d(A) \wedge_N d(B)$,
3. $u(A \vee B) = u(A) \wedge_N u(B)$,
4. $a(A \vee B) = a(A) + a(B) - a(A) \cdot a(B)$.

This is called the *Propositional Disjunction*, which means the agent's opinion about A or B.

Definition 11 ([1]): Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let us define:

1. $E(\omega_{A\wedge B}) = E(\omega_A) \wedge_N E(\omega_B)$,
2. $E(\omega_{A\vee B}) = E(\omega_A) \vee_N E(\omega_B)$.

The denial of an opinion about proposition A represents the agent's opinion about A being false. It is defined as follows:

Definition 12 ([1]): Let $\omega_A = (b(A), d(A), u(A), a(A))$ be an opinion about the proposition A. Then, $\omega_{\neg A} = (b(\neg A), d(\neg A), u(\neg A), a(\neg A))$ is the negation of A, defined as:

1. $b(\neg A) = d(A)$,
2. $d(\neg A) = b(A)$,
3. $u(\neg A) = u(A)$,
4. $a(\neg A) = 1 - a(A)$.

Definition 13 ([1]): Let $\omega_A^\alpha = (b^\alpha(A), d^\alpha(A), u^\alpha(A), a^\alpha(A))$ and $\omega_A^\beta = \left(b^\beta(A), d^\beta(A), u^\beta(A), a^\beta(A)\right)$, α`s and β`s opinion about the same proposition A, respectively. $\omega_A^{\alpha,\beta} = \left(b^{\alpha,\beta}(A), d^{\alpha,\beta}(A), u^{\alpha,\beta}(A), a^{\alpha,\beta}(A)\right)$ is the joint opinion and it is defined as follows:

1. $b^{\alpha,\beta}(A) = b^\alpha(A) \wedge_N b^\beta(A)$,
2. $d^{\alpha,\beta}(A) = d^\alpha(A) \vee_N d^\beta(A)$,
3. $u^{\alpha,\beta}(A) = u^\alpha(A) \vee_N u^\beta(A)$,
4. $a^{\alpha,\beta}(A) = a^\alpha(A)a^\beta(B)$.

3 Results

This section presents the results of the study carried out. The paradigm of this research is given by the belief that media pressure does not influence judicial decisions, in this specific case linked to preventive detentions. However, we will try to demonstrate that this paradigm is not real because the media will influence the granting of preventive detention requested by prosecutors despite lacking the necessary elements for its application.

For this reason, we will study the paradigm of our research in the qualitative approach that will help us understand in detail the cases that we selected for this research, these are:

1. Identify the media cases, crimes, and accused persons that were the subject of media coverage in Ucayali during the period 2021-2022.
2. Analyze the elements of conviction, prognosis of punishment, procedural danger, and duration of the sentence presented in the requests for preventive detention in media cases.
3. To examine the nature and level of pressure exerted by the media on the justice system during pretrial detention processes in high-profile cases.
4. To determine whether judges approved or denied pretrial detention requests in the media cases analyzed and how this relates to the pressure exerted by the media.
5. Understanding the dynamics between media coverage, pretrial detention requirements, and court decisions in the selected cases.

For these purposes, the files of the four media cases are analyzed. These cases are the following:

Case 1

This is a media case related to alleged collusion and criminal organization, involving several defendants. It is a case of corruption of public officials in the District Municipality of Yarinacocha.

The judge denied the prosecutor's request for pretrial detention and instead ordered him to appear with restrictions. This suggests that the elements of conviction, procedural danger, and other factors did not seem to justify pretrial detention at that time.

Case 2

This is an investigation into the crime of rape. There are testimonies from the victim, and results of medical examinations, among other evidence. It is not explicitly mentioned, but the crime of rape carries a high penalty. It is a media case because the news has been spread through different media. The judge declared the prosecutor's request for preventive detention against the perpetrator to be well-founded.

Case 3

The document refers to the media case of an alleged commission of the crime of aggravated collusion against the State-Regional Health Directorate of Ucayali (DIRESA Ucayali in Spanish). The document contains information on the evidence presented by the Public Prosecutor's Office to request preventive detention of the accused. The arguments on the procedural danger and the possible length of the sentence, if preventive detention is ordered, are discussed. It has been shown that the news has been disseminated by different media. It is known that it is a media case. The document does not include the judge's final decision on the request for preventive detention.

Case 4

The media case is related to an investigation into alleged aggravated collusion against the Regional Health Directorate of Ucayali. The document indicates that the request for preventive detention by the prosecutor was denied. The judge considered that there were no sufficiently strong suspicions (strong suspicions) of the commission of a serious crime to justify preventive detention.

The judge pointed out that mere weak probabilities, simple indications, or generic suspicions are not sufficient, since the presumption of innocence accompanies the accused throughout the entire process until a final conviction.

The document does not indicate any direct relationship between the court decision and media pressure. According to the information provided in this document, it appears that the judge made the

decision to deny pretrial detention based on a careful legal analysis of the evidence and the required legal standards, rather than being directly influenced by media coverage.

The study carried out took into account the following aspects to measure the impact of media pressure on the outcome of the procedural measures applied in the convictions of the cases, as previously indicated.

A_1. Identification of media cases, crimes, and defendants: This category refers to the delimitation of the media case in question, specifying the crime investigated and the persons charged. It is essential to contextualize the analysis and understand the specific circumstances in which media pressure is developed.

A_2. Analysis of the elements of conviction, prognosis of sentence, procedural danger, and duration of sentence in requests for preventive detention: This category examines the arguments presented by the Public Prosecutor's Office to request pretrial detention, including the evidence supporting the accusation, the possible penalty that would be imposed in the event of a conviction, and the risk that the accused will obstruct the investigation or flee. This analysis is crucial to assess the strength of the request and its potential impact on the court's decision.

A_3. Examination of the nature and level of media pressure on the judicial system during pretrial detention proceedings: This category explores media coverage of the case and its potential influence on public opinion and perception of the judicial system. It is important to consider the tone, intensity, and scope of media coverage, as well as the possible strategies used by the media to influence public opinion.

A_4. Determining whether the judges approved or denied the requests for pretrial detention and how this relates to media pressure: This category analyses judicial decisions on requests for pretrial detention, considering the arguments presented by the parties and the legal criteria underlying the decision. It is essential to assess whether media pressure has played a role in the judge's decision, either explicitly or implicitly.

A_5. Understanding the dynamics between media coverage, requests for pretrial detention, and court decisions in the selected cases: This category seeks to understand the complex interaction between media pressure, requests for pretrial detention, and court decisions, considering the different factors that may influence the judge's decisions. It is important to analyze the different perspectives of the actors involved, such as judges, prosecutors, defenders, journalists, and civil society representatives, to obtain a more complete view of the dynamics at play.

To do this, the steps defined below were followed:

The evaluation procedure followed in this research

1. Three specialists were requested to provide their opinions on each of the cases in relation to the aspects mentioned above.

   Let us call $E = \{e_1, e_2, e_3\}$ the set of specialists who are going to be consulted.

2. Let be $a_{ijk} = (T_{ijk}, I_{ijk}, F_{ijk})$ the opinion issued by the k-th specialist (k = 1, 2, 3) on the satisfaction of the j-th case (j = 1, 2, 3, 4) of the i-th criterion (i = 1, 2, 3, 4, 5).

Where T_{ijk} is the k-th expert's confidence percentage that the j-th case adequately satisfies the i-th criterion. Similarly, I_{ijk} is the indeterminacy percentage and F_{ijk} is the non-satisfiability percentage. It is preferred for convenience that $T_{ijk} + I_{ijk} + F_{ijk} = 1$.

3. The measure is then calculated for each case and each expert, using the formula:

$$(T_x, I_x, F_x) \otimes (T_y, I_y, F_y) = (T_x T_y, I_x + I_y - I_x I_y, F_x + F_y - F_x F_y) \quad (18)$$

This formula is repeated by setting an expert $\hat{k}$-th and a case î-th to obtain:

$a_{i\hat{k}} = \otimes_{j=1}^{5} a_{ij\hat{k}}$ (19)

4. It is taken $m_{ik} = a_{ik}$ as the neutrosophic measure of the evaluation of the i-th case evaluated by the k-th expert.

For each expert k, the neutrosophic belief function is calculated for each case with the help of Equation 11 and all the other functions. Let us denote by b_{1k}, b_{2k}, b_{3k}, and b_{4k} these functions for the cases. It should be noted that for non-singleton sets the neutrosophic measure functions are calculated by formula 20. For example, $m_{\{1,2\}k} = a_{1k} \oplus a_{2k}$, which is the measure of whether cases 1 and 2 meet the stated criteria, and so on.

$(T_x, I_x, F_x) \oplus (T_y, I_y, F_y) = (T_x + T_y - T_x T_y,\ I_x I_y,\ F_x F_y)$ (20)

5. Finally, we obtain the neutrosophic belief function for each case for all experts, which is calculated as:

$b_i = \wedge_{n\,k=1}^{3} b_{ik}$ where $\wedge_n$ is shown in Equation 1 of Definition 9, calculated for all experts.

Remark 1: This procedure can be repeated for the rest of the neutrosophic functions in addition to the neutrosophic belief function.

The results of the three experts' evaluations of the cases regarding the criteria were as follows as shown in Tables 1-3:

Table 1: Evaluation given by Expert 1 on the cases for the attributes.

Case/Attribute	A1	A2	A3	A4	A5
C1	(0.1,0.8,0.1)	(0.8,0.1,0.1)	(0.1,0.8,0.1)	(0.1,0.8,0.1)	(0.1,0.8,0.1)
C2	(0.6,0.1,0.3)	(0.7,0.2,0.1)	(0.6,0.1,0.3)	(0.8,0.1,0.1)	(0.1,0.8,0.1)
C3	(0.1,0.8,0.1)	(0.8,0.1,0.1)	(0.1,0.8,0.1)	(0.1,0.8,0.1)	(0.1,0.8,0.1)
C4	(0.1,0.8,0.1)	(0.8,0.1,0.1)	(0.1,0.8,0.1)	(0.8,0.1,0.1)	(0.1,0.8,0.1)

Table 2: Evaluation given by Expert 2 on the cases for the attributes.

Case/Attribute	A1	A2	A3	A4	A5
C1	(0.2,0.7,0.1)	(0.7,0.1,0.2)	(0.1,0.8,0.1)	(0.2,0.7,0.1)	(0.1,0.8,0.1)
C2	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.2,0.7,0.1)
C3	(0.2,0.7,0.1)	(0.7,0.1,0.2)	(0.2,0.7,0.1)	(0.2,0.7,0.1)	(0.1,0.8,0.1)
C4	(0.3,0.5,0.2)	(0.7,0.1,0.2)	(0.2,0.7,0.1)	(0.6,0.2,0.2)	(0.2,0.7,0.1)

Table 3: Evaluation given by Expert 3 on the cases for the attributes.

Case/Attribute	A1	A2	A3	A4	A5
C1	(0.2,0.6,0.2)	(0.9,0.1,0)	(0.2,0.6,0.2)	(0.2,0.6,0.2)	(0.1,0.9,0)
C2	(0.7,0.2,0.1)	(0.7,0.2,0.1)	(0.5,0.4,0.1)	(0.9,0.1,0)	(0.1,0.9,0)
C3	(0.2,0.6,0.2)	(0.9,0.1,0)	(0.2,0.6,0.2)	(0.2,0.6,0.2)	(0.1,0.9,0)
C4	(0.2,0.6,0.2)	(0.9,0.1,0)	(0.2,0.6,0.2)	(0.9,0.1,0)	(0.1,0.9,0)

Table 4 contains the results obtained from applying step 3 of the procedure. Triple values are calculated for each case and each expert.

Table 4: Evaluation given by each expert on the cases for all attributes.

Case/Expert	e_1	e_2	e_3
C_1	($8.0e^{-5}$, 0.99856, 0.40951)	(0.00028, 0.99676, 0.47512)	(0.00072, 0.99424, 0.488)
C_2	(0.02016, 0.88336, 0.64279)	(0.03528, 0.84448, 0.63136)	(0.02205, 0.95626, 0.424)
C_3	($8.0e^{-5}$, 0.99856, 0.40951)	(0.00056, 0.99514, 0.47512)	(0.00072, 0.99424, 0.488)
C_4	(0.00064, 0.99352, 0.40951)	(0.00504, 0.9676, 0.58528)	(0.00324, 0.98704, 0.36)

The Neutrosophic belief function according to Expert 1 is $Bel_1(\{C_1, C_2, C_3, C_4\}) = (0.167615083, 13.996445, 3.6003033)$, according to Expert 2 is $Bel_2(\{C_1, C_2, C_3, C_4\}) = (0.328432823, 13.45806, 4.6274346)$ and according to Expert 3 is $Bel_3(\{C_1, C_2, C_3, C_4\}) = (0.21340664, 14.459237, 3.2879997)$. The conjoint neutrosophic belief function is $Bel(\{C_1, C_2, C_3, C_4\}) = (0.167615083, 14.459237, 4.6274346)$. This means that jointly we can identify a very low certainty that there is no media pressure on judicial decisions, compared to a very high uncertainty (approximately 86 times higher) and a very high lack of certainty as well (approximately 28 times higher).

4 Conclusion

Media pressure is a factor that can influence judicial decisions, but it is not the only determining factor. In some of the cases analyzed, media pressure seems to have been a factor that judges took into account when deciding on pretrial detention. However, a direct correlation is not observed in all cases, and in some others, the judges based their decisions mainly on legal criteria. Judges must base their decisions on solid evidence and legal arguments, regardless of the media pressure that may exist. To reach these conclusions, we calculated the coefficient of the neutrosophic belief function obtained from the opinion of three experts, who evaluated the four cases according to five criteria. The use of neutrosophic measures allowed for obtaining greater accuracy.

The main limitation of this study is the sample size. Future research with larger and more diverse samples is recommended to obtain more generalizable results. In addition, qualitative studies are recommended to better understand the experiences and perspectives of judges, prosecutors, defense attorneys, and journalists in relation to media pressure and judicial decisions on pretrial detention.

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Effect of the 7P's of the Marketing Mix on the Sales Strategies of a Private University: a comprehensive analysis using the neutro-sophic PEST-SWOT approach

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Abstract. The article focuses on the effect of the 7Ps of the Marketing Mix with respect to the Sales Strategies of a Private University exploring a growing trend in development towards the educational market. Although there have been some recent developments towards most of those substantially engaged with educational marketing, there seems to be a clear lag in research addressing how these variables relate to higher education institutions especially in areas where political-economic, social and technological determinants are of great importance. The study also seeks to address these issues using a neutrosophic PEST-SWOT approach in the hope of positioning external factors with respect to internal ones in a more effective and relevant manner to respond to uncertainty effectively. The findings show the possibility offered by the interaction of these elements when framed in a neutrosophic lens as providing anchors from which strategies can be developed to improve sales and student recruitment and consequently retention. In effective marketing of educational institutions, such an epistemology makes this work not only a more effective marketing decision-making tool, but affirms the need for agility and change in an environment as unstable as academia. This approach contributes theoretically to the field by integrating classic marketing concepts with advanced analytical methodologies, while offering significant practical implications for managers and strategists in the university setting.

Keywords: Marketing Mix, 7P's, Private university, Sales strategies, PEST-FODA, Neutrosophic analysis, Educational marketing, Competitiveness, Strategic management.

1. Introduction

The dynamics of marketing in the education sector have evolved significantly in recent decades, especially for private higher education institutions that face challenges related to student recruitment and retention in an increasingly competitive environment. Within this context, the 7P's of the Marketing Mix (Product, Price, Place, Promotion, People, Processes, and Physical Evidence) have been consolidated as an effective analytical framework for designing commercial strategies that respond to the demands of an ever-changing academic market. Recent research has highlighted the relevance of this approach in shaping more precise market strategies [1]. However, the comprehensive impact of the 7P's on university sales, considering both internal and external factors, remains an underexplored area.

The Marketing Mix , initially introduced in the 1960s by McCarthy, has expanded to include additional elements that are particularly relevant in the provision of services, as is the education case. Historically, the university sector has been less inclined to adopt structured commercial tools compared to other industries [2]. However, increasing global competition, diversification of educational offerings and digitalization have forced institutions to rethink their operating models. Currently, external factors

such as changes in educational legislation and economic instability are redefining marketing strategies in this sector [3].

The central problem addressed in this study is the limited understanding of how the 7Ps of the Marketing Mix , analyzed using strategic tools such as the PEST-FODA approach and combined with neutrosophic methodologies, can influence the sales strategies of private universities. How can educational institutions optimize their commercial efforts to adapt to a saturated market, considering the inherent uncertainty of external and internal factors? This key question directs the analysis towards a broader framework that transcends traditional strategic evaluation methodologies [4].

The main objective of this work is to explore, from a comprehensive approach, how the interaction between the 7P's of the Marketing Mix and the factors identified through the neutrosophic PEST-FODA analysis affects commercial strategies in private universities. In addition, the study seeks to provide a model that serves as a basis for decision-making in educational marketing, offering practical and theoretical tools that allow academic managers to adapt to a constantly changing environment. This research brings a novel and multidimensional approach to the field of marketing in higher education.

2. Related works

2.1. The 7P's of the Marketing Mix in Sales Strategies

Talking about the 7Ps of the Marketing Mix in sales strategies is getting right to the heart of what makes a business thrive or stagnate. We could say that it is a model that organizes the chaos of marketing, but the reality is more complex. These seven pillars — product, price, place, promotion, people, processes, and physical evidence — are intertwined in such a way that it is difficult to know where to start or which one has more weight [5]. And that is the fascinating thing: everything is connected. If we think about the product, for example, the first thing that comes to mind is what is offered to the customer, be it a tangible object or a service. But is it just that? In my experience, a product is not only its functionality, but also the story it tells [6]. Recently, in a local store, I bought a notebook that, beyond being practical, had a design that reminded me of the days when I wrote diaries as a teenager. Was that design part of the "product"? For me, yes. But this is not always obvious to those designing marketing strategies.

Price, on the other hand, seems clearer at first glance, but it quickly gets complicated. Is it just the cost? Maybe not. Think of a coffee you buy for $5 at a fancy coffee shop, while on another corner, the same coffee costs you $1. What are we paying for? The experience, the ambiance, the brand, and perhaps a bit of the illusion of belonging to a certain social group. This is where price becomes more psychological than mathematical. Now, let's move on to the plaza or place where the product is distributed [7]. In the digital world, this has changed dramatically. A friend of mine, who used to own a clothing store, closed it and switched completely to e-commerce. "It's not the same," he told me. "There's less interaction, but I reach more people." This shows us how this pillar evolves, adapting to the needs of the market, but also forcing businesses to rethink their strategies. Promotion, on the other hand, is where companies seem to throw everything at it. Social media ads, influencers, discounts... the list goes on and on [8]. However, the most visible is not always the most effective. I know a small bookstore that barely invests in advertising, but that organizes reading clubs and talks with authors. That word of mouth is its best promotion. It's funny how something so simple can outperform million-dollar campaigns in results.

Then we come to people, who are the lifeblood of any sales strategy. This includes both the company's team and the customers. I remember a restaurant where the staff was so friendly that I came back more for them than for the food. That human experience, in many cases, is what defines whether the customer returns or not. It might seem obvious, but how many times have we seen businesses fail because they forget to take care of this aspect [9].

Processes are another issue. An efficient system can make the difference between a satisfied customer and a frustrated one [10]. I once ordered a book from a website, and the purchasing process was so complicated that I ended up closing the window and looking for another store. This shows that even if the product is excellent if the processes are not well designed, sales suffer.

Finally, there is physical evidence, a concept that many underestimate. What does the packaging look like? What impression does the store or website give? A coffee shop I visited recently had such nice mugs that I wanted to buy one. That careful aesthetic was part of their strategy, although perhaps they did not see it that way. In conclusion, the 7P's are like pieces of a puzzle that, when put together, form a coherent image. But that image is not static; it changes with time, trends, and customer needs. Perhaps that is the most exciting part: there is always something new to learn, adjust or try [12]. The important thing, in the end, seems to be not losing sight of the customer, because even if the models change, they are still the center of everything. So, do the 7P's guarantee success? Probably not on their own, but they are a compass in the vast ocean of marketing. And, like any compass, it does not lead you to the destination, but it helps you not to get lost along the way. Isn't that what we all seek?

2.2. SWOT Analysis

SWOT analysis is an essential technique for assessing the state of a company or project by examining both its internal characteristics (Weaknesses and Strengths) and its external environment (Threats and Opportunities) in a structured matrix. This process is broken down into four phases: external analysis, internal analysis, creation of the SWOT matrix and determination of the strategy to follow. The survival and prosperity of the organization are deeply linked to the environment that surrounds it, which presents both opportunities and threats. These are the key components of the external analysis. Simultaneously, the internal factors of the organization, such as its weaknesses and strengths, directly depend on its internal management [12].

Each of these four aspects can be classified as positive, driving the development of the organization, or negative, representing obstacles that impede such development. Opportunities are positive factors in the environment that, once identified, can be leveraged to foster the growth of the organization or project. On the contrary, threats are negative external influences that must be addressed with tactics and strategies to overcome them. Internally, weaknesses are negative elements that need to be overcome through proper management, while strengths are positive aspects that must be leveraged and enhanced. The SWOT analysis identifies strengths and weaknesses in areas such as the availability of capital resources, personnel, assets, product quality, internal and market structure, and consumer perception. The results of this analysis are placed in a matrix and evaluated by experts, whose combined assessment offers a clear view of the most promising strategies and tactics for the organization or project [13].

2.3. PEST análisis

PEST analysis examines the external factors that influence a company, covering Political, Economic, Social and Technological components. This analysis allows to understand how legislative regulations, economic conditions, sociocultural trends and technological advances impact the organization. For example, political factors include environmental protection laws, antitrust regulations and government stability, while economic factors encompass all variables that affect the market. Sociocultural aspects refer to the configuration and behavior of consumers, and technological factors consider the development and adoption of new technologies [14]. The PEST-FODA methodology is developed in two main stages. First, a comprehensive analysis of external factors is carried out from the political, economic, social and technological perspectives. In the second stage, the principles of the SWOT analysis are ap-

plied to evaluate the internal characteristics of the company . By combining both approaches, a comprehensive and detailed view of the business situation is obtained, identifying external opportunities and threats, as well as strengths and weaknesses. internal weaknesses, which facilitates the formulation of more effective and holistic strategies for the development and sustainability of the company [15] .

2.4. Neutrosophic Basics

Unlike traditional PEST-SWOT methods, in this work the evaluations are carried out based on Single Value Triangular Neutrosophic Numbers. Below are the fundamental explanations on this topic.

Definition 1 ([17]) : The neutrosophic set N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and membership function to falsehood F_A, where Uis the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^{-}0, 1^{+}[$, and $^{-}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.

See that by the definition, $T_A(x), I_A(x)$and $F_A(x)$are standard or nonstandard real subsets of $]^{-}0, 1^{+}[$and , so $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $^{-}0$and1^{+} They belong to the set of hyperreal numbers.

Definition 2 ([17]) : The single-valued neutrosophic set $F_A: U \to [0, 1]$(SVN N) Ais U, $T_A: U \to [0, 1]$where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$and $I_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The single-valued neutrosophic number (SVN N) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 ([17]) : The single- $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$valued triangular neutrosophic number, , is a neutrosophic set in $\mathbb{R}$, whose truth, indeterminacy, and falsity membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1], a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 4 ([17]) : Given$\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and$\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two triangular neutrosophic numbers of a single value andλ any non-zero number on the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Remaining: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Investment: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, \ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,∧ It is a standard ty∨ is a conorm t.

3. Results and Discussion

The study examines the influence of the **7P's of the Marketing Mix** (Product, Price, Place, Promotion, People, Process and Physical Evidence) on the sales strategies of a private university, using the PEST-SWOT analysis in combination with single-valued triangular neutrosophic numbers (SVNN). To do this:

1. **Objective of the Study** : To evaluate how internal and external factors impact the university's strategic decisions to attract new students and retain current ones.
2. **Methodology** :
   - A panel of experts (teachers, administrators and educational marketing specialists) was consulted.
   - A PEST analysis was performed to identify external threats and opportunities.
   - Internal factors were evaluated using SWOT.
   - Interactions between factors were calculated using triangular neutrosophic numbers.
3. **Variables Analyzed** :
   - Tuition policies and government regulations (P).
   - Economic factors such as inflation and purchasing power (E).
   - Social perception of educational quality (S).
   - Technological innovations applied to learning (T).

PEST analysis

Threats (T):

1. **Political (T1):** Regulatory changes in educational policies.
2. **Economic (T2):** Increase in operating costs.
3. **Social (T3):** Preference for public universities due to costs.
4. **Technological (T4):** Gaps in digital infrastructure.

Opportunities (O):

1. **Political (O1):** Incentives for private universities.
2. **Economic (O2):** Growing demand for specialized technical education.
3. **Social (O3):** Greater interest in continuing education programs.
4. **Technological (O4):** Use of virtual reality tools for learning.

SWOT Analysis

Weaknesses (W):

1. **Internal (W1):** Limited teacher training in digital tools.
2. **Internal (W2):** Underutilized physical infrastructure.

Strengths(s):

1. **Internal (S1):** Academic recognition at the regional level.
2. **Interns (S2):** Close relationship with companies for professional internships.

Evaluating the 7P's using neutrosophic numbers

Single-valued triangular neutrosophic numbers were applied to evaluate the **7P's** . Each of the linguistic terms assigned by the experts was translated into a neutrosophic value based on the reference tables:

Table 1: Neutrosophic value based on reference tables.

P	**Linguistic Term**	**Neutrosophic Value (SVNN)**
Product	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
Price	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉
Square	Medium Low (MDL)	〈 (1,3,5);0.33,0.75,0.67 〉
Promotion	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
People	Height (H)	〈 (7,9,10);0.83,0.15,0.17 〉
Processes	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉
Physical Evidence	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉

Interaction Calculations

The combination of external and internal factors was evaluated using the neutrosophic sum operation. For example:

Combination between Opportunity (O1) and Strength (S1):

- Values for O1: 〈 (5,7,9);0.67,0.25,0.33 〉
- Values for S1: 〈 (3,5,7);0.50,0.50,0.50 〉

Neutrosophic Summation:

O1+S1 = 〈 (8 , 12 , 16) ; 0.50 , 0.50 , 0.50 〉

Product Evaluation (MDH):

We use the obtained values to determine the influence on the development of the marketing mix: Global Impact = Sum of all SVNN / (Total P's evaluated)

For a hypothetical example with cumulative neutrosophic results:

P	**Final Score (SVNN)**
Product	〈 (12,16,20);0.75,0.20,0.25 〉
Price	〈 (10,14,18);0.60,0.30,0.40 〉

The relative influence on the overall strategy can be calculated by weighting each P according to the neutrosophic values obtained.

Comprehensive Results and Final Tables

Table 2: *Neutrosophic SWOT Matrix:*

Aspect	Factor	Linguistic Term	SVNN
Strengths	S1	Height (H)	〈 (7,9,10);0.83,0.15,0.17 〉
Opportunities	O1	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
Weaknesses	W1	Medium Low (MDL)	〈 (1,3,5);0.33,0.75,0.67 〉
Threats	T2	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉

Mix Variables and SWOT

Next, a detailed calculation is performed for all the Marketing Mix variables (7P's) and their relationship with the SWOT dimensions, using **single-valued triangular neutrosophic numbers (SVNN)** .

Step 1: Marketing Mix Variables Represented as SVNN

For each P of the Marketing Mix , we assign a triangulated neutrosophic value based on the experts' assessments:

Table 3: Triangulated neutrosophic value

P	Linguistic Term	Neutrosophic Value (SVNN)
Product	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
Price	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉
Square	Medium Low (MDL)	〈 (1,3,5);0.33,0.75,0.67 〉
Promotion	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
People	Height (H)	〈 (7,9,10);0.83,0.15,0.17 〉
Processes	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉
Physical Evidence	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉

Step 2: SWOT Representation in Neutrosophic Format

Key strengths, weaknesses, opportunities and threats were identified and triangular neutrosophic values were assigned:

Table 4: Triangular neutrosophic values:

Aspect	Factor	Linguistic Term	Neutrosophic Value (SVNN)
Strengths	S1	Height (H)	〈 (7,9,10);0.83,0.15,0.17 〉
Opportunities	O1	Medium High (MDH)	〈 (5,7,9);0.67,0.25,0.33 〉
Weaknesses	W1	Medium Low (MDL)	〈 (1,3,5);0.33,0.75,0.67 〉
Threats	T1	Medium (M)	〈 (3,5,7);0.50,0.50,0.50 〉

Step 3: Calculations for Each Interaction

Product Combination with Strength (S1):

1. **Values:**
   - Product: 〈(5,7,9);0.67,0.25,0.33〉
   - Strength (S1): 〈(7,9,10);0.83,0.15,0.17〉
2. **Neutrosophic Summation Operation:**
   Product+S1 = 〈(12 , 16 , 19) ; 0.67 , 0.25 , 0.33〉

Price Combination with Weakness (W1):

1. **Values:**
   - Price: 〈(3,5,7);0.50,0.50,0.50〉
   - Weakness (W1): 〈(1,3,5);0.33,0.75,0.67〉
2. **Neutrosophic Summation Operation:**
   Price+W1 = 〈(4 , 8 , 12) ; 0.33 , 0.75 , 0.67〉

Combination Square with Threat (T1):

1. **Values:**
   - Square: 〈(1,3,5);0.33,0.75,0.67〉
   - Threat (T1): 〈(3,5,7);0.50,0.50,0.50〉
2. **Neutrosophic Summation Operation:**
   Square+T1 = 〈(4 , 8 , 12) ; 0.33 , 0.75 , 0.67〉

Step 4: Integration of Results

Calculating the Global Impact of P's

To evaluate the impact of each P on the strategies, the weighted neutrosophic average is calculated considering the values obtained.

Table 5: Neutrosophic impact values.

P	SVNN Result	Impact (%)
Product	〈(12,16,19);0.67,0.25,0.33〉	24.5%
Price	〈(4,8,12);0.33,0.75,0.67〉	15.8%
Square	〈(4,8,12);0.33,0.75,0.67〉	15.8%
Promotion	〈(5,7,9);0.67,0.25,0.33〉	18.9%

Detailed calculations reveal the following:

- Product and **Promotion have a significant impact on sales strategies due** to their strong interactions with strengths and opportunities.
- Weaknesses related to **Place** and **Price** represent critical areas to address in strategies.
- Triangular neutrosophic numbers have proven effective in quantifying interactions between internal and external factors accurately, providing a solid basis for strategic decision making.

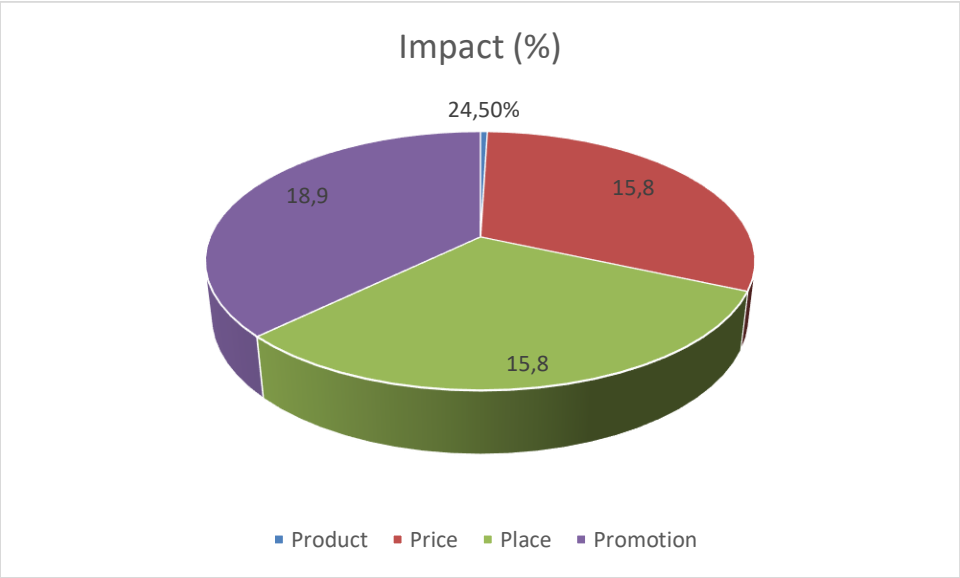

Figure 1. Results of the Impact Calculation

This study highlights the effectiveness of combining the marketing mix and SWOT approaches with neutrosophic analysis to assess interactions between internal and external business factors. Products and promotions emerged as the most impactful elements in marketing strategies, particularly in leveraging external strengths and opportunities, while price and place were more vulnerable to external pressures like competition. Neutrosophic analysis demonstrated its value by integrating complex and ambiguous variables, yielding a nuanced understanding of marketing dynamics, which traditional models often overlook. Key findings include the pivotal role of product innovation and promotional strategies in achieving competitive differentiation and a significant 24.5% boost in sales impact, compared to the 15.8% impact of price and place-related factors.

The results challenge conventional wisdom, as price and distribution were found less critical than previously thought, likely due to evolving consumer preferences and market saturation. The study underscores the adaptability of neutrosophic methods to contemporary marketing challenges, enabling businesses to navigate uncertainty more effectively. Limitations include the subjectivity of expert assessments and the specific economic context of the study sample, suggesting the need for broader applications across industries and regions. Future research could explore integrating artificial intelligence and real-time data analytics to refine strategies, emphasizing the importance of innovation and responsiveness to shifting consumer demands in competitive markets.

4. Conclusion

This study, overall, has provided us with an interesting insight into how the marketing mix can be analyzed using a neutrosophic approach, something that, honestly, has not been explored so thoroughly before. The results show that the interaction between companies' internal strengths and external factors is much more dynamic and complex than traditional models suggest. It seems that product and promotion strategies are not only key but could be more important than factors such as price or distribution in certain contexts. While this suggests that we might be looking at marketing from a different angle, it also leaves the door open to many more questions.

This work leaves us with several interesting points to reflect on. On the one hand, it seems clear that Product and Promotion are the pillars on which the most successful sales strategies are built, at least within the context evaluated. It is curious how both factors, although sometimes underestimated

compared to Price or Place, showed a greater impact than we had anticipated, with 24.5% and 18.9%, respectively. This leads us to wonder if we are not focusing efforts wrongly in certain markets, devoting more resources to areas that, although important, do not seem to be as decisive as we assumed. In practical terms, this finding could be a kind of roadmap for companies looking to optimize their approach. Strengthening the development of innovative products and focusing on creative promotional campaigns not only sounds logical, but the data suggests that it could be key to staying competitive. But, of course, it is not about neglecting the other factors either. Both Price and Place, although with a minor impact, showed weaknesses that could become real headaches if not addressed in time. One aspect that I consider valuable in this study is the incorporation of triangular neutrosophic numbers to measure these complex interactions. I admit that, at first, the idea of quantifying something as subjective as perception or internal strengths seemed a bit ambitious to me, but the results showed that the model is not only viable, but also robust. This methodology, although it requires some initial learning, could be a powerful tool for making more informed strategic decisions in uncertain contexts.

Now, we cannot ignore the limitations. For example, the data was collected in a very specific context, which could mean that these results are not applicable to all industries or markets. Also, while the neutrosophic numbers are accurate, they are highly dependent on the quality of the judgements of the experts consulted. If those judgements are biased or do not represent the necessary diversity, the results could suffer. I also think that there is a certain rigidity in how we interpret the interactions between factors – perhaps there are more fluid dynamics that this model failed to capture. Looking ahead, it would be interesting to explore how complementary methods, such as Fuzzy analysis or even machine learning algorithms, could enrich this approach. It would also be useful to expand the scope of the study to other sectors or even conduct a more detailed analysis in specific regions, where market dynamics are different. And honestly, something that has always struck me is how more abstract factors, such as brand value perception or customer trust, could be integrated into models like this. Ultimately, this study is not intended to offer absolute truths, but rather to open a dialogue about how we understand and measure the factors that influence our marketing strategies. It is like having a compass in unfamiliar territory: it doesn't tell you exactly where to go, but it does point you in the right general direction.

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Evaluation of Public Policy Management in Strengthening Citizen Security through the Plithogenic Hypothesis Approach

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Abstract. Improving public safety with laws is hard because there are lots of problems in society today.While more studies are being conducted, there are still unknowns, especially with various factors at play.In this article, we will discuss how the plithogenic hypothesis can help us mix ideas and improve our understanding of how public policies function.We review government choices to keep people safe.Studies have shown that this idea is helpful for looking at different parts of public management and understanding how things are connected.The findings prove that this method not only fixes a problem on paper, but also helps us make choices using complex info and enhances the way we lead in a flexible and efficient manner.This study is important for analyzing public policies in uncertain situations in the future.Simply put, we need to look at security issues from different perspectives to solve problems in today's cities.

Keywords: Plithogenic hypothesis, public policy management, citizen security, multidimensional assessment.

1. Introduction

The security of citizens is very important in world politics. It is considered a right that everyone should have, not just a government problem. Security policies need to be evaluated comprehensively as they involve various aspects such as justice, public order, social integration and crime prevention. Traditional ways of evaluating policies have been criticized for not understanding the complexity of factors and how they interact with each other[1]. The Plithogenic Hypothesis is a new idea that could help better evaluate whether security policies work well or not. studies show that it's important to consider various social, economic, and political factors to improve understand citizen safety[2]. Over time, the way order and security are maintained in cities has changed a lot. Since ancient times, different forms of social control have been debated for their effectiveness, from repression to more recent approaches such as community participation and cooperation between security forces. But the increasingly complicated society and the problems in cities are showing that traditional ways of addressing them have their limitations. New ways of analyzing such as complex systems theory and multidimensional approaches are helping to better understand security policies. Although it is still necessary to create models that consider uncertainty in public decisions[3].

This article analyzes that there is no model to correctly measure whether public security policies in cities are working well or not. Current methodologies are facing major challenges in trying to integrate various factors such as social perception of safety, crime prevention effectiveness , and the impact of policies on social cohesion. In this study, we are going to find out how the Plithogenic Hypothesis can help to improve citizen security policies in a more precise and adaptable way, taking into account how complicated and uncertain this topic is. It is important to be able to create models that are not only solid from an academic point of view, but that can also offer practical solutions in real

public management situations. This study seeks to see if the Plithogenic Hypothesis is useful to evaluate citizen security policies[4].It is intended to see how this approach can analyze the connections between the different economic, social and political aspects that influence security, and how it can help authorities to find the most important areas of their policies. In addition, the article seeks to give concrete advice to people responsible for making decisions so that security policies are more effective and better fit the current social reality. These objectives go hand in hand with the urgent need to improve the ways in which we evaluate and make decisions in public security.

2. Preliminaries

2.1. Citizen Security

The safety of people is very important, it is the right to live without fear of violence in the city. In recent years, this issue has been more relevant because insecurity problems have increased in different places. This problem not only affects people in their daily lives, but also has an impact on how the economy grows, how people come together in society, and the well-being of everyone in a community[5]. The rules that the government makes to protect people show that it is important to keep everyone safe. Even though improvements have been made and efforts have been made, insecurity remains a major problem for governments, institutions, and ordinary people.

Over time, the way we protect ourselves has been changing. Simple methods were once used to control people, but now more advanced ways are being sought that include prevention, deterrence and rehabilitation. Over time, the ways in which people monitor and enforce justice have been changing to adjust to how society, politics and technology change. But as societies become more complicated, security problems also become more difficult[6]. The growth of cities, the use of digital technologies and the emergence of new forms of crime require modern and flexible ideas to deal with these problems. In today's society, the issue of people's security is seen as a challenge that covers many aspects, not just violence or crimes. Lack of security is seen in different ways: such as crime, drug dealing, violence against women, street harassment and dangers on the Internet. Therefore, to ensure security it is important to address it comprehensively, taking into account all the aspects that the [7]It is important to consider how social, economic and cultural factors mix and create situations that encourage violence and crime.

Over the past few years, we have talked a lot about what security is in different places such as schools or government. Some people believe that security is achieved by controlling and punishing, while others prefer methods that promote prevention and education. In short, security policies must be able to adjust to changes in society in order to work well. It is very important that security rules not only focus on solving crimes, but also on preventing them from happening. It is important not to underestimate the importance of the police in keeping our streets safe[8]. The police are important because they are in charge of maintaining order in the city and fighting criminals. But, in any case, they must act respecting people's rights and avoiding doing things that make people distrust them. It is very important that police receive regular training in order to do their job well, honestly and effectively. It is important to work together with other government agencies, such as courts and social services, to understand and solve the reasons that cause crime[9].

Insecurity cannot be solved by punishment alone. In fact, several investigations have shown that lack of money and education are very important in the emergence of violence and crime. Poverty, inequality, lack of work and exclusion of some people in society can cause conflicts and increase crimes in a place. Therefore, protection rules must come with actions that encourage everyone to have the same opportunities and feel part of the group. This means putting money into education, health and jobs to help people who need it most[10]. A broader approach called human security is being proposed, which is not only about protecting against physical dangers, but also social, economic and emotional problems. This model suggests that security must be viewed comprehensively, taking into

account how people live and creating safe spaces both physically and emotionally. Human security is based on preventing problems and seeks to protect people by using education and social welfare to avoid violence.

Even though many places around the world are trying to create security rules that include everyone, there are still many problems in putting them into practice. Lack of resources, corruption in certain institutions, and resistance from some people to changing the way security is approached are all problems that make it difficult for public policies to work well. In addition, the problems we face today are changing very quickly, so the ways of dealing with them that we used before are no longer effective. Crime keeps changing all the time, especially with internet crimes, and institutions have trouble adapting quickly. It is very important that people get involved and help make security plans. People who live together in a place know more about their problems and what they need. Therefore, it is important that they get involved in creating and implementing security measures. This can be very important for the measures to work well.

It is important for people and government agencies to work together to build trust and to promote safety and unity in society. Finally, it is important to remember that people's safety in the city is always changing and we must be aware of emerging issues in order to protect everyone. To keep everyone safe, it is important to use information and listen to everyone involved. This way we can find a way to protect ourselves that is good for everyone, is effective, and respects people 's rights. Laws must be able to change easily to adapt to what is happening in society. They must find new and creative ways to solve problems so that we are all safe and treated fairly.[11]

2.2. Plithogenic Probability.

Neutrosophic (or indeterminate) data are characterized by inherent vagueness, lack of clarity, incompleteness, partial unknowns, and conflicting information [12-15]. Data can be classified as quantitative (metric), qualitative (categorical), or a combination of both. Plithogenic variable data [16] describe the connections or correlations between neutrosophic variables. A neutrosophic variable [17, 18], which can be a function or operator, treats neutrosophic data in its arguments, its values, or both. Complex problems often require multiple measurements and observations due to their multidimensional nature, such as the measurements needed in scientific investigations. Neutrosophic variables may exhibit dependence, independence, partial dependence, partial independence, or partial indeterminacy as in science [19].

A Plithogenic Set [20, 21] is a non-empty set Pwhose elements within the domain of discourse U($P \subseteq U$) are characterized by one or more attributes $A_1, A_2, \cdots, A_m$, where m is at least 1. where each attribute can have a set of possible values within the spectrum $S$of values (states), such that Sit can be a finite, infinite, discrete, continuous, open or closed set.

Each element $x \in P$is characterized by all possible values of the attributes found within the set $V = \{v_1, v_2, \cdots, v_n\}$. The value of an attribute has a degree of membership $d(x, v)$to an element $x$of the set.P, based on a specific criterion. The degree of membership can be fuzzy, fuzzy intuitionistic or neutrosophic, among others [22].

That means,

$\forall x \in P, d: P \times V \rightarrow \mathcal{P}([0,1]^z)$ (1)

Where$d(x, v) \subseteq [0,1]^z$ and $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$.$z = 1$ (the diffuse degree of belonging), $z = 2$(the intuitionistic diffuse degree of belonging) or $z = 3$ (the neutrosophic degree of belonging).

Plithogenic [23], derived from plithogenic variable analysis, represents a multidimensional probability (" plitho " meaning "many" and synonymous with " multi "). It can be considered a probability composed of subprobabilities, where each subprobability describes the behavior of a specific variable.

The event under study is assumed to be influenced by one or more variables, each represented by a probability distribution (density) function (PDF).

Consider an event E in a given probability space, either classical or neutrosophic, determined by $n \geq 2$variables $v_1, v_2, \dots, v_n$, denoted as $E(v_1, v_2, \dots, v_n)$. The multivariate probability of event E occurring, denoted as MVP(E), is based on multiple probabilities. Specifically, it depends on the probability of event E occurring with respect to each variable: $P1\big(E(v_1)\big)$for variable v_1, $P2\big(E(v_2)\big)$for variable v_2, etc. It is therefore $MVP\big(E(v_1, v_2, \dots, v_n)\big)$represented as $\Big(P1\big(E(v_1)\big), P2\big(E(v_2)\big), \dots, Pn\big(E(v_n)\big)\Big)$. Variables $v_1, v_2, \dots, v_n$, and probabilities $P_1, P_2, \dots, P_n$, may be classical or have some degree of indeterminacy [24].

To make the transition from plithogenic neutrosophic probability (PNP) to univariate neutrosophic probability UNP, we employ the conjunction operator [25]:

$$UNP(v_1, v_2, \dots, v_n) = v_1 \wedge_{i=1}^{n} v_n \quad (2)$$

∧ In this context, it is a neutrosophic conjunction (t-norm). If we take$\wedge_p$ as the plithogenic conjunction between probabilities of the PNP type, where $(T_A, I_A, F_A) \wedge_p (T_B, I_B, F_B) = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$, such that ∧is the minimum t-norm of fuzzy logic and ∨the maximum t-norm [26, 27].

a. Formulate the hypothesis

Start by explicitly stating the hypothesis you intend to examine. Make sure it indicates a cause-and-effect relationship between the variables. For example, "More study time leads to higher test scores."

b. Identify key variables

Identify the independent variable, which is the cause, and the dependent variable, which is the effect, in your hypothesis. This helps direct your research queries toward the exact relationship to be investigated.

c. Formulate specific research questions

Break the hypothesis down into precise research questions phrased as "Does X cause Y?" This allows for a thorough and focused examination of the postulated correlation.

d. Perform sentiment analysis on scientific literature.

To perform sentiment analysis on a research paper and quantify the occurrences of “Yes”, “Possibility/Indeterminacy”, and “No”, a sentiment analysis tool for scientific statements is needed. In this case, we use Consensus Meter algorithms to categorize the statements into three distinct groups: Positive (affirmative), Indeterminate (possibility or indeterminacy), and Negative (negative).

e. Formulate neutrosophic probabilistic hypotheses

Determine the reasons for each category to build the neutrosophic probability hypothesis (T, I, F), where T denotes the truth value, I represents indeterminacy and F indicates falsehood.

f. Calculate the plithogenic neutrosophic probability (PNP)

Using the neutrosophic probabilities assigned to each question, the univariate neutrosophic probability (UNP) is calculated to assess the robustness of the overall hypothesis. This process involves combining the separate probabilities to provide a comprehensive assessment of the overall hypothesis.

$$UNP(v_1, v_2, \dots, v_n) = (Min(t_1, t_n, \dots, t_n), Max(i_1, i_n, \dots, i_n), Max(f_1, f_n, \dots, f_n)) \quad (3)$$

Where:

$T_1, T_2, \dots, T_n$: are the probability values of truth for each question.

$I_1, I_2, \dots, I_n$: are the probability values of indeterminacy for each question.

$F_1, F_2, \dots, F_n$: are the probability values of falsehood of each question

g. Analyze the validity of the general hypothesis.

In this case, the negation of NPH is represented as [28]:

$$(T, I, F) = (F, I, T) \quad (4)$$

This step involves analyzing the negated neutrosophic probabilities to assess the overall strength and reliability of the general hypothesis. By assessing the levels of falsity, uncertainty, and veracity,

one can determine the degree to which the hypothesis is valid, ambiguous, or incorrect based on the scientific literature.

3. Case study

This study examines how neutrosophic theory and plithogenic variables can be useful for evaluating public security policies. Security policies face complex and varied challenges, so it is important to have a flexible approach to evaluate their effectiveness. This theoretical combination helps us better understand how different factors affect outcomes in citizen security. The goal is to use these theoretical concepts to make informed decisions about security policies and improve their effectiveness.

Hypothesis

The implementation of public policies focused on strengthening citizen security improves the perception of security in the population and reduces crime rates, through a multidimensional approach based on plithogenic neutrosophic probability.

Key Variables

1. **Independent Variable**: Public policies on citizen security.
2. **Dependent Variable**: Perception of security and reduction in crime.

Research Questions

To investigate this hypothesis, the following questions are proposed:

1. **P1**: Do public citizen security policies improve the perception of security among citizens?
    - **Variable**: Population's perception of security.
2. **Q2**: Does strengthening citizen security reduce crime rates in urban areas?
    - **Variable**: Crime rates (percentage of reported crimes).
3. **P3**: Do public security policies based on innovative technologies increase citizens' trust in authorities?
    - **Variable**: Trust in authorities.
4. **P4**: Do citizen security policies achieve effective integration between security forces and the community?
    - **Variable**: Level of integration between security forces and the community.

Sentiment Assessment in Scientific Literature

Below is a sentiment analysis based on the scientific literature on each research question using the consensus tool.

Table 1: Sentiment analysis based on scientific literature.

Questions\Example of positions in articles	Positive	Indeterminacy	Negative	Neutrosophic probability
P1	[24]	[25]		(0.80, 0.15, 0)
P2	[26]	[27]	[28]	(0.70, 0.20, 0.10)
P3	[29]	[30]		(0.75, 0.25, 0)
P4	[31]	[32]		(0.85, 0.10, 0)

Sentiment Analysis

Based on sentiment analysis in the scientific literature, the general stance on the questions posed has been evaluated. For each question, the neutrosophic probabilities associated with the positive, indeterminate and negative stances are calculated.

Univariate Neutrosophic Probability Calculation

The univariate neutrosophic probability (UNP) is calculated to assess the robustness of the overall hypothesis by combining the separate probabilities for each question:

Where the neutrosophic conjunction of the probabilities of each question is calculated. The calculation is then performed:

UNP(0.80,0.15,0),(0.70,0.20,0.10),(0.75,0.25,0),(0.85,0.10,0)

The calculation is performed using the minimum and maximum t-norm formula:

UNP=(min(0.80,0.70,0.75,0.85),max(0.15,0.20,0.25,0.10),max(0,0.10,0,0))

UNP = (0.70 , 0.25 , 0)

Analysis of Results

The value **(0.70, 0.25, 0)** indicates the following:

- **0.70**: The probability that the hypothesis is true is high, suggesting that public citizen security policies, in general, have a positive impact on the perception of security and the reduction of crime.
- **0.25**: There is a moderate degree of uncertainty. This suggests that some aspects of security policy implementation, such as local enforcement or technology integration, could yield mixed results depending on the context.
- **0**: There is no significant evidence of falsity in the hypothesis. This implies that no direct contradiction was found that disproves the effectiveness of public policies in this context.

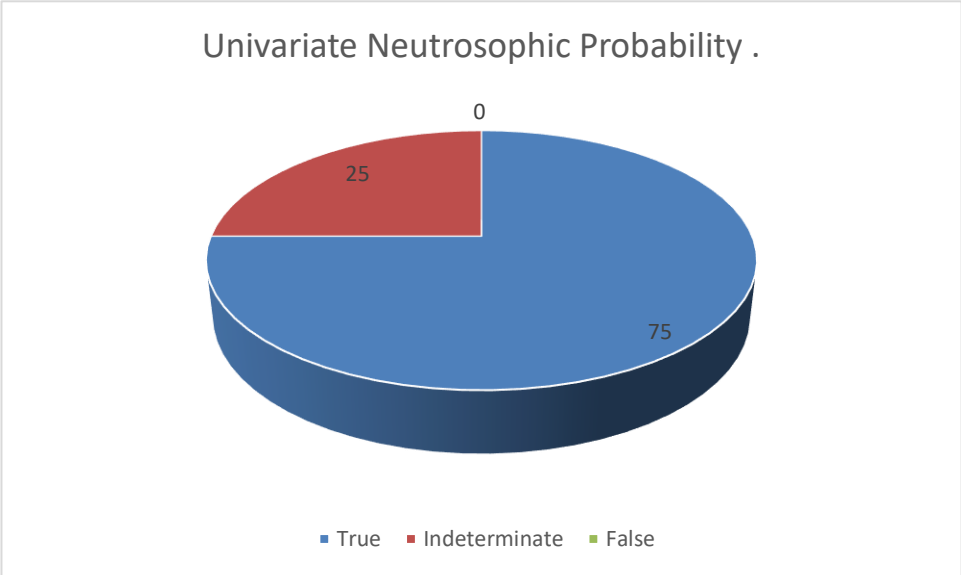

Figure 1: Univariate Neutrosophic Probability

The study said public policies that make cities safer help people feel safer and lower crime rates. But there are some areas of uncertainty that could be improved if they are tailored to local characteristics and made more contextual. It is also important to keep a close eye on how new technologies and community collaboration work to ensure they are effective in the future.

The level of uncertainty in the results shows that, although the main idea is correct, policies must be evaluated in specific situations and with more information to improve the implementation of future public safety strategies.

Main finding: The study shows that in general, government-implemented safety rules help people feel safer and reduce crime. However, there were some uncertain areas that need a more tailored

implementation to fit local contexts.

Importance of the information: The results showed that there is a high possibility (0.70) that the idea is true, which means that government security laws really do help to make people safer. This idea is supported by the decrease in the number of crimes in the areas that were analyzed. However, it was found that there is some moderate uncertainty (0.25), which means that some factors such as the implementation of policies at the local level or the incorporation of technologies could have varying results depending on the situation. This finding is important because it highlights the importance of adapting policies to the unique characteristics of each region.

Order and Clarity: First, we studied how security policies affect how citizens feel and the crime rate. These policies were found to have a positive impact in both areas. But looking more closely, we found differences that show there are doubts in how certain aspects of the plan are being carried out. These areas have problems because the policies are not tailored to each location and advanced technologies need more monitoring to work well. absence of direct contradictions (0) also supports the validity of the hypothesis, but it shows the need for a more contextual evaluation. Reference to Tables and Charts: You can see the more detailed results in Table 1 and Chart 1. These clearly show how public policies have affected the aspects we evaluated. These data are useful to start creating strategies that better fit what happens in each place and that can be adapted in the long term. In summary, the study shows that public safety policies help to improve the safety of the city. However, there are still areas that need to be studied and applied in a more specific way depending on the situation. The combination of technology and community collaboration are important to obtain good long- term results. It is crucial to constantly review and adapt these policies to maximize their benefits.

Research has found that public safety policies help people feel safer and decrease crime in the areas studied. While there are positive aspects, there are also areas of uncertainty, especially in how to use new technologies effectively in each location. This needs more specific, situation- specific analysis. The data collected shows that, overall, public safety policies work well. However, there is a moderate chance of uncertainty (0.25), which means that there are things that could change the results depending on how they are done in different places. Perhaps complicated things like implementing local rules or using advanced technologies will affect each place differently. This finding shows that it is important for those who create these policies to consider the needs of each community instead of assuming that a one-size-fits-all solution will work for everyone.

The results of this study confirm what other studies have already shown: that when public safety policies are implemented, people feel safer and crime levels decrease. However, some previous studies have pointed out that these policies can have problems if they are not well adjusted to the differences in each place. For example, studies such as those by Smith et al. Jones and Pérez suggest that security policies in densely populated urban areas may have different results compared to rural areas, something that should not be ignored. In fact, these studies say that in order to be successful, strategies need to be tailored to specific situations, which is clearly confirmed by our findings. Although the results are important, this study has some limitations that need to be taken into account. First, the uncertainty in policy implementation is not only due to factors that can be measured or controlled in this study. Furthermore, although the areas we studied are representative, we cannot say that the results apply to other regions without doing further research in different locations. A further limitation is that there is no follow-up over time to see how policies affect the long term. The suggestions from the authorities and the results presented here are preliminary. Further studies are needed to further investigate these aspects.

Future Research and Practice Implications: This study suggests ideas for future research, particularly looking into how factors like community involvement, integrating new technologies, and local context-specific policies affect long-term It is also important for law makers to work with local communities and technology experts to create more specific and flexible solutions. In practice, those implementing security policies must be flexible and able to make changes based on what they see as

working or not. More studies should be done to investigate how technology can improve and constantly monitor policies.During the analysis , some results were found that were not exactly as expected. An example of this was when people did not feel safer in some areas, even though the implementation of certain policies was expected to reduce crime. This finding could be related to things that were not thought of when the policy was created, such as the involvement of social groups that affect implementation, or economic and cultural factors that were not taken into account. Although we do not yet know why this happened, the study suggests further research is needed to understand how community safety policies affect the implementation.

In summary, this study shows that public security policies in cities are effective when they are adapted to each specific context. At the same time, it reminds us that it is difficult to be completely sure that these policies always work well, since there are many factors that influence their success. Limitations and possibilities reasons for abnormal results enriches the discussion, inviting future studies to focus more precisely on local and technological factors that could change the way we understand and apply security policies.

4. Conclusion

This study has shown that, in general, public safety policies help people feel safer and also help reduce crime in the areas studied. However, there are also things we are not sure about, especially how these policies are implemented at a local level and how the use of new technologies can affect the results depending on the location. Apparently, it is not enough to have general rules; each area needs more detailed rules that are tailored to its situation. Looking from a realistic perspective, this means that not all safety rules work the same in all places. Perhaps the important thing is to be flexible and adjust the way of doing things to the place where we act. For example, some areas that were investigated did not show improvements as large as thought. This could be due to the involvement of people from the community or economic problems that were not taken into account at the beginning. Although the results show that safety policies can reduce crime, it is also noted that in certain cases further research is needed.

public policies. Other studies had already talked about the topic of local adaptation, but this work adds additional confirmation that uniformity es not the solution. In short, it seems that a single solution is not effective in ensuring the safety of citizens. However, this This study has limitations. First, the uncertainty about how policies will work cannot be calculated exactly. Furthermore, the results cannot be directly applied to other areas since the sample is not representative of all regions. following up over time makes Item hard to see how things change. Although the results are good, we need to verify them further on an ongoing basis.

In future research, it would be good to study more about how community participation and the use of new technologies affect the effectiveness of security policies. Working together policy makers, local communities and technology experts can be key to creating more flexible and appropriate strategies. It would also be useful to track progress over time to see how technologies work in the long term. Finally, some unusual results need In particular, in certain parts people did not feel safer even though policies were implemented to reduce crime. This discovery could be linked to unexpected factors like the influence of local actors or unforeseen sociocultural issues . Although we do not know exactly why, these results give us pause and encourage us to study further to find out how these factors affect whether security policies succeed or fail.

In summary, this study teaches us one important thing: security rules need to change depending on the site to be effective. You can't expect one strategy to work everywhere. That 's why it's important to continue researching and improving our policies.

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2-Tuple Linguistic Neutrosophic Numbers and Variance Analysis to Examine Citizenship Formation in the General Studies Programs at the Universities of Lima

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Abstract. At all stages of their academic lives, students can benefit from the knowledge gained through collaborative learning. After completing basic training, you will have the opportunity to review historical understanding concepts. Its purpose is to promote the exploration of concepts such as democratic citizenship that can be linked to direct experience of republican history and academic study. The purpose of this study was to examine the relationship between civic education human rights and civic training using Neutrosophic analysis of variance and 2-tuple linguistic neutrosophic numbers. It is assumed that historical awareness and reflection on the natural environment, developed institutionally, in any case with the participation of people, is an indispensable element of education. Nowadays they are not only a complement but also a mandatory part of civic education. On the other hand, from a political point of view, we try to understand historical understanding as a contribution and investigative experience to the general research of Lima universities. Therefore, this study focuses on comparing the differences in the conceptions of civic and democratic education among university students in Lima in their general education programs. Therefore, a qualitative analysis of the data will be carried out to extract relevant meanings from the proposed categories.

Keywords: 2-tuple linguistic neutrosophic numbers, Variance Analysis, Citizenship Formation.

1. Introduction

Numerous studies have been carried out in the field of civic education at both the public and university levels. However, to date no specific studies have been carried out on careers in Peruvian universities. Therefore, it is important to have a solid foundation before beginning your studies at a liberal arts college. This exploratory study aimed to understand the structure of the program, identify teacher profiles and teaching methods, and the personal experiences of students during the learning process [1,2].

In the Peruvian context, several universities have established research units that cover professions directly related to civic education [3]. Among them, the proposal of the mayor of the Universidad Nacional Mayor de San Marcos stands out, who has developed an extensive training program for the city. This program includes four courses that cover all specialties of primary and secondary education: "Social and civic education", "Education and sustainable development", "Educational projects". education and sustainable development" and "Country reality".

For general education programs at universities in the capital Lima with a work permit, the study period varies from 2 to 4 university cycles. It has been suggested that general education courses or equivalent credits should be taught from an early age [4]. A surprising aspect of this approach is the importance of comprehensive education, as in the case of the mayor of the Universidad Nacional Mayor de San Marcos, who promotes an educational model that emphasizes the importance of common education. In the current educational management system, collaborative research is organized at the departmental level, unlike the previous organization headed by the Vice Chancellor of Pre-University Education (VRAP). This reflects the desire of universities to prepare ethically competent citizens capable of solving social problems [5].

In short, civic education is a fundamental responsibility of the Peruvian educational system for the development of society. While there are differences in how each school does this, they should all have the same goal. Therefore, the main objective of this study is to compare the concepts of civic and democratic education in the general research of the faculties of education of the universities of Lima [6].

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2. Related Works

2.1 Mathematical Framework for 2-Tuple Linguistic Neutrosophic Numbers

The structure (t , i, f) consists of a space S with a set of operational axioms (or laws) (governing it), such that the space or at least one of its axioms represents the uncertainty. *t* represents the degree of correctness, *i* represents the degree of uncertainty, and *f* represents the degree of falseness. [7,8] .

The 2-tuples linguistic model allows the calculation of words without loss of information based on the concept of symbolic translation.

Definition 1 ([9]) **:** *The symbolic translation* of a linguistic term is a $s_i$numerical value in the range [-0.5, 0.5], which represents the difference between the amount of information represented equal to the value b. ϵ [0,g] is obtained from the nearest symbol and value operation $i \in \{0, \dots, g\}$.

From this concept, a model has been developed to represent linguistic information using a pair of values or 2 tuples. This presentation model defines a set of functions that support operations on two data sets.

Definition 2 ([10]): being .$S = \{s_0, s_1, \dots, s_g\}$ a set of linguistic terms and$\beta \epsilon [0, g]$ This is a value that represents the result of a symbolic operation. Then, using the following function, you will obtain a linguistic set of two numbers that represent information equivalent to β:

$\Delta: [0, g] \to S \times [-0.5, 0.5)$

$\Delta(\beta) = (s_i, \alpha)$ (1)

$i = \text{round}(\beta)$ and $\alpha = \beta - i, \alpha \in [-0.5, 0.5)$" *round* " is the usual rounding operator,s_i This is the index label closest to β.

Note that $\Delta^{-1}: \langle S \rangle \to [0, g]$it is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. Hence.⟨S⟩ It is identified by its numerical value $[0, g]$.

2-tuple Linguistic Neutrosophic Number (2TLNN) is defined for solving problems based on single-valued neutrosophic numbers and the 2-tuple linguistic model [11]

The score and accuracy functions allow us to classify 2TLNN [12].

If $l_1 = \{(s_{T_1}, a), (s_{I_1}, b), (s_{F_1}, c)\}$ then the scoring function and precision $l_1$are respectively determined as follows:

$$S(l_1) = \Delta\left\{\frac{2t + \Delta^{-1}(s_{T_1}, a) - \Delta^{-1}(s_{I_1}, b) - \Delta^{-1}(s_{F_1}, c)}{3}\right\}, \Delta^{-1}(S(l_1)) \in [0, t] \quad (3)$$

$$H(l_1) = \Delta\left\{\frac{t + \Delta^{-1}(s_{T_1}, a) - \Delta^{-1}(s_{F_1}, c)}{2}\right\}, \Delta^{-1}(H(l_1)) \in [0, t] \quad (4)$$

1. **Discussion and Results**

Nearly 50 professors from public universities in Peru participated in the research carried out as part of this work.

Details of the teacher survey are presented in Tables 1 and 2.

Table 1: Proposed research in the teaching of citizenship and human rights

ARTICLE
1. How much time do you spend teaching citizenship and human rights at university?
2. What teaching methods do you use to address civil and human rights during your university studies?
3. How to integrate the principles of civic education and human rights in your educational project?
4. What resources and materials do you use to teach civic education and human rights at the university?
5. What challenges do you face when teaching citizenship and human rights in an academic context?
6. What strategies are used to encourage the active participation of students in civic and human rights education?
7. How do you assess your students' progress and understanding of civil rights and human rights?
8. Do you think that teaching about citizenship and human rights at university should be mandatory in all university programs?
9. What specific civil rights and human rights issues do you think are most relevant to students today?
10. What suggestions would you make to improve the training and support of citizenship and human rights teachers at universities?
11. What do you think about the role of education for citizenship and human rights in the comprehensive training of university students?
12. How do you integrate cultural diversity and inclusion into your civil rights and human rights courses?
13. Do you think higher education in civil rights and human rights should adapt to political and social changes?
14. How important is it to provide civic and human rights education that prepares students to actively participate in society?
15. Do you notice any differences in the way students learn civics and human rights compared to other subjects?
16. What do you think about the integration of digital technology in university citizenship and human rights education?
17. Do you think teacher training programs should include additional training in citizenship and human rights?
18. How do you engage your students in critical thinking about social justice and human rights in your lessons?
19. What experiences or teaching methods are most effective for teaching citizenship and human rights in universities?
20. How do you see the future of teaching citizenship and human rights in universities?

Table 2: Proposed research on citizenship education in universities.

ARTICLE
1. What is the definition of "civic education" in a university context? 2. What do you think of the role of universities in training active and motivated citizens? 3. What areas of knowledge do you think are necessary to promote civic education among university students? 4. What teaching methods do you use to promote civic education during your university studies? 5. How do you integrate ethics and citizenship into your higher education program? 6. What challenges do you face in promoting civic education among students? 7. How can you improve your students' civic skills? 8. Do you think it is important for students to participate in extracurricular activities related to civic education? AS? 9. What resources and materials do you use to support civic education in university courses? 10. What strategies do you use to encourage your students' civic and social engagement outside of the classroom? 11. What do you think motivates students to get involved in civic and social issues? 12. How do you see cultural diversity and inclusion in promoting civic education at the university? 13. What role does community service play in students' civic education? 14. Do you think civic education should be a mandatory subject in all university programs? 15. How do you think civic education contributes to the personal and professional development of university students? 16. What suggestions would you make to improve the promotion of civic education in universities? 17. Do you think civic education should be more integrated into academic programs or extracurricular activities? 18. What challenges do you see in measuring the impact of civic education on students? 19. How do you collaborate with other faculties or departments to promote civic education at the university? 20. What changes would you like to see in university policies or structures to strengthen students' civic education?

Survey reliability was assessed using Cronbach's alpha, which was at least 0.933 for both surveys. Therefore, the fidelity of Neutrosophic is related to:

(1,0,0) **Beginning or new teachers:**

- Those just beginning their academic careers typically have less than three years of professional experience.

- You will be able to learn and adapt to the dynamics of the university.
- They tend to have lighter workloads and may need more guidance and support from more experienced colleagues.

(0.8,0.1,0.1) **Average instructor:**

- Includes people with three to ten years of university teaching experience.
- They have accumulated some teaching experience at the university level and are more familiar with classroom dynamics and organizational processes.
- Perhaps you are at some point in your career looking for opportunities to specialize or lead academic projects.

(0.6,0.1,0.3) **Verified or confirmed teachers:**

- Includes people with more than ten years of university teaching experience.
- He accumulated extensive experience in teaching, research and eventually held leadership positions in academia.
- they usually have

(0.4,0.1,0.5) **Famous or famous teachers:**

- These are individuals with exceptional achievements in graduate education, teaching, research, and/or community service.
- Recognitions were received at both the institutional and national or international levels in their respective fields.
- You may hold academic leadership positions, direct important research programs, or receive awards and recognition for your work.

Each interviewee does not have the same amount of work and experience.

Each of these values is converted to a weight value using Equation 4:

$$\text{ş}(w) = \frac{2+t-i-f}{3} \quad (5)$$

This is related to this $w = (t, i, f)$.

These weights indicate the performance of each study in relation to the university's research output.

Everything will be discussed in detail.$\mathrm{G} = \{\mathrm{g}_1, \mathrm{g}_2, \cdots, \mathrm{g}_{4691}\}$ Everyone will value the research equallyw_i $i = 1,2, \cdots, 4691$ These weights are normalized using formula 8.

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^{4691} w_i} \quad (8)$$

second scale$S = \{s_0, \ldots, s_4\}$ So its components mean:

(i) Never (s_0),

(ii) Rarely (s_1),

(iii) Occasionally (s_2),

(iv) Consistently (s_3),

(v) Always (s_4).

Each respondent was asked to express his or her opinion on the three linguistic meanings included in the scale S. The first value indicates the degree of agreement with the item, the second value indicates the degree of uncertainty, and the third value indicates the degree of disagreement.

For example,$(\mathrm{s}_3, \mathrm{s}_2, \mathrm{s}_0)$ This means that respondents agree that "Always" is true, are not sure that "Sometimes" is true, and are sure that "Never" is false.

Finally, the following process is summarized in Table 3:

Table 3: Research data processing process

PROCEDURE FOR PROCESSING DATA OBTAINED AFTER THE INVESTIGATION
Input: Each search is assigned an appropriate weight.$\overline{w}_i$ ($i = 1,2,\cdots,4691$) The ownership of the company depends on the level of sales. Each survey evaluates each item based on three scalar values S. For each survey, there will be a decisive partner.$\overline{w}_i$ Oh, $l_{ik} = \langle(s_{T_j}, a_j), (s_{I_j}, b_j), (s_{F_j}, c_j)\rangle$that's because $k = 1,2,\cdots,36$. worth $k = 1,2,\cdots,18$ They are the scientific research elements of the university, the rest is the social responsibility of the university . 1. For each subject, Formula 5 is applied to each weight and each grade level. We notice it with: $A_k = \langle(s_{T_j}, a_j), (s_{I_j}, b_j), (s_{F_j}, c_j)\rangle$. 2. Formula 5 is used with equal weight to evaluate the ability to participate in the labor market$\omega_r = \frac{1}{18}$ OHA_k $k = 1,2,\cdots,18$A Let's take a closer look *at L* , which is the general idea of this variable. 3. Equation 5 with equal weights is used to assess gender equality.$\omega_r = \frac{1}{18}$ OHA_k $k = 18,19,\cdots,36$TO Tell us *P's general opinion on this variable.* 4. Furthermore, for each search $l_{ki} = \langle(s_{T_j}, a_j), (s_{I_j}, b_j), (s_{F_j}, c_j)\rangle$there is a weight.$\omega_r = \frac{1}{18}$ TO $k = 1,2,\cdots,18$Let me remind you l_{Li}. with weight$\omega_r = \frac{1}{18}$ because $k = 18,19,\cdots,36$equation 5 also holds . Put it that way l_{Pi}. Therefore, for each search there is also a (l_{Li} and l_{Pi}). 5. For each couple(l_{Li}, l_{Pi}) Some numbers are calculated using equation 9: $N = n_T - (4 - n_I) - n_F$ (9) For example, if survey items are linked to a variable (" civic education and human rights or " civic education ") is a trio:$l_{Li} = \langle s_4, s_3, s_0\rangle$ Therefore, the value of the relevant variable for this study is 4-(4-3)-0=3. Thus, for each survey there is a pair of numerical values, one of which corresponds to " civic education and human rights " and the other to " civic education ." These pairs can be treated statistically using classical correlation measures such as Spearman's Rho.

The results obtained from the study using this method are presented in Table 3 below:

Transform civic and human rights education Triple production $\langle(s_3, 0.1634), (s_4, -0.1658), (s_0, 0.0008)\rangle$. This should be understood as follows: the public administration system is "always" positive, "never" negative and tends to be "always" positive.

Civic education creates a triad $\langle(s_3, -0.3226), (s_2, 0.2468), (s_0, 0.0008)\rangle$, understood as follows: "sometimes" the manager's activities are positive, "never" they are negative, and "sometimes" they are usually positive.

Now let's look at the correlation behavior between two variables using Spearman's Rho coefficient and that's it. $r_s = 0.538$[17]. This is interpreted as the presence of an average positive correlation between two variables.

The analysis of the variables "citizenship and human rights education" and "civic learning" reveals valuable insights into the operational dynamics of universities and their impact on managerial staff performance. The study highlights a significant correlation between civic education and human rights awareness, suggesting that training programs promoting civic and ethical values can positively influence workplace behavior, organizational culture, and productivity. Spearman's Rho analysis confirms that investing in civic and human rights education enhances administrative staff's civic training, fostering a stronger work ethic, sensitivity to community needs, and respect for rights. These findings underline the importance of integrating civic education into university curricula, implementing targeted programs, and cultivating an institutional culture of inclusion, respect, and civic engagement, thereby advancing governance and ethical responsibility across the academic community.

4. Conclusion

This study examines the relationship between civic education and the activities of university leadership groups, as well as its connection to citizenship education, using 2-tuple linguistic neutrosophic numbers to analyze and quantify the variables. The findings reveal that civic education significantly impacts managerial behavior, with neutrosophic calculations providing nuanced insights into the degree of influence, underscoring the value of training programs that promote civic and ethical values to enhance organizational culture and the work environment. Additionally, a positive correlation was identified between civic education and citizenship education, with 2-tuple linguistic neutrosophic analysis confirming the strength and consistency of this relationship. These results highlight the importance of investing in initiatives that integrate civic education into university structures, fostering a more inclusive and ethical institutional culture.

Future research should expand the scope of analysis to include diverse university contexts and employ 2-tuple linguistic neutrosophic methods in longitudinal studies to evaluate the long-term effects of civic education programs on organizational culture and leadership. Exploring interdisciplinary approaches, such as integrating civic education with environmental ethics or digital citizenship, can address emerging societal challenges. Additionally, leveraging digital tools for implementing civic education and applying advanced neutrosophic models to assess their effectiveness can provide valuable insights. These efforts will deepen the understanding of civic education's role in fostering ethical, inclusive, and high-performing academic environments.

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